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Pattern variation at different Prandtl Number in Natural convective MHD flow past a low -heat -resistance sheet with internal heating: A Numerical Study

[1]Ashok Kumar., [2]S. R.Ansari., P.Alam[3]
[1,3]*Research Scholar, Department of Mathematics, HNB Garhwal University Srinagar (INDIA)*
[2]*Professor, Department of Mathematics, HNB Garhwal University Srinagar (INDIA)*

Abstract: In this paper an attempt has been taken to understand the variation of prandtl number for MHD natural convective fluid flow past a low heat resistance sheet with internal heating. The boundary layer flow in viscous media is presented in terms of physical model which is transformed in the set of Coupled Ordinary differential equation using similarity transformation. The set of differential equation solved numerically. The main emphasis is given on the Varity of prandltl number with quantitative change of internal heating. The velocity, temperature and stream function is plotted w.r.t to different physical parameter.

I. INTRODUCTION

The stretching sheet problem in thermo-fluid dynamics remains a topic of significant interest due to ever-increasing applications in metal spinning, hot rolling, polymer processing, paper production, extrusion of plastic sheets, glass blowing, continuous casting of metals, materials manufacturing etc. In all these areas, the quality of the final product depends strongly upon the rate of heat transfer at the stretching surface. Numerous theoretical and computational studies of stretching sheet flow phenomena have appeared in literature [1,2, 3]. There are large number of practical situations in which convection is driven by internal heat source in the porous media. The wide applications of such convections occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid [4], who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Bejan [5] studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil [6] studied the effect of non-uniform boundaries temperatures on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. In a variety of applications in metallurgy, surface coating operations, crystal synthesis etc, the fluid may be electrically-conducting and under the influence of a transverse magnetic field which interacts with the flow and temperature fields. Such external magnetic fields have been successful in controlling transport mechanisms in similar regimes and generally decelerate flow and simultaneously reducing heat transfer rates i.e. suppressing thermal convection. Hydromagnetic stretching flows have been studied theoretically by Chakrabarti and Gupta [7] and also kumari et al [8] for the linear case.

Mukhopadhyay[9] performed an analysis to investigate the effects of the thermal radiation on the unsteady mixed convection flow and heat transfer over a porous stretching surface in a porous medium. Numerical simulations of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum were conducted by Nithyadevi and Yang [10].

Yih[11] numerically analyzed the effect of the transpiration velocity on the heat and mass transfer characteristics of the mixed convection about a permeable vertical plate embedded in a saturated porous medium under the coupled effects of thermal and mass diffusion. Elbashbeshy[12] studied the effect of the surface mass flux on the mixed convection along a vertical plate embedded in a porous medium.

Gaikwad et al.[13] investigated the onset of the double diffusive convection in a two-component couple of the stress fluid layer with the Soret and Dufour effects using both linear and nonlinear stability analyses. Ambethkar[14] studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction. Alam et al.[15] studied the Dufour and Soret effects on a steady MHD combined free-forced convective and mass transfer flow past a semi-infinite vertical plate. Alam and Rahman[16] investigated the Dufour and Soret effects on the mixed convection flow past a vertical porous flat plate with variable suction. Postelnicu[17] discussed influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering the Soret and Dufour effects. In the present paper, we consider the magneto hydrodynamic laminar boundary layer heat transfer of a low heat restance sheet embedded in viscous regime solve using finite difference methods. Such study has not appeared so far in the technical literature and constitutes a more realistic attempt to simulate the complex flows encountered in industrial engineering systems.presented graphically

II. MATHEMATICAL MODEL

Let us consider the problem of cooling of a low-heat-resistance sheet that moves downwards in a viscous fluid when the velocity of the fluid far away from the plate is equal to zero. The variation of surface temperature is linear. The flow configuration and coordinate system is shown in Fig.1. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of linear momentum. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. Electric field is assumed to exists and both viscous and magnetic dissipation are neglected. The Hall Effect, viscous dissipation and the joule heating term are neglected. Under these assumption along with the Bousineque approximation, the boundary layer equation for the problem.

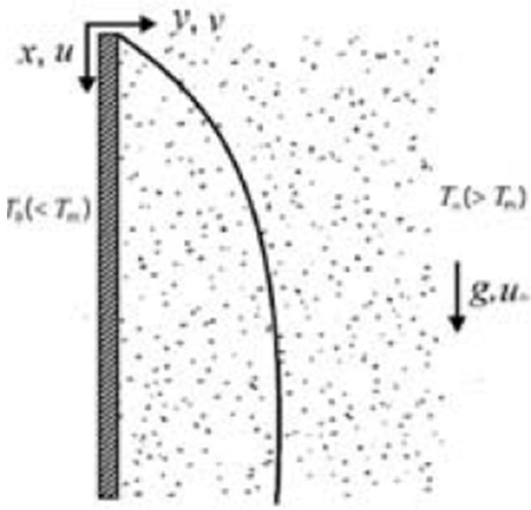

Figure 1 Physical Model of the Problem

III. GOVERNING EQUATION

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) + g\beta(T - T_\infty) + J \times B$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + Q_0(T - T_\infty)$$

Where J is Current density

Neglecting the displacement current , the Maxwell equation and Ohm's law becomes

$$div\ B = 0, Curl\ B = \mu_e J , Curl\ E = -\frac{\partial B}{\partial t}$$

Where B is magnetic field strength

$$J = \sigma (E + V \times B)$$

Where σ is electrical conductivity

and μ_e is the magnetic permeability

E is the electric field

The imposed and induced electric field are assume to be negligible under the assumption of low magnetic Reynolds number

$$J \times B = -\sigma \mu_e^2 H_0^2 u$$

i.e equation reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\left(\frac{\partial^2 u}{\partial y^2}\right) + g\beta(T - T_\infty) - \sigma \mu_e^2 H_0^2 u$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} + Q_0(T - T_\infty)$$

Subject to the boundary conditions

$$u = 0, \quad v = 0, \text{ at } y = 0$$

$$u \to 0, \quad T \to \infty \text{ as } y \to \infty,$$

$$\psi = [g\beta(T - T_\infty)v^2 x_0^3]^{\frac{1}{4}} f(\eta),$$

$$T = T_\infty + (T - T_\infty)\left[\frac{x_0}{x_0 - x}\right]^3 \theta(\eta),$$

$$\eta = \left[\frac{g\beta(T - T_\infty)x_0^3}{v^2}\right]\frac{y}{(x_0 - x)},$$

$$f''' - (f' + M)f' + \theta = 0,$$

$$\frac{1}{\Pr}\theta'' - 3f'\theta + Q\theta = 0.$$

The boundary conditions (4) becomes

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) \to 0,$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0,$$

Where $M = \sigma B_0^2 B^{-1} \rho^{-1}$ the magnetic is filed (hartmen number) and Q is internal heating parameter or number called

IV. NUMERICAL COMPUTATION

The Spectral collocation method is adopted to find the numerical solution of the nonlinear coupled differential Equations (8)-(10) under the boundary condition (11)-(12). The comparison is also made with the finite difference technique which is available in

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literature.

V. RESULT AND DISSCUSSION

The variation of velocity and temperature profile with stream function is plotted for different value of Prandtl number (Pr) and with different value of Internal heating Q is 1 and 5 respectively. In entire study the hartmen number M is fixes at1. In Fig (2) the stream line is plotted for different value of Pr =0.1, 1 and 10 respectively. The characteristic length if fixed 25 for this study. It is found in the profile of streamline is smooth and it increased as the characteristic length increased. We can also observed that as the value of Pr is changed and it is increased to 1 then the stream line fall down drastically in sense of magnitude. In continuation of this Fig (3) is plotted for the different value of internal heating which is fixed at 5. In this figure it has been observed that the stream line is very similar to the previous one. Which shows that the internal heating is not much effective in this way? From both the figure it is also observed that at low value of η around 0.8 the profile of $f(\eta)$ is coincide which shows, When Pr is small, it means that heat diffuses very quickly compared to the velocity (momentum). This means the thickness of the thermal boundary layer is much bigger than the velocity boundary layer for liquid metals. Here the effectiveness of Pr is shown over a low heat resistance sheet.

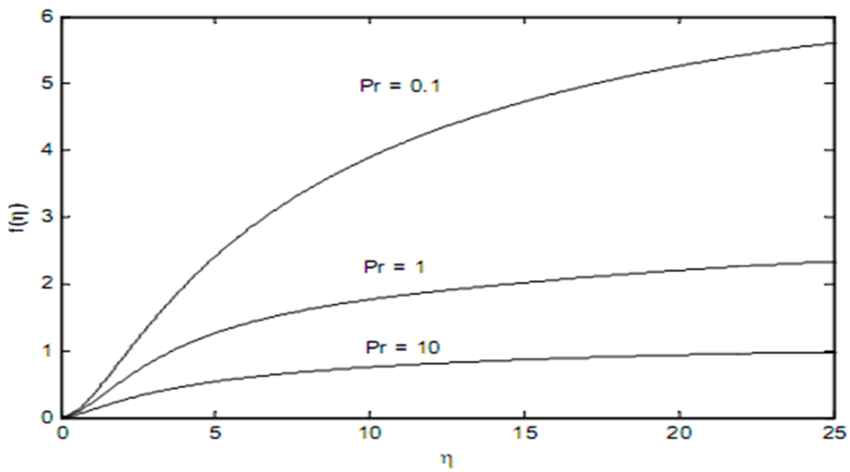

Figure 2 Variation of the f(η) for different values of Pr at Q=1

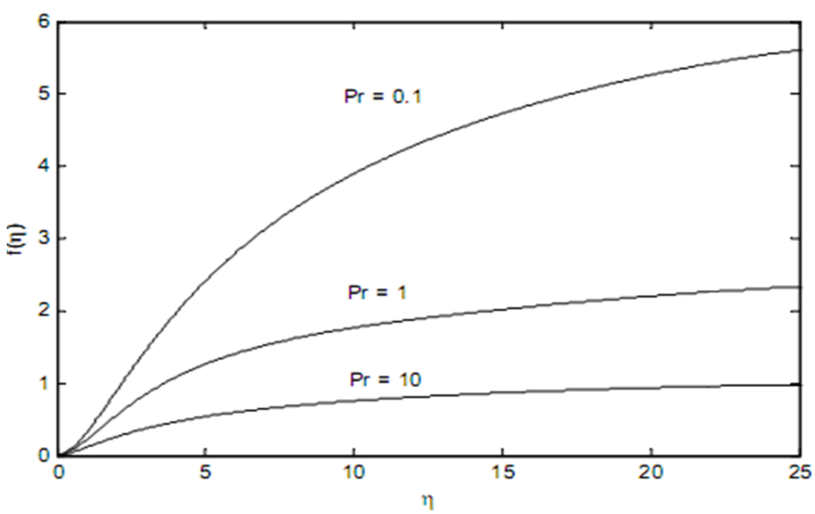

Figure 3 Variation of the f(η) for different values of Pr at Q=5

The variation of velocity is plotted in figure 4 and 5 respectively. In figure 4 it can observed that there is a drastic jump is recorded in the small interval of $0 < \eta < 3$ thereafter it decreases smoothly and asymptotically converges around $\eta < 15$ at the same time there is two different profile is plotted for the different values of Q. The figure 4 is plotted for Q=1 wherever the figure 5 is plotted for the Q=5 respectively. In this similar passion it has been found that there is no much difference is observed in changing of Q. This variation is shows that the internal heating is not much effected in the case of low heat resistance. The above study is also conclude that the resistance in heating work as flow stopper .

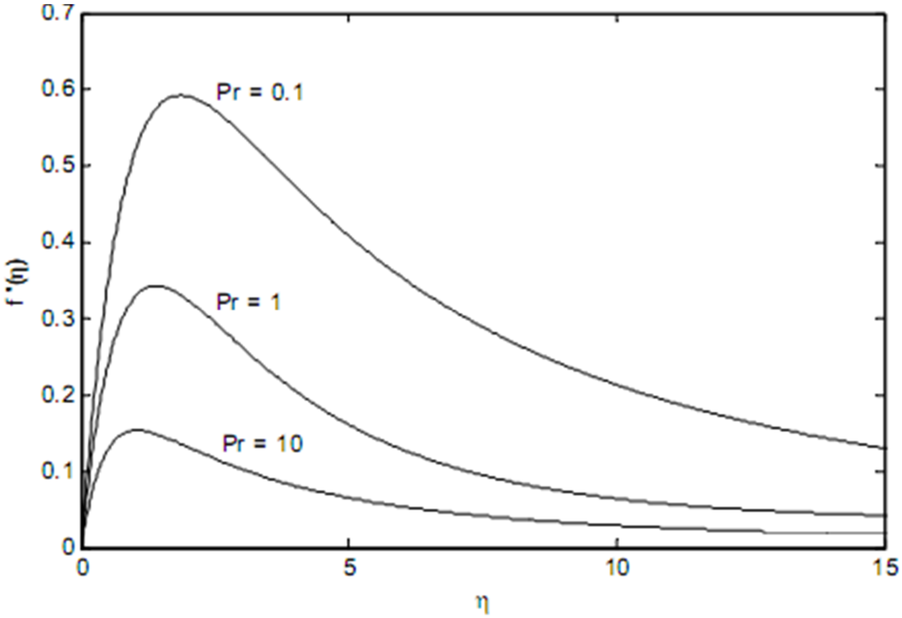

Figure 4 Variation of the Velocity for different values of Pr at Q=1

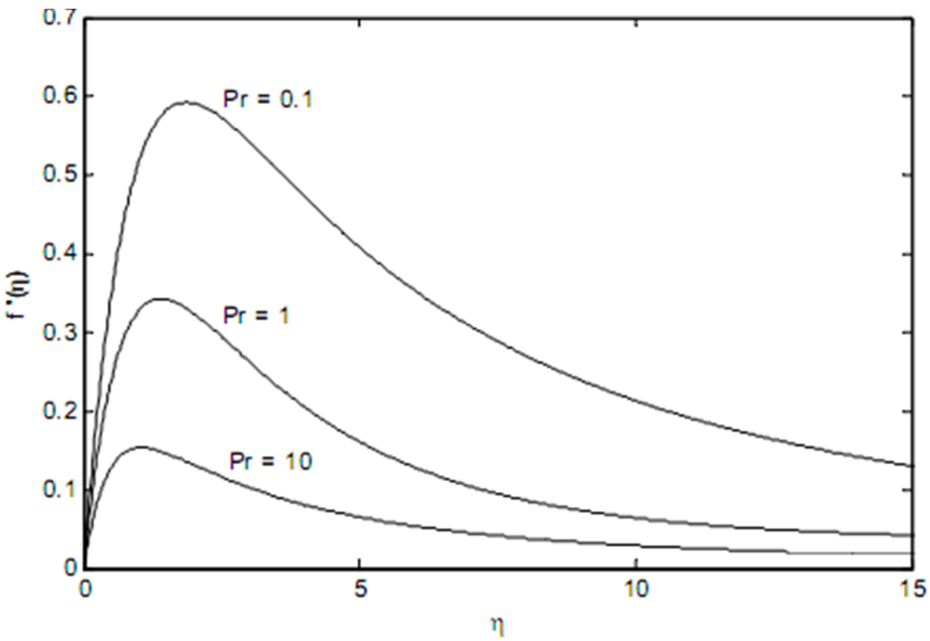

Figure 5 Variation of the Velocity for different values of Pr at Q=5

Finally the temperature is plotted in figure 6 and figure 7 respectively for the similar two different value of internal heating Q=1 and 5 respectively. The profiles shows that in increment of Pr shows the increment in temperature. There is also important point to observe that as we seen the velocity that, temperature is very large in the small domain of $0 < \eta < 3$ and rapidly fall down around $\eta < 2.9$. The profile is also concluding that the normal of each slope of temperature profile is common. In continuation of above

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observation it is also conclude that the profile become flat as $\eta > 10$and its nearly become parallel between characteristic length 10 to 15. The maximum value of velocity is around 0.6 which is obtained for Pr=0.1. The pattern variation in both the figure 6 and 7 for Q =1 and 5 is very similar and hence it can conclude that the internal heating is not much effected in this case. In the entire study the magnetic number is also fixes is at 1, which help resistance to drag the internal heating during the flow mechanics. Overall the different mode of study is help full to understand the pattern variation for the further more look at the study some quantitative analysis is also made in this manuscript.

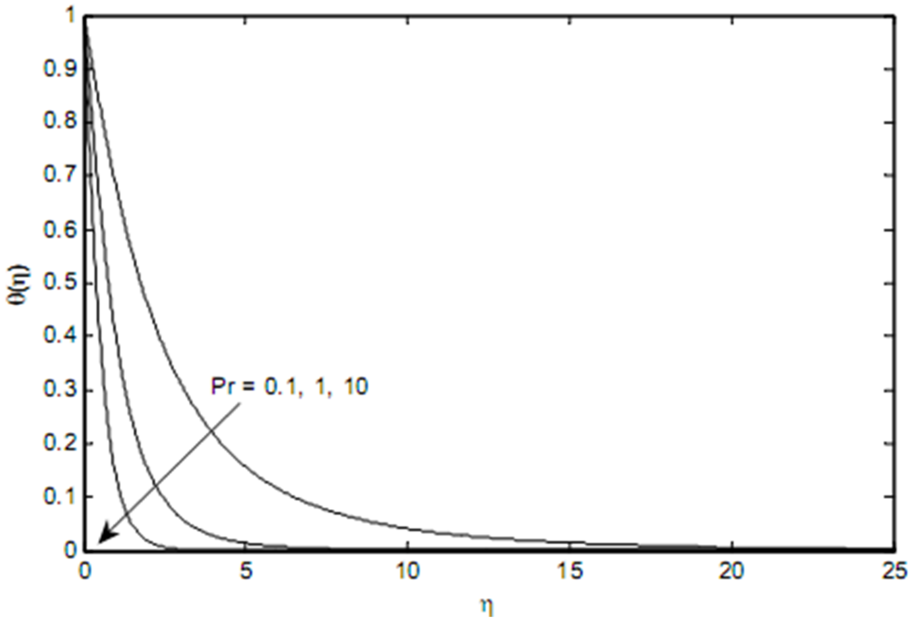

Figure 6 Variation of the temperature profiles θ(η) for different values of Pr at Q=1

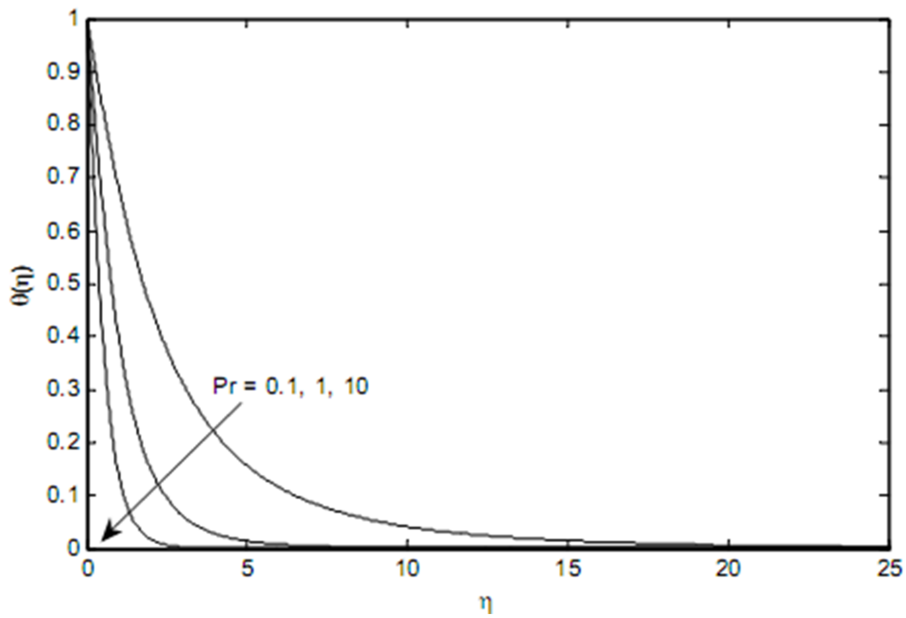

Figure 7 Variation of the temperature profiles θ(η) for different values of Pr at Q=5

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For the further investigation along the numerical method validation the Table 1 and 2 is also made forhe comparative study. The result is compared with the result available in literature

PR	F.Md.Ali et.al (2009)	FDM	SCM
0.001	1.1223	1.1234813212	1.1234811111
0.01	1.0634	1.0629446214	1.0629449065
0.1	0.9241	0.9238258391	0.9238258890
1	0.6932	0.6933185213	0.6933185334
10	0.4471	0.4469506880	0.44695060909
100	0.2645	0.2642557805	0.26425009878
1000	0.1513	0.1512267821	0.15122677811
10000	0.0856	0.08523782130	0.08523700098

TABLE 2: VALUES 0F $f''(0)$ AS COMPARED TO PUBLISH RESULT

In the table 1 we discussed and compare the value of velocity gradient at different prandtl number from the published work of Ali 2009 in which the desired order of accuracy we , which is shown in column 3. along with there is another observation we observed which is that as we increasing the values of Pr the velocity gradient is keep on decreasing which is support of our plotted that rate change of velocity should decrease while the velocity decreases' another compassion is also made which shows in the next table

PR	F.Md.Ali et.al (2009)	FDM	SCM
0.001	0.0480	0.0454324901	0.0454321506
0.01	0.1358	0.1354647102	0.1354640098
0.1	0.3501	0.3488734103	0.3488736781
1	0.7699	0.7623100452	0.762310000
10	1.4971	1.4936173072	1.4936178071
100	2.7472	2.7447645671	2.7447647766
1000	4.9366	4.8923015681	4.8923010916
10000	8.8134	8.7634526171	8.7634525643

TABLE 2: VALUES 0F $-\theta'(0)$ AS COMPARED TO PUBLISH RESULT

In the final table of this work we compare our result again with ali in[15] but for the value of temperature gradient in which we found that maximum heat transfer is occurred for the higher order fluid i.e the fluid which has high prandtl number has maximum heat transfer

VI. CONCLUSION

From the above study we conclude the following

A. The Spectral collocation method gives much similar result as we obtained in Finite difference method

B. Both gives more accurate solution rather than the Keller box technique which is used by the ali [15]

C. Every profile (Velocity or temperature) has shown an asymptomatic behavior and converges for the large value of η

D. W.r.t the characteristic length all profiles are nearly flat while keep on increasing Pr

E. The velocity profiles are found toincrease to a certain maximum point and then reduce asymptotically to zero. As Prandtl number increases, the temperature profile and the thermal boundary layer thickness decrease.

F. The internal heat source is not effected in case of low heat resistance sheet

G. The magnetic number helps in dragging the flow mechanism.

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