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Dimensional Analysis for Determining Optimal Discharge and Penstock Diameter in Reaction Turbines

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Abstract: The objective is to analyze the penstock diameter and discharge dimensionally to get optimization of dimensions of penstock and discharge through it. This dimensional analysis will give the general insights to minimize the consumption of water while producing hydro electric power. The analysis mainly based on the penstock's geometric and hydraulic characteristics, hydraulic head, and the desired power production. Minimizing water consumption for energy production may be effective to the availability of water for other purposes such as irrigation and navigation. The analysis in this paper carried out from various dimensionless relationships between power production, flow discharge, and head losses which were derived previously by various authors. As mentioned in the analysis it was found that for minimizing water consumption, the ratio of head loss to gross head should remained not more than 15.6%. Taking into consideration various dimensional constants and friction factors the formulation in analysis is explained. Making iterative calculations based on derivation given by different authors the maximum and optimum diameter as well as optimum discharge with respect to head loss is calculated. An example of application on an existing 2x12 MW Hydro Power Project is presented for determining optimal flow discharge and optimal penstock diameter for reaction turbine by dimensional analysis.
Keywords: Hydropower, penstock, optimal flow, gross head, dimensional analysis, gross head, turbine.

I. INTRODUCTION

India's economically exploitable and viable hydroelectric potential is estimated to be 148,701 MW. An additional 6,780 MW from smaller hydro schemes (with capacities of less than 25 MW) is estimated as exploitable. 56 sites for pumped storage schemes with an aggregate installed capacity of 94,000 MW have also been identified. In central India, the hydroelectric power potential from the Godavari, Mahanadi, Nagavali, Vamsadhara and Narmada river basins has not been developed on a major scale due to potential opposition from the tribal population. .As per report from 'World Energy Council' the hydropower capacity is often categorized as 'gross theoretical capacity', the capacity of hydropower generation possible if all natural water flows contained as many 100% efficient turbines as possible; 'technically exploitable capacity', the amount of gross theoretical capacity possible within the limits of current technology; and 'economically exploitable capacity', the capacity possible within the constraints of current technology and local economic conditions. There are three types of hydropower stations: 'run of river', where the electricity is generated through the flow of a river'; 'reservoir', where power is generated through the release of stored water; and 'pumped storage', where stored water is recycled by pumping it back up to a higher reservoir in order to be released again. Hydropower facilities installed today range in size from less than 100 kW to greater than 22 GW, with individual turbines reaching 1000 MW in capacity.

The public sector accounts for 92.5% of India's hydroelectric power production. The private sector is also expected to grow with the development of hydroelectric energy in the Himalayan mountain ranges and in the northeast of India. The hydropower generation is highly capital-intensive mode of electricity generation but being renewable source of energy with no consumables involved; there is very little recurring cost and hence no high long term expenditure. It is cheaper as compared to electricity generated from coal and gas fired plants. The life cycle analysis of hydropower shows as cleanest electricity technology with a low carbon footprint, excellent energy pay back ratio, feasible for mass storage of electricity and an opportunity for development when social and environmental impacts are dealt with properly. Also hydropower plays a key role in power systems due to its flexibility and reliability and in the present scenario; its importance has further increased because of the large scale addition of variable renewable energy power in the form of solar and wind energy in the power system. The projects have potential to meet power requirements of remote and isolated areas. These factors make small hydel as one of the most attractive renewable source of grid quality power generation. Apart from the benefit of increase in installation of power generation in the state and eventually overall capacity addition in the country, there is a series of socio-economic activities in the project area which help in overall development of the area, by providing sustainable economic activity, employment opportunity and inherent potential of developing entrepreneurs.

Further, a number of projects are located in remote sites in states which do not have enough demand for electricity that presents geographical constraints in developing requisite transmission infrastructure for evacuation. This paper is presented partially as dimensionless relationships between flow discharge, power production and head losses. On second hand, these relationships are given to get general insights on determining optimal flow discharge and optimal penstock diameter. Finaly, an example of application while designing impulse turbines is presented. The conclusion has summarized with key results in the end of the paper.

II.   LITERATURE REVIEW

Author Ling Zhu, A. S. Leon in 2014 derived various dimensionless relationships between power production, flow discharge and head losses to get general understanding on determining optimal flow discharge and optimal penstock diameter. Resulting that, for minimizing water consumption, the ratio of head loss to gross head should remained not more than 15.6%. He derivate the dimensionless penstock's geometric constant for both impulse and reaction turbine. In different research paper a different author compared previously available formulae for penstock design to review their suitability. He introduces a new method for optimum design of penstock based on minimizing the total head loss having friction and other losses. Considering total head loss, friction losses it has been formulated by Darcy Weisbach formula. These relations have been used for various hydro electric power projects having varying capacity to calculate optimum diameter. From this new method, as the penstock diameter increased in the range, it resulted in the net saving in cost of earlier penstock cost. For the design and manufacturing of hydro turbine runners a collaborative design methodology is developed. In-house MATLAB codes, the design of runner blade to get the desired head and efficiency depends on the correction of runner shape with trial-error is carried out. The hydraulic performance of turbine depends on the shape of the different parts. For accurate result and to achieve hydraulic expectation, CFD analysis and advanced manufacturing tools are must. An association present the best practice for surge tank, penstocks and tunnels; showing innovations in technology, proper condition assessments, and improvements in operation and maintenance practices which can contribute in maximizing the overall plant performance and reliability. Explanation on penstock internal surface roughness contributes in head loss which can be reduced an increase in efficiency. Water flows through penstock from the intake to the generator, gives head loss to the system by hydraulic friction and geometric change in bends, contractions, and expansions.Moko Antony in 2015 conducts a literature review on specifications and design parameters to design a penstock for Kengen Sagana Hydro Power Station. After field data review for designing, the component was derived due to poor performance of the existing penstock. The total head losses from ductile iron and its subsequent efficiency determined then compared to that of upvc and steel penstock, it was seen that ductile iron is not efficient material. At this optimum penstock, power has a very reasonable increase and taking a new material has used with more efficiency, it is cost worthy. Available power could be estimated from head loss and the rate of flow.

III.   DIMENSIONAL ANALYSIS

The electric power P in Watts (W) can be determined by the following equation

$$P = \eta \, \gamma \, Q \, (H_g - h_L)$$

Where,

$\gamma = (\rho \times g)$ specific weight of water in $kg/(m^2 \times s^2)$,

Q = Flow discharge in m^3/s,

Hg = Gross head in m,

h_L = Sum of head losses in m,

ρ = Water density in kg/m^3,

g = Acceleration of gravity in m/s^2,

η = overall hydroelectric efficiency, the product of turbine efficiency (η_t) and generator efficiency (η_g).
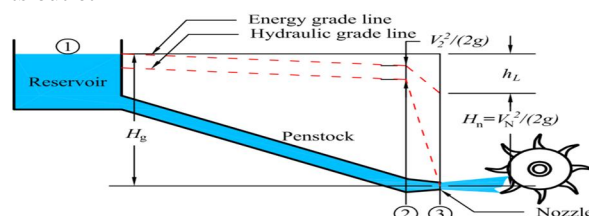
A_d = draft tube cross-sectional area at its outlet

Fig.1 Sketch of an Impulse Turbine

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The sum of head losses can be written as

$$h_L = \frac{Q^2}{2gA^2} \left[f\frac{L}{D} + \sum k\,1-2 + k_N \left(\frac{A2}{Ad}\right)^2 \right] \qquad \text{… for impulse turbine}$$

$$h_L = \frac{Q^2}{2gA^2} \left[f\frac{L}{D} + \sum k\,1-2 + \left(\frac{A2}{Ad}\right)^2 \right] \qquad \text{… for reaction turbine}$$

The expression in the bracket is dimensionless and denoted as

$$C_L = f\frac{L}{D} + \sum k\,1-2 + k_N \left(\frac{A2}{Ad}\right)^2 \qquad \text{… for impulse turbine}$$

$$C_L = f\frac{L}{D} + \sum k\,1-2 + \left(\frac{A2}{Ad}\right)^2 \qquad \text{… for reaction turbine}$$
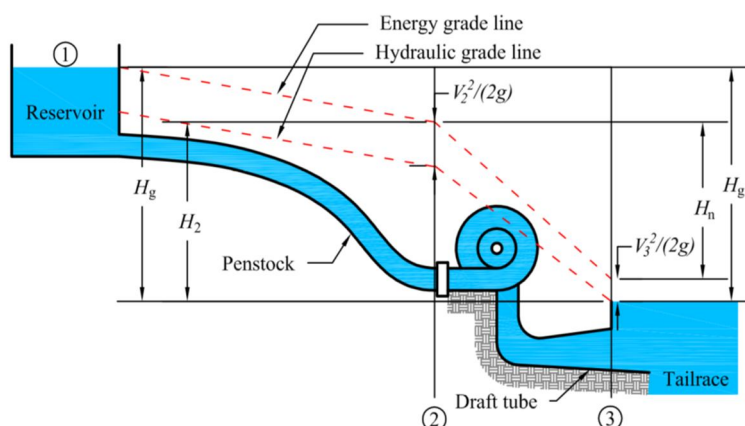

Fig.2 Sketch of Reaction Turbine

Hence, from above constants the total head losses can be written as

$$P = \eta\,\gamma\,Q\left(H_g - C_L \frac{Q^2}{2gA^2}\right)$$

Obtained dimensionless relationship between power and discharge is

$$P_+ = \eta\left[\frac{3}{2}Q_+ - C_L\left(\frac{A3}{A2}\right)^2 Q_+^{\,3}\right]$$

Now, taking β the product of C_L and $(A_3/A_2)^2$

$$\beta = (A_N/A_2)^2 \left[f\frac{L}{D} + \sum k\,1-2 + k_N\,(A_2/A_N)^2 \right] \qquad \text{… for impulse turbine}$$

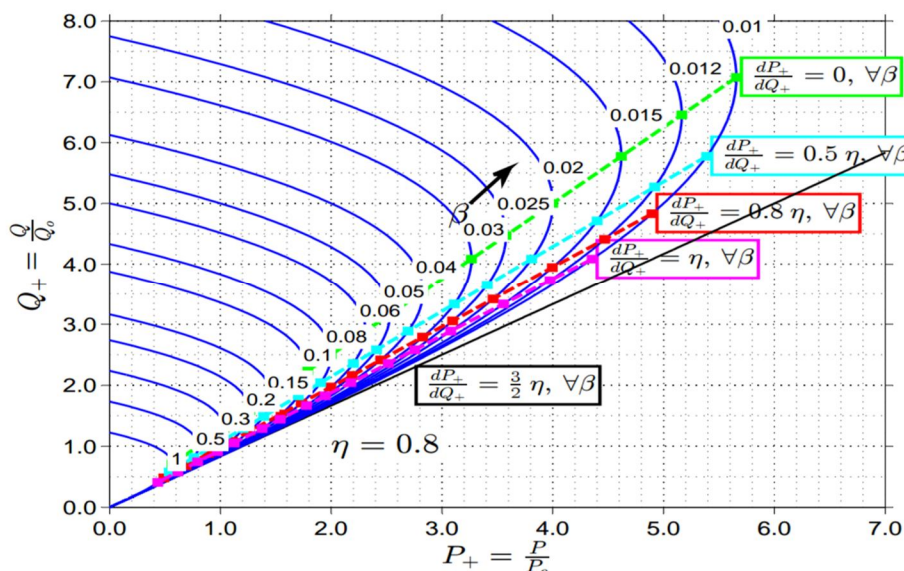$$\beta = (A_d/A_2)^2 \left[f\frac{L}{D} + \sum k\,1-2 + (A_2/A_d)^2 \right] \qquad \text{… for reaction turbine}$$

Fig.3 Dimensionless discharge ($Q+$) versus dimensionless power ($P+$) and a typical range of β for impulse turbines

The variation of P_+ with respect to Q_+ for a fixed β can be obtained by differentiating P_+ with respect to Q_+. The maximum dimensionless power for a fixed β can be obtained by setting dP_+/dQ_+ equal to zero.
The power will maximum when

$$(Q_+)_{max} = \sqrt{\frac{1}{2\beta}}$$

The maximum dimensionless power for a fixed β is

$$(P_+)_{max} = \eta\sqrt{\frac{1}{2\beta}}$$

In applications the value of β ranges between 0.01 to 1.0 for impulse turbines and from 10 to 1000 for reaction turbines. Same as C_L range from 1 to 100 for both impulse and reaction turbines.
To minimize water consumption to get a required amount of hydropower, it is necessary that dP_+/dQ_+ is close to its maximum value $(3/2)$ η. So as the optimal lower limit of dP_+/dQ_+ is set to 0.8η.
    Substituting $dP_+/dQ_+ = 0.8\eta$ which gives the upper limit for the dimensionless terms as follows

Dimensionless flow discharge $(Q_+)_{opt\ upper} = \sqrt{\dfrac{7}{30\beta}}$

Corresponding dimensionless power $(P_+)_{max} = \eta\,\dfrac{19}{15}\sqrt{\dfrac{7}{30\beta}}$
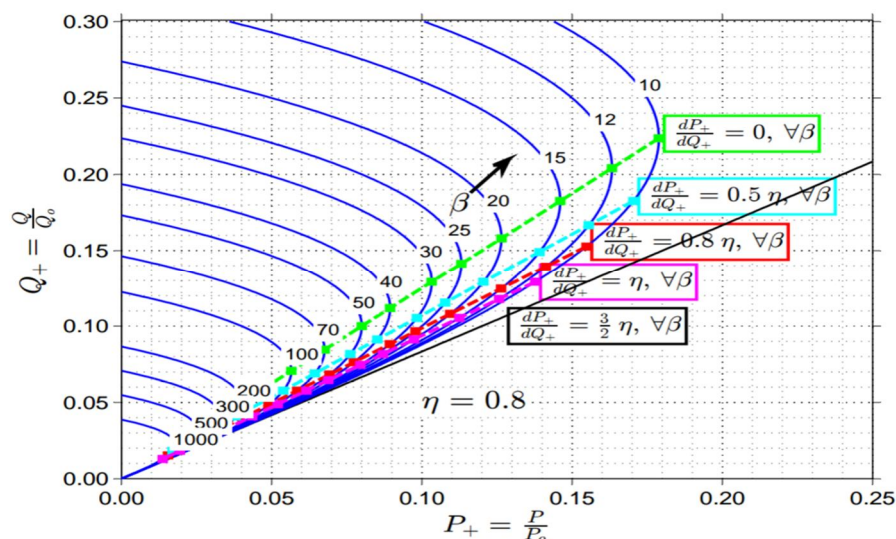

Fig.4 Dimensionless discharge (Q+) versus dimensionless power (P+) and a typical range of β for reaction turbines

The optimal dimensionless head loss ($hL+ = hL/Hg$) obtained by taking the optimal upper limit for the flow discharge $Q_+ = [7/(30\beta)]^{1/2}$ in the below equation

$$h_{L+} \leq \frac{2}{3}\beta\,Q_+{}^2$$

By substituting it will be

$$h_{L+} \leq \frac{7}{45} = 15.6\%$$

Above equation shows, for minimizing water consumption, the ratio of head loss to gross head should not exceed 15.6%. While designing a turbine, it is important to specify either the flow discharge or the desired electric power. These terms are presented as below

A. *Case A: P is specified*
If P is specified, the optimal upper limit of flow discharge will be

$$Q_{opt} = \frac{45}{38}\left(\frac{P}{\eta\rho H g}\right)$$

The optimal penstock diameter or geometric constant C_L can be calculated from

$$\frac{(C_L)opt}{A^2} \leq \frac{14}{45}\frac{g\,Hg}{Q^2}$$

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B. Case B: Q is specified

If Q is specified, the optimal upper limit of power will be

$$P_{opt} = \frac{38}{45} \eta \gamma H g \, Q$$

Same as case A the optimal penstock diameter or geometric constant C_L can be calculated from

$$\frac{(CL)opt}{A2} \leq \frac{14}{45} \frac{g \, Hg}{Q2}$$

IV. APPLICATION OF ANALYSIS FOR REACTION TURBINE

The site data from an existing hydro Power Project are as given below,

1) Type of turbine = Kaplan
2) Hydroelectric efficiency (η) = 0.95%
3) Gross head (Hg) = 25 m
4) Penstock length (L) = 800 m
5) Ratio of penstock cross-sectional area to nozzle cross-sectional area at its outlet (A_2/A_N) = 16
6) Nozzle Velocity coefficient (C_V) = 0.985
7) Sum of local losses in penstock due to entrance, bends, penstock fittings and gates (Σk_{1-2}) = 1.5
8) Roughness height of penstock material ($\mathcal{E}$)   = 0.045 mm (commercial steel)
9) Kinematic viscosity ($\sqrt{}$)   = 10-6 m2/s
10) Ratio of penstock cross-sectional area to draft tube cross-sectional area at its outlet (A_2/A_d) = 1/3

Calculating the maximum discharge for determining the maximum values of power production, the values for *β* should be ranges from 10 to 1000 as mentioned per above analysis

Taking *β* = 10, the maximum discharge for given hydroelectric system will be

$$(Q_+)_{max} = \sqrt{\frac{1}{2\beta}}$$
$$= 0.224$$

The maximum power will be

$$(P_+)_{max} = \eta \sqrt{\frac{1}{2\beta}}$$
$$= 0.201$$

Now, for minimizing water consumptions the dimensionless optimal flow discharge and corresponding produce power will be

$$(Q_+)_{opt \, upper} = \sqrt{\frac{7}{30\beta}}$$
$$= 0.153$$

Dimensionless optimal power will be

$$(P_+)_{max} = \eta \frac{19}{15} \sqrt{\frac{7}{30\beta}}$$
$$= 0.174$$

As per given analysis, for this dimensionless optimal discharge and dimensionless power, the dimensionless head loss should satisfy the inequality equation i.e. < 15.6%

$$h_{L+} \leq \frac{2}{3} \beta \, Q_+{}^2$$
$$= 0.156$$
$$= 15.6\%$$

For designing the optimal penstock diameter for optimal discharge the geometric constant C_L needs to be calculated from given data available at site

We will go for numbers of iterative values of *β* between 10 to 1000 and calculating for different values of flow discharge and power produce

Table: 4.1 iterative calculations for values of β from 10 to 1000

β	Maximum Discharge and Production		Minimum Water Consumption		Optimal Head Loss
	Q+ max	P+ max	Q+ opt	P+ opt	Hl
10	0.224	0.212	0.153	0.184	15.6%
60	0.091	0.087	0.062	0.075	15.6%
110	0.067	0.064	0.046	0.055	15.6%
160	0.056	0.053	0.038	0.046	15.6%
210	0.049	0.046	0.033	0.040	15.6%
260	0.044	0.042	0.030	0.036	15.6%
310	0.040	0.038	0.027	0.033	15.6%
360	0.037	0.035	0.025	0.031	15.6%
410	0.035	0.033	0.024	0.029	15.6%
460	0.033	0.031	0.023	0.027	15.6%
510	0.031	0.030	0.021	0.026	15.6%
560	0.030	0.028	0.020	0.025	15.6%
610	0.029	0.027	0.020	0.024	15.6%
660	0.028	0.026	0.019	0.023	15.6%
710	0.027	0.025	0.018	0.022	15.6%
760	0.026	0.024	0.018	0.021	15.6%
810	0.025	0.024	0.017	0.020	15.6%
860	0.024	0.023	0.016	0.020	15.6%
910	0.023	0.022	0.016	0.019	15.6%
960	0.023	0.022	0.016	0.019	15.6%

A. Case A: Q is Specified

Let the design flow for given reaction turbine, $Q = 55.8 \text{ m}^3/\text{sec}$

The optimal penstock diameter or geometric constant C_L can be calculated

$$\frac{(CL)opt}{A2} \leq \frac{14}{45} \frac{g\,Hg}{Q2}$$
$$= 0.0245 \text{ m}^{-4}$$

But, $C_L = f\frac{L}{D} + \sum k\,1 - 2 + (\frac{A2}{Ad})^2$
$$= 4.18$$

From above equation we can determine the optimal diameter for penstock
$$A^2 = 13.07 \text{ m}^2$$
$$D = 4.08 \text{ m}$$

Now,
Checking for dimensional optimal head loss

$$h_L = C_L \frac{Q2}{2g\,Hg\,A2}$$
$$= 0.1526$$
$$= 15.26\% < 15.6\%$$

Finally, calculating for the optimal power for specified optimal flow discharge

$$P_{opt} = \eta\,\gamma\,Q\,(H_g - C_L \frac{Q2}{2gA^2})$$
$$= 12303.898 \text{ Kw}$$

Table: 4.2 iterative calculations for different values of flow discharge Q

If Q is specified				
Q in m3/s	CL	P in kW	D in m	hL
20.70	5.70	4564	2.48	21.21%
24.60	5.36	5424	2.71	19.95%
28.50	5.10	6284	2.92	18.96%
32.40	4.88	7144	3.11	18.16%
36.30	4.70	8004	3.29	17.49%
40.20	4.54	8864	3.46	16.91%
44.10	4.41	9724	3.63	16.42%
48.00	4.30	10583	3.78	15.99%
51.90	4.19	11443	3.93	15.60%
55.80	4.10	12303	4.08	15.26%
59.70	4.02	13163	4.22	14.95%
63.60	3.94	14023	4.36	14.68%
67.50	3.88	14883	4.49	14.42%
71.40	3.81	15743	4.62	14.19%
75.30	3.75	16603	4.74	13.97%
79.20	3.70	17463	4.86	13.77%
83.10	3.65	18323	4.98	13.59%
87.00	3.61	19183	5.09	13.42%
90.90	3.56	20043	5.21	13.26%

B.   Case B: P is specified

Let assume Power P = 12000 kW

Then optimal upper limit of flow discharge will be

$$Q_{opt} = \frac{45}{38} \left(\frac{P}{\eta \rho H g} \right)$$
$$= 61.05 \text{m}^3/\text{sec}$$

Same as case A

The optimal penstock diameter or geometric constant C_L can be calculated from

$$\frac{(CL)opt}{A^2} \leq \frac{14}{45} \frac{g \, Hg}{Q^2}$$
$$= 4.18$$
$$A^2 = 14.296 \text{ m}^2$$

$$D = 4.26 \text{ m}$$

Table: 4.3 iterative calculations for different values of power P

If P is specified			
P in kW	Q in m3/s	D in m	hL
20043	101.98	5.52	15.56%
19183	97.60	5.40	15.56%
18323	93.23	5.27	15.56%
17463	88.85	5.15	15.56%
16603	84.48	5.02	15.56%
15743	80.10	4.89	15.56%
14883	75.72	4.75	15.56%
14023	71.35	4.61	15.56%
13163	66.97	4.47	15.56%
12303	62.60	4.32	15.56%
11443	58.22	4.17	15.56%
10583	53.85	4.01	15.56%
9724	49.48	3.84	15.56%
8864	45.10	3.67	15.56%
8004	40.72	3.49	15.56%
7144	36.35	3.29	15.56%
6284	31.97	3.09	15.56%
5424	27.75	2.88	15.56%
4564	23.22	2.63	15.56%

Hence,

From all these analysis and calculations for an existing hydroelectric plant, it is found that for producing 12000kW of power the required optimal flow discharge should be 61.05m^3/sec where as the optimal penstock diameter will be select 4.26 m which satisfy the inequality equation.

Minimizing water consumption for getting optimal discharge the dimensionless head loss should not exceed 15.6% that is satisfied for given hydro power system.

V. CONCLUSION

These calculations give the clarifications on the designing of penstock diameter for hydro power system by minimizing the flow discharge. From both cases its shows relative outputs from iterative calculation which satisfy the inequality equation. Analysis for optimal discharge, penstock diameter, optimal head loss and power produce are calculated and compare which giving analytical values as required objectives.

For an existing hydro power system the optimal discharge is determined from given data and the desired power outputs also achieved. Analysis for optimal penstock diameter gives the optimal values of penstock diameter which satisfy the inequality equation. Water consumption is minimized by optimizing flow discharge through the penstock. An existing hydro power having penstock diameter of 5m is been optimized to 4.26m achieving power output by satisfying the dimensionless head loss condition i.e. dimensionless head loss should not exceed 15.6%.

This paper gives the additional clarification and backup data for the dimensional analysis of penstock diameter which was derived, analyzed and calculated by different authors from different streams. This analysis is dependent on some hydraulic and geometric characteristics of penstock, total hydraulic head and required power outputs.

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