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Study of Cubi-Linear (Non-Conforming) Element

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Abstract: This paper aims the study of Cubi-Linear element of Finite Element Method. It is eight-nodded non-conforming element. Since many researches have been carried out in area of conforming element, this paper includes study of non-conforming element. Cubi-Linear element is having combination of 4 nodes on its x-axis and 2 nodes o y-axis. In this study, initially the behaviour of Cubi-Linear Element is scrutinised and later its behaviour is compare with classic approach of theory and standard software viz. STAAD Pro. The comparison of Deflection and Bending Moment of beam element are conducted. The comparison of primary and secondary unknowns are conducted to study the convergence rate and accuracy of Cubi-Linear Element.
Keywords: Cubi-Linear Element, Non-Conforming Element, Finite Element Method, Eight Nodded Element, Computer Programming, STAAD Pro, Auto-meshing.

I. INTRODUCTION

The finite element method is computer aided mathematical technique to obtain approximate numerical solution to the abstract of equation that predict the response of physical system under eternal influence like displacement, shear force, bending moment in case of structural problems. Because of its diversity and flexibility as an analysis tool, Finite Element Method is getting much attention in field of engineering. In real, any structures are made up from infinite numbers of sub-region but in FEM, the finite numbers of sub regions are considered. Many times in practise, it is laborious to analyse complex structures in terms of geometry, material non-linearity etc. with conventional methods like Moment distribution method, Slope deflection method, Kani's method, Matrix method. In such cases, Finite Element method has proven efficient and accurate. When it is applied with computer programming, many lengthy steps can be eliminate and quick solution can obtain.

A computer program is developed to analyse beam element having different support condition. A computer program, which able to perform auto meshing will be developed for beam with different support conditions. In this paper, Finite Element Method is adopted with FORTRAN computer language. The FORTRAN programming language is specially used for numerical computation and scientific mathematical calculation purpose. Through this, it is easy to store required output in text format.

II. OBJECTIVE

The main objective of this research is to study the behaviour of Cubi-linear (non-conforming) element, later it has been used to analyse beams having different support conditions. Finally, validation of its behaviour has been carried out through standard software STAAD Pro and classical or theoretical approach of theory. The exact analysis of beam is a complex problem so program based on Finite Element Method is used to analyse the beam. Hence, Cubi-linear element is used for meshing to account for the linear stress distribution along depth and cubic stress distribution along longitudinal direction.

III. SCOPE

To solve primary and secondary unknown, following steps to be followed:

A. Understand the problem and select appropriate displacement function. Whether structure is 1D, 2D or 3D.
B. Discretise the structure or member (meshing of structure).
C. Calculate elemental properties with help of selected displacement function (member stiffness matrix).
D. Calculate loading condition with help of shape function (load vector for each member).
E. Calculate global elemental properties (stiffness matrix and load vector for whole structure).
F. Calculate primary unknowns (slope, deflection, S.F., B.M.) with help of simultaneous equation (by applying equilibrium).
G. Calculate secondary unknowns (Stresses and strain) with calculated primary unknowns.

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The computer program is developed which will perform specific task as follows:
When the beam specification (Length, width, depth and material properties) with different support condition is entered into the program, it will discretize beam into defined manner from first loop onwards and then answer is stored, in second loop beam is discretize into further elements and answer is compared, and based on accuracy next loop will run. Further, program will discretize beam into elements loop wise in both directions, and node number of element is auto-generated along with its co-ordinates. In auto-meshing module for analysing beam, different support condition can be provided and required unknowns can be calculated, and analysis of deep beam will be involved. Scope will be limited to determination of deflection of deep beam.

IV. CONFORMING ELEMENT

The basic concept of conforming Finite Element is it is having symmetry in numbers of node on its x-axis and y-axis. There are many basic conforming elements in Finite Element Method. Basic one-dimensional elements are linear as (fig.1), which can be having two nodes, three nodes, four nodes etc. Similarly, in two-dimensional elements, there are Constant Strain Triangle (CST) with three nodes, Linear Strain Triangle (LST) with six nodes and Quadrilateral element with four nodes, eight nodes, nine nodes as in (fig.2). There is Brick Element in three-dimensional finite element having eight nodes as (fig.3).

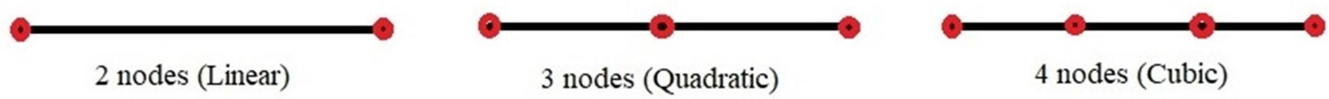

Figure - 1: One Dimensional Elements

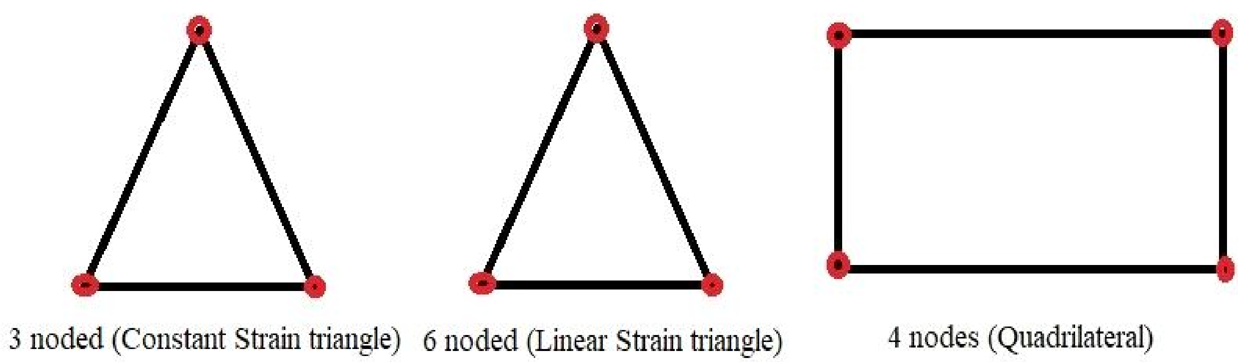

Figure - 2: Two Dimensional Elements

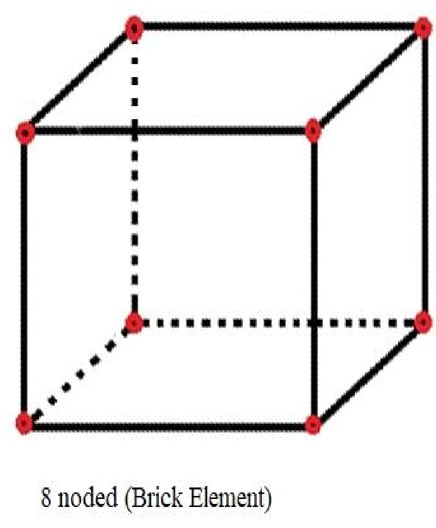

Figure - 3: Three Dimensional Element

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V. NON-CONFORMING ELEMENT

In non-conforming elements, no symmetry is observed in terms on numbers of nodes in x-axis and y-axis. This paper includes Cubi-Linear non-conforming element with is having combination of eight nodes (fig). In basic quadrilateral element, there are four nodes are available i.e. node 1, 7, 8, 2 at (-1,-1), (1,-1), (1,1) and (-1,1). By adding another four nodes at (-1/3,-1), (1/3,-1), (1/3,1) and (-1/3,1), Cubi-Linear element can be generated. There are four nodes in x-axis that shows cubical stress variation in length of member and two nodes on y-axis that shows linear stress variation along depth of member.

A. Shape Function of Cubi-Linear element (Using Lagrange's Method)

$\mathbf{N_1} = \frac{(9\xi^3-9\xi^2-\xi+1)(\eta-1)}{32}$; $\mathbf{N_2} = \frac{(9\xi^3-9\xi^2-\xi+1)(-\eta-1)}{32}$; $\mathbf{N_3} = \frac{(27\xi^3-9\xi^2-27\xi+9)(-\eta+1)}{32}$; $\mathbf{N_4} = \frac{(27\xi^3-9\xi^2-27\xi+9)(\eta+1)}{32}$

$\mathbf{N_5} = \frac{(27\xi^3+9\xi^2-27\xi-9)(\eta-1)}{32}$; $\mathbf{N_6} = \frac{(27\xi^3+9\xi^2-27\xi-9)(-\eta-1)}{32}$; $\mathbf{N_7} = \frac{(9\xi^3+9\xi^2-\xi-1)(-\eta-1)}{32}$; $\mathbf{N_8} = \frac{(9\xi^3+9\xi^2-\xi-1)(\eta+1)}{32}$

Here N_1, N_2, N_3,… N_8 are Shape functions of nodes 1, 2, 3,…8 respectively.

B. Constitutive Matrix [D]

This matrix denotes plane stress/strain condition. In this study, plane stress condition is adopted.

$[D] = E/\,(1 - \mu^2)\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu)/2 \end{bmatrix}$; E = Modulus of Elasticity of section ; μ = Poisson's ratio

C. Matrix of Co-Ordinates of Nodes [q]

$[q] = \begin{bmatrix} \xi 1 & \xi 2 & \xi 3 & \xi 4 & \xi 5 & \xi 6 & \xi 7 & \xi 8 \\ \eta 1 & \eta 2 & \eta 3 & \eta 4 & \eta 5 & \eta 6 & \eta 7 & \eta 8 \end{bmatrix}$; ξi & ηi = co-ordinates of nodes 1, 2 … 8 in x and y-axis respectively.

$$[q^T] = \begin{bmatrix} \xi 1 & \eta 1 \\ \xi 2 & \eta 2 \\ \xi 3 & \eta 3 \\ \xi 4 & \eta 4 \\ \xi 5 & \eta 5 \\ \xi 6 & \eta 6 \\ \xi 7 & \eta 7 \\ \xi 8 & \eta 8 \end{bmatrix}$$

D. Jacobi's Matrix [J]

$$[J] = \begin{bmatrix} \frac{\partial N1}{\partial \xi} & \frac{\partial N2}{\partial \xi} & \cdots & \frac{\partial N7}{\partial \xi} & \frac{\partial N8}{\partial \xi} \\ \frac{\partial N1}{\partial \eta} & \frac{\partial N2}{\partial \eta} & \cdots & \frac{\partial N7}{\partial \eta} & \frac{\partial N8}{\partial \eta} \end{bmatrix} * \begin{bmatrix} \xi 1 & \eta 1 \\ \xi 2 & \eta 2 \\ \vdots & \vdots \\ \xi 7 & \eta 7 \\ \xi 8 & \eta 8 \end{bmatrix} = \begin{bmatrix} J11 & J12 \\ J21 & J22 \end{bmatrix}$$

$[J^{-1}] = \frac{1}{|J|}\begin{bmatrix} J22 & -J12 \\ -J21 & J11 \end{bmatrix}$; |J| = determinate of [J]

E. Geometric Matrix [G]

$$[G] = \begin{bmatrix} \frac{\partial N1}{\partial \xi} & 0 & \frac{\partial N2}{\partial \xi} & 0 & \cdots & \frac{\partial N7}{\partial \xi} & 0 & \frac{\partial N8}{\partial \xi} & 0 \\ \frac{\partial N1}{\partial \eta} & 0 & \frac{\partial N2}{\partial \eta} & 0 & \cdots & \frac{\partial N7}{\partial \eta} & 0 & \frac{\partial N8}{\partial \eta} & 0 \\ 0 & \frac{\partial N1}{\partial \xi} & 0 & \frac{\partial N2}{\partial \xi} & \cdots & 0 & \frac{\partial N7}{\partial \xi} & 0 & \frac{\partial N8}{\partial \xi} \\ 0 & \frac{\partial N1}{\partial \eta} & 0 & \frac{\partial N2}{\partial \eta} & \cdots & 0 & \frac{\partial N7}{\partial \eta} & 0 & \frac{\partial N8}{\partial \eta} \end{bmatrix}$$

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F. Strain Displacement Matrix [B]

$$[B] = [A] * [G] = [B] = \begin{bmatrix} B1\,1 & B1\,2 & \cdots & B1\,15 & B1\,16 \\ B2\,1 & B2\,2 & \cdots & B2\,15 & B2\,16 \\ B3\,1 & B3\,2 & \ldots & B3\,15 & B3\,16 \end{bmatrix}$$

G. Member Stiffness Matrix [k]

$$[k] = t * |J^{-1}| * [B^T] * [D] * [B]$$

$$[k_1] = \begin{bmatrix} K1\,1 & K1\,2 & \cdots & K1\,15 & K1\,16 \\ K2\,1 & K2\,2 & \ldots & K2\,15 & K2\,16 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ K15\,1 & K15\,2 & \ldots & K15\,15 & K15\,16 \\ K16\,1 & K16\,2 & \cdots & K16\,15 & K16\,16 \end{bmatrix}; [k_2] = \begin{bmatrix} K1\,1 & K1\,2 & \cdots & K1\,15 & K1\,16 \\ K2\,1 & K2\,2 & \ldots & K2\,15 & K2\,16 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ K15\,1 & K15\,2 & \ldots & K15\,15 & K15\,16 \\ K16\,1 & K16\,2 & \cdots & K16\,15 & K16\,16 \end{bmatrix}$$

H. Assemble Stiffness Matrix [K]

$$[K] = \begin{bmatrix} K1\,1 & K1\,2 & \cdots & K1\,27 & K1\,28 \\ K2\,1 & K2\,2 & \ldots & K2\,27 & K2\,28 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ K15\,1 & K15\,2 & \ldots & K27\,27 & K27\,28 \\ K28\,1 & K28\,2 & \cdots & K28\,27 & K28\,28 \end{bmatrix}$$

I. Deflection/ Deformation at Each Nodes {δ}

$$\{\delta\} = [K^{-1}] * [F] = \begin{bmatrix} \delta 1 \\ \delta 2 \\ \vdots \\ \delta 23 \\ \delta 24 \end{bmatrix}; \text{ where Force vector } [F] = \begin{bmatrix} F1 \\ F2 \\ \vdots \\ F15 \\ F16 \end{bmatrix} \text{ forces on each nodes.}$$

δ1, δ2, … δ24 are the deformation are node 1, 2 … 24 respectively.

VI. ANALYSIS OF DEEP BEAMS

Deep beams can be defined as structural elements carries a significant amount of load to the supports in different load conditions. The strain is not linear, and shear deformations is more significant in compare to pure bending, and needed to be calculated.

According to IS-456 (2005) Cl.29.1, A beam shall deemed as a deep beam when the ratio of effective span to overall depth, is less than: (i) 2.0 for a simply supported beam; and (ii) 2.5 for a continuous beam.

According to ACI-31- R14 Cl.9.9.1, Deep beams are members that are loaded on one face and supported on the opposite face such that strut-like compression elements can develop between the loads and supports and that satisfy: (i) Clear span does not exceed 4 times the overall member depth h and (ii) Concentrated loads exist within a distance 2h from theface of the support.

The elementary theory of bending for simple beams may not be applicable to deep beams even under the linear elastic assumption. A deep beam is in fact a vertical plate subjected to loading in its own plane. The strain or stress distribution across the depth is no longer a straight line, shear deformation have to be accounted which can be neglected in simple beams. The analysis of a deep beam should therefore, treated as a two dimensional plane stress problem, and two-dimensional stress analysis methods should be used in order to obtain a realistic stress distribution even for a linear elastic solution.

A. Meshing

Analysis of deep beam is done by meshing the beam in both x- direction and y- direction where Nx indicates number of parts of beam in x- direction and Ny indicates number of parts of beam in y- direction. In first iteration of meshing, parts in x-direction will be two and parts in y-direction will be one. Thus, Nx = 2 and Ny = 1. In this iteration, procedure of finite element will be perform by program and it will give output of required unknowns at each nodes and each elements. In second iteration, value of Nx will increase by two and value of Ny by one. Thus Nx = 4 and Ny = 2 again whole procedure will repeated itself up to achievement of required accuracy. It should be noted that, with increase in value of Nx and Ny total number of nodes will also increase and it can calculated by following. Through total numbers of nodes, elemental properties of each element is calculated.

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1) Total no. of nodes = (3*Nx+1)*(Ny+1)
2) Auto numbering formulae = [4*(iel-1) + (j-1) + k]

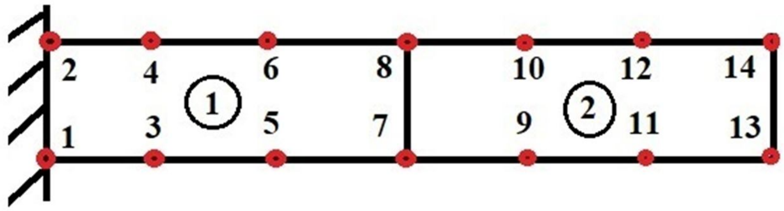

Figure - 4: Iteration 1, Nx = 2, Ny = 1

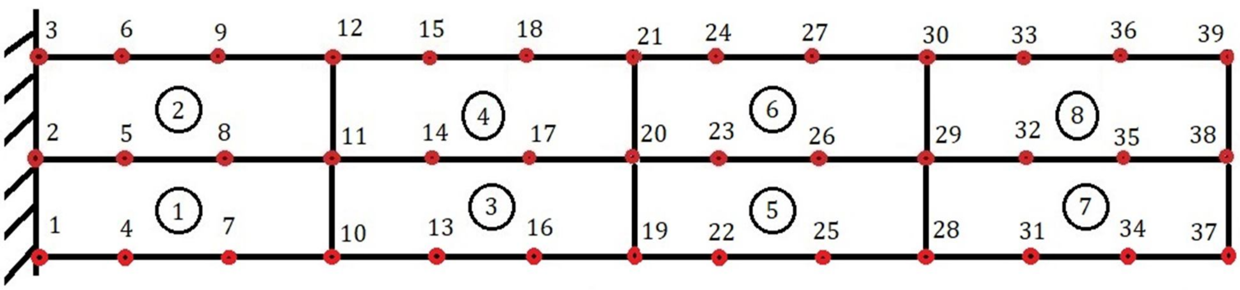

Figure - 5: Iteration 2, Nx = 4, Ny = 2

VII. VALIDATION OF STUDY

For the purpose of validation a standard examples are taken which are analysed using the developed program and with a widely accepted software namely STAAD.Pro and classical approach of theory. The results of three are compared and based on that conclusion can be given.

1) *Example – 1:* Simply Supported beam with Central Point Load

P (Load)	10 KN
Location Of Point Load	1.5m
L (Length)	3 m
B (Thickness)	0.10 m
D (Depth)	1.5 m
E (Modulus of Elasticity)	2*10^7 N/mm^2
μ (Poisson's Ratio)	0.3

Table - 1: Beam data for Example 1

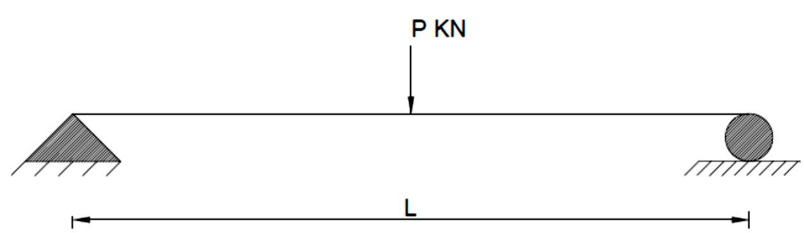

Figure - 6: Simply Supported Beam with Central Point Load

Comparison of Deflection(mm) for Example - 1					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi-Linear	0.0000	0.0067	0.0098	0.0067	0.0000
STAAD	0.0000	0.0070	0.0100	0.0070	0.0000
Theory	0.0000	0.0068	0.0100	0.0068	0.0000

Table - 2: Deflection value for Example 1

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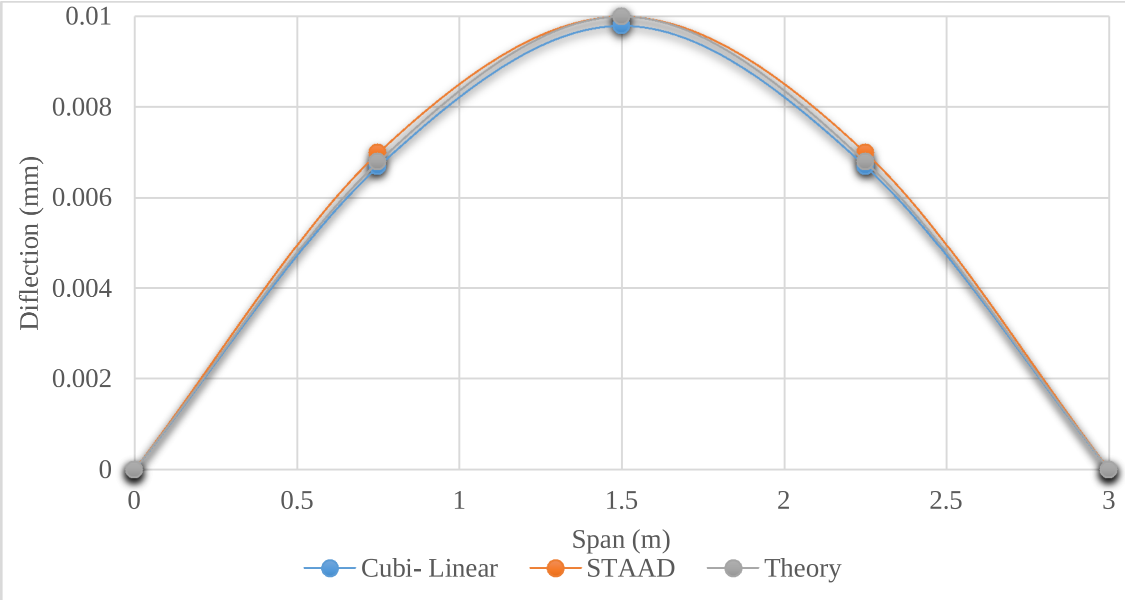

Figure - 7: Comparison of Deflection for Example 1

Comparison of Bending Moment(KNm) for Example - 1					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi-Linear	-0.097	-3.754	-7.495	-3.754	-0.097
STAAD	0.000	-3.750	-7.500	-3.750	0.000
Theory	0.000	-3.750	-7.500	-3.750	0.000

Table - 3: Bending Moment value for Example 1

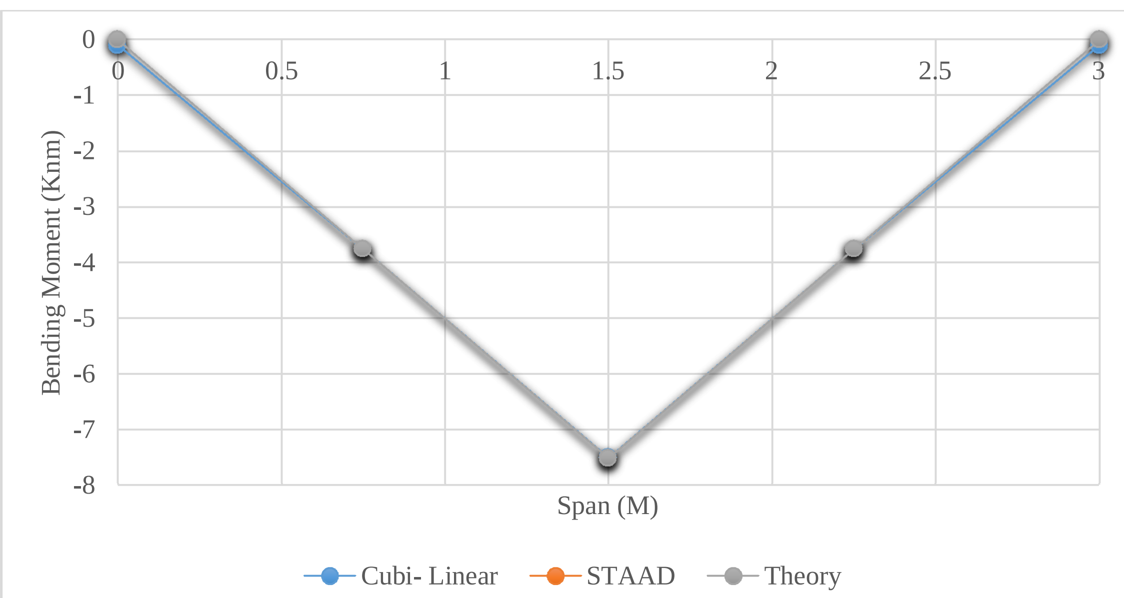

Figure - 8: Comparison of Bending Moment for Example 1

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2) Example – 2: Simply Supported beam with UDL

P (Load)	10 KN/m^2
L (Length)	3 m
B (Thickness)	0.10 m
D (Depth)	1.5 m
E (Modulus of Elasticity)	2*10^7 N/mm^2
μ (Poisson's Ratio)	0.3

Table - 4: Beam data for Example 2

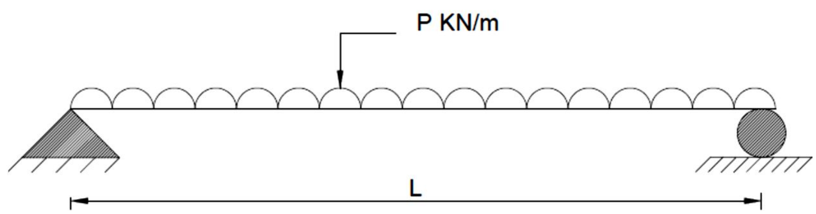

Figure - 9: Simply Supported Beam with UDL

Comparison of Deflection(mm) for Example - 2					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi- Linear	0.0000	0.0128	0.0186	0.0128	0.0000
STAAD	0.0000	0.0130	0.0190	0.0130	0.0000
Theory	0.0000	0.0133	0.0187	0.0133	0.0000

Table - 5: Deflection value for Example 2

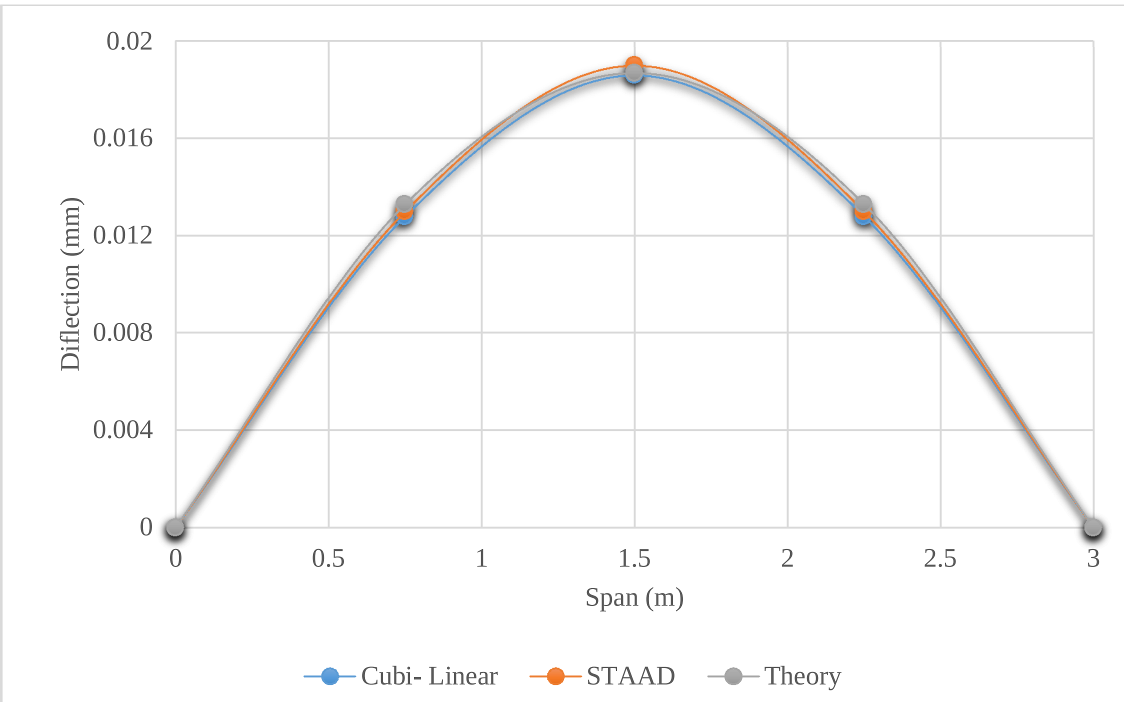

Figure - 10: Comparison of Deflection for Example 2

Comparison of Bending Moment(KNm) for Example - 2					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi- Linear	-0.2111	-8.400	-11.324	-8.400	-0.2111
STAAD	0.000	-8.430	-11.250	-8.430	0.000
Theory	0.000	-8.437	-11.250	-8.437	0.000

Table 6: Bending Moment value for Example 2

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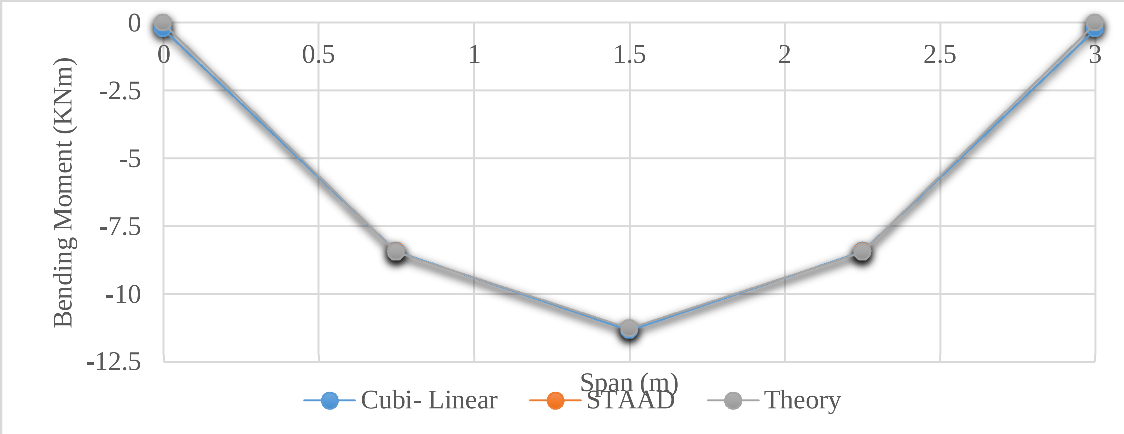

Figure - 11: Comparison of Bending Moment for Example 2

3) Example – 3: Simply Supported beam with UDL and Central Point Load

P (Load)	10 KN m^2, 10 KN
Location Of Point Load	1.5m
L (Length)	3 m
B (Thickness)	0.10 m
D (Depth)	1.5 m
E (Modulus of Elasticity)	2*10^7 N/mm^2
μ (Poisson's Ratio)	0.3

Table - 7: Beam data for Example 3

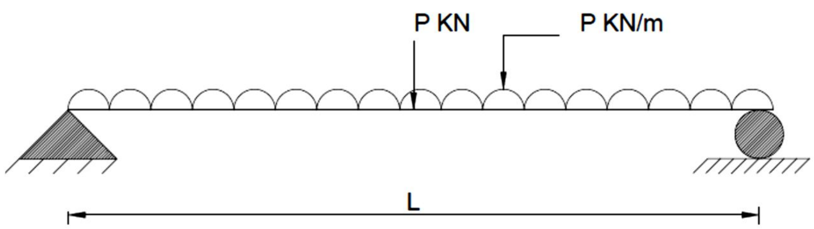

Figure - 12: Simply Supported Beam with UDL and Central Point Load

Comparison of Deflection(mm) for Example - 2					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi- Linear	0.0000	0.0198	0.0281	0.0198	0.0000
STAAD	0.0000	0.0200	0.0290	0.0200	0.0000
Theory	0.0000	0.0201	0.0287	0.0201	0.0000

Table - 8: Deflection value for Example 3

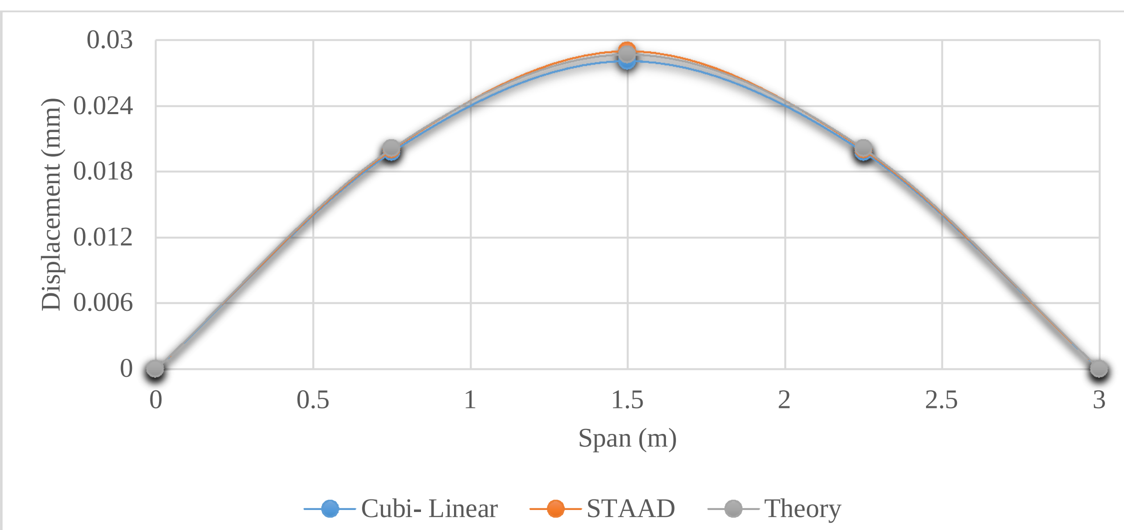

Figure - 13: Comparison of Deflection for Example 3

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Comparison of Bending Moment(KNm) for Example - 1					
Span	0m	0.75m	1.5m	2.25m	3m
Cubi- Linear	-0.281	-12.150	-18.822	-12.150	-0.281
STAAD	0.000	-12.180	-18.750	-12.180	0.000
Theory	0.000	-12.187	-18.750	-12.187	0.000

Table 9: Bending Moment value for Example 3

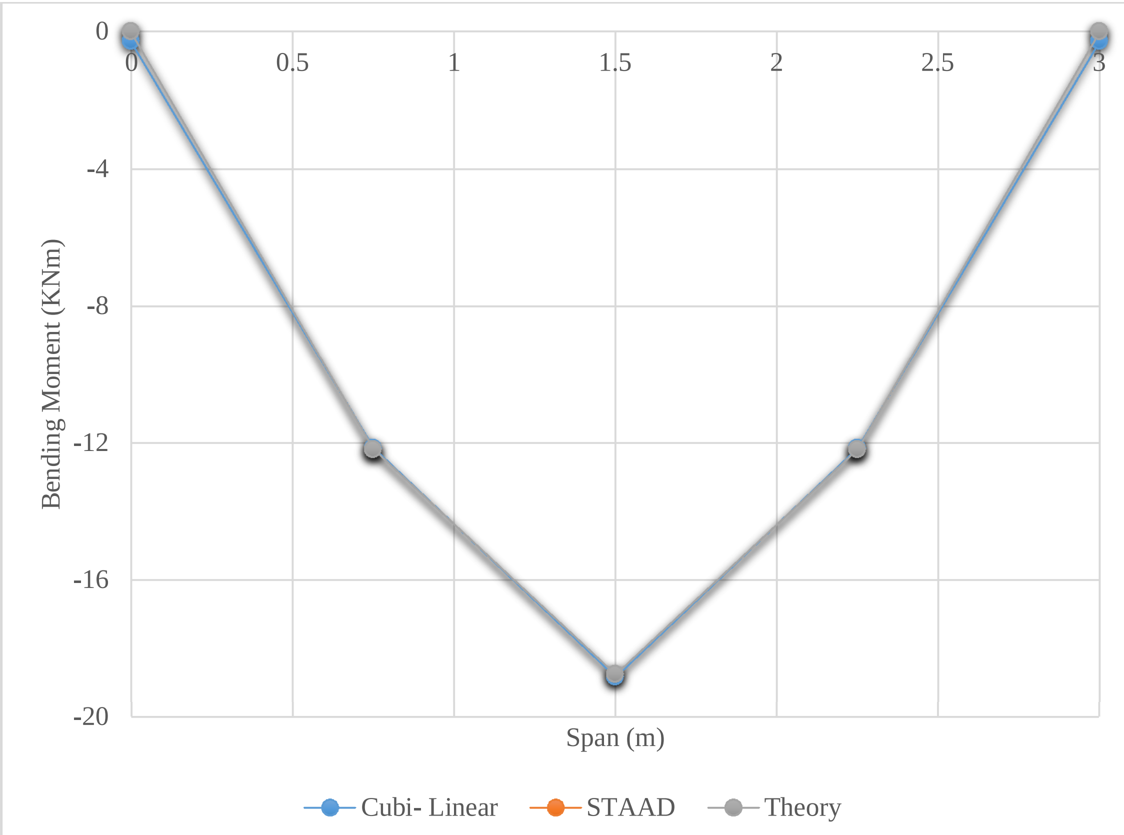

Figure - 14: Comparison of Bending Moment for Example 3

VIII. RESULTS & CONCLUSION

A. Conclusion

1) *Faster Convergence Rate:* For the analysis of shallow and deep beam, Cubi-Linear element converges faster than basic conforming quadrilateral element, which reflects in computation of primary unknown i.e. deflection and bending moment of any section. The variation of primary and secondary unknowns are negligible after two to three iterations. The variation of unknowns are linear in both axis in case of quadrilateral element where as in Cubi-Linear element it is cubical along x-axis and linear along y-axis.
2) *Method Of Analysis:* In general practice, small deflection of beams are neglected in many software which shows shear stresses are zero since they are based on Stiffness Member Approach method, sometimes these deflection could be critical for design purpose. While in developed program, no terms are neglected and meshing along depth of beam is adopted higher accuracy.

IX. ACKNOWLEDGMENT

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