International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177
Volume 7 Issue V, May 2019- Available at www.ijraset.com

Interacting Two Fluid Viscous Dark Energy Models in Scalar-Tensor Theory of Gravitation

V P Kadam
Dept of Math, Gopikabai Sitaram Gawande College , Umarkhed- 445206, India

Abstract: In this paper, we study a class of solution of Saez-Ballester scalar tensor theory with barotropic fluid and bulk viscous fluid for the new class of Bianchi model. We consider two cases of an interacting and non-interacting two fluid and obtained general results. The exact solution of the field equations are obtained for three different form of scale factor. The Physical and geometrical aspects of the models are discussed.

Keywords: Saez-Ballester theory, dark energy, two-fluid scenario.

I. INTRODUCTION

Recently there has been an immense interest in scalar – tensor theories of gravitation proposed by Brans and Dicke (1961), Nordvedt (1970), and Saez and Ballester (1986). Among them Brans-Dicke and Saez-Ballester scalar-tensor theories are considered to be viable alternatives to general relativity. Brans-Dicke theory includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez-Ballester scalar – tensor theory metric is coupled with a dimensionless scalar field in a simple manner. A brief review of Saez-Ballester theory is given by Reddy et al. [1]. The field equations of Saez-Ballester theory [2] for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^{n}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -KT_{ij} \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^{n}\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{1.2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress tensor of matter and comma and semicolon denote partial and covariant differentiation respectively.

Today, this is well established reality that expansion of Universe is accelerating. One of the observational foundations for the big bang model of cosmology was observed expansion of the universe. Measurement of the expansion rate is a critical part of the study, and it has been found that the expansion rate is very nearly "flat". That is, the universe is very close to the critical density, above which it would slow down and collapse inward toward a future "big crunch". One of the great challenges of astronomy and astrophysics is distance measurement over the vast distances of the universe. Since the 1990s it has become apparent that type Ia supernovae offer a unique opportunity for the consistent measurement of distance out to perhaps 1000 Mpc. Measurement at these great distances provided the first data to suggest that the expansion rate of the universe is actually accelerating. That acceleration implies an energy density that acts in opposition to gravity which would cause the expansion to accelerate. In 1998 and early in1999, High-z Supernova Search teams observed an accelerating expansion of the universe (Riess et al.[3];Permutter et al [4]). The current universe is not only expanding but also accelerating ,is confirmed by various results including Permutter et al. [5,6]; Riess et al.[7]; Garnavich et al.[8]; Schmidt et al.[9]; Tonry et al.[10]; Clocchiatti et al.[11]; fluctuation of cosmic microwave background radiation de Bernardis et al [12]; Large scale structure Spergel et al [13]; Tegmark. et al. [14]; Seljak et al.[15]; Adelman-McCarthy et al [16]; Wilkinson Microwave anisotropy probe (WMAP) Bennett. et al.[17] . Dark energy is the most accepted theory to explain observations since the 1990s that indicate that the universe is expanding at an accelerating rate. In the standard model of cosmology, dark energy currently accounts for 73% of the total mass-energy of the universe, (P.J.E.Peebles [18]). The dark energy can be characterized by the equation of state (EoS) parameter ω_D , defined by $\omega_D = \frac{p_D}{\rho_D}$ which is negative for DE, where ρ_D and p_D are energy density and fluid pressure respectively (Carroll and Hoffman [19]). The simplest candidate for the dark energy is a cosmological constant.

The introduction of viscosity into cosmology has been investigated from different view points (Gron [20], Padmanabhan & Chitre [21]: Barrow [22]; Zimdahi [23],Farzin et al [24]). Misner [25,26] noted that the "measurement of the isotropy of the cosmic background radiation represents the most accurate observational datum in cosmology". An explanation of this isotropy was provided by showing that in large class of homogeneous but anisotropy universe, the anisotropy dies away rapidly. It was found that the most important mechanism in reducing the anisotropy is neutrino viscosity at temperatures just above 10^{10} K (when the universe was about 1 s old: cf. Zel'dovich and Novikov [27]). The astrophysical observations also indicates some evidences that cosmic media is not a perfect fluid (Jaffe et al.[28]) and the viscosity effect could be concerned in the evolution of the universe (Brevik & Gorbunova,[29];Brevik et al.[30];Cataldo et al.[31]).On the other hand, in the standard cosmological model, if the EoS parameter ω is less than -1 so called phantom, the universe shows the future finite time singularity called the Big Rip (Caldwell et al.[32]; Nojiri et al.[33].) or cosmic Doomsday. Several Mechanisms are proposed to prevent the future big rip, like by considering quantum effects terms in the action (Nojiri & Odintsov [34]; Elizalde et al.[35]), or by including viscosity effects for the universe evolution (Meng et al. [36]). A well known result of the FRW cosmological solutions, corresponding to Universes filled with perfect fluid and bulk viscous stresses, is the possibility of violating dominant energy condition (Barrow [37,38]; Folomeev & Gurovich [39]; Ren & Meng [40]; Nojiri & Odintsov [41]). Setare [42-44] and Setare and Saridakis [45] have studied the interacting models of dark energy in different context. Interacting new agegraphic viscous dark energy with varying G has been studied by Sheykhi and Setare [46].

In this paper ,we study the evolution of dark energy parameter within frame work of Bianchi Type $-III, V, VI_0$ and VI_h cosmological models filled with two fluid Viscous dark Energy. Also, we consider both interacting and non- interacting cases.

II. THE METRIC AND FIELD EQUATIONS

We consider the diagonal form of the metric of general class of Bianchi cosmological models as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2x} dy^2 + C^2 e^{-2mx} dz^2 \qquad (2.1)$$

Where A, B, C are functions of t only. The metric (2.1) corresponds to a Bianchi type III model for $m = 0$, Bianchi type IV model for $m = 1$ & Bianchi type VI_0 model for $m = -1$.

The energy momentum tensor for a two fluid source is given by

$$T_i^{\,j} = (\rho + \bar{p}) u_i u^j - \bar{p}\delta_i^{\,j} \qquad (2.2)$$

Where T_j^i represent two fluid energy momentum tensor of bulk viscous dark energy and is barotropic fluid, together with

$$u^i u_i = -1 \qquad (2.3)$$

and

$$\bar{p} = p - \xi u^i_{;i} \qquad (2.4)$$

Where ρ is the energy density; p is the pressure, ξ is the bulk-viscous coefficient, and u^i is the four-velocity vector of the distribution. Here after the semicolon denotes covariant differentiation.

The universe field with bulk viscous fluid, from equation (2.2) one find

$$T_1^1 = T_2^2 = T_3^3 = -\bar{p}, T_4^4 = \rho \text{ and } T = \rho - 3\bar{p} \qquad (2.5)$$

The average scale factor 'a' of Bianchi type model defined by equation (2.1) is defined as

$$a = (ABC)^{\frac{1}{3}} \qquad (2.6)$$

The spatial volume V is given by

$$V = a^3 = (ABC) \qquad (2.7)$$

The expansion factor θ is defined by $\theta = u^i_{;i}$. Hence equation (2.3) leads to

$$\bar{p} = p - 3\xi H \tag{2.8}$$

Where H is Hubble's constant defined by

$$H = \frac{a_4}{a} \tag{2.9}$$

The Bianchi identity for the bulk viscous fluid distribution $G^i_{i;j} = 0$ leads to

$T^i_{j;i} = 0$ which yields to

$$\rho u^i + (\rho + \bar{p})u^i_{;i} = 0 \tag{2.10}$$

Which leads to

$$\rho_4 + 3H(\rho + \bar{p}) = 0 \tag{2.11}$$

The set of field equations for metric (2..1) in Seaz-Ballester theory of gravitations are

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{m}{A^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -K\bar{p} \tag{2.12}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4}{A}\frac{C_4}{C} - \frac{m^2}{A^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -K\bar{p} \tag{2.13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4}{A}\frac{B_4}{B} - \frac{1}{A^2} - \frac{\omega}{2}\phi^n {\phi_4}^2 = -K\bar{p} \tag{2.14}$$

$$\frac{A_4}{A}\frac{B_4}{B} + \frac{A_4}{A}\frac{C_4}{C} + \frac{B_4}{B}\frac{C_4}{C} - \frac{(1+m+m^2)}{A^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 = K\rho \tag{2.15}$$

$$(1+m)\frac{A_4}{A} - \frac{B_4}{B} - \frac{mC_4}{C} = 0 \tag{2.16}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{n}{2}\frac{{\phi_4}^2}{\phi} = 0 \tag{2.17}$$

$$\rho_4 + (\rho + \bar{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{2.18}$$

Where suffices denotes differentiation with respect to t.

Above field equations in the form of $H, \sigma \,\&\, q$ can be written as

$$K\bar{p} = H^2(2q-1) - \sigma^2 + \frac{(1+m+m^2)}{3A^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 \tag{2.19}$$

$$K\rho = 3H^2 - \sigma^2 - \frac{(1+m+m^2)}{A^2} + \frac{\omega}{2}\phi^n {\phi_4}^2 \tag{2.20}$$

Where $\bar{p} = p_m + \bar{p}_D$ and $\rho = \rho_m + \rho_D$. Here p_m and ρ_m are pressure and energy density of barotropic fluid and p_D and ρ_D are pressure and energy density of dark fluid respectively.

The equations of state (EoS) for the barotropic fluid ω_m and dark fluid ω_D are given by

$$\omega_m = \frac{p_m}{\rho_m} \tag{2.21}$$

&

$$\omega_D = \frac{\bar{p}_D}{\rho_D} \tag{2.22}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177
Volume 7 Issue V, May 2019- Available at www.ijraset.com

Respectively.

Let us introduce the dynamical scalars such as expansion parameter θ , shear scalars σ^2 and the mean anisotropy parameter A_m as usual

$$\theta = u^i_{;i} = 3H \tag{2.23}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left\{\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right\} - \frac{\theta^2}{6} \tag{2.24}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{2.25}$$

Where $$H = (\ln a)^* = \frac{a_4}{a} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{2.26}$$

For the general class of Bianchi metric equation (2.1), these dynamical scalars have the forms

$$\theta = (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = 3H \tag{2.27}$$

III. SOLUTIONS

Recently, Akarsu and Dereli [47] proposed a special law for the deceleration parameter which is linear in time with a negative slope. The linearly varying deceleration parameter q defined as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}(\frac{1}{H}) - 1 = -kt + s - 1 \tag{3.1}$$

Where k & $s \geq$ are positive constants.

Above equation gives the law of variation for average scale factor a as

$$a = (slt + b)^{\frac{1}{s}}, \quad k = 0, s > 0 \tag{3.2}$$

$$a = de^{lt} \quad , \quad k = 0, s = 0 \tag{3.3}$$

$$a = fe^{\frac{2}{s}\tanh^{-1}\left(\frac{kt}{s} - 1\right)} \quad k > 0, s > 1 \tag{3.4}$$

Where b, d and f are constants of integration.

A. Case I

when $k = 0, s > 0$ and $C = V^\alpha$ where α is any constant number then from equation (2.7), (2.16) and (3.2), we get

$$A = (slt + b)^{\frac{3 - 3\alpha + 3m\alpha}{s(m+2)}}, \tag{3.5}$$

$$B = (slt + b)^{\frac{3 + 3m - 3\alpha - 6m\alpha}{s(m+2)}} \tag{3.6}$$

$$C = (slt + b)^{\frac{3\alpha}{s}} \tag{3.7}$$

$$\phi = \left\{\left(\frac{n+2}{2(s-3)l}\right)(slt + b)^{\frac{s-3}{s}}\right\}^{\frac{2}{n+2}} \tag{3.8}$$

The directional Hubble parameter H_1, H_2 and H_3 having the values

$$H_1 = \left\{\frac{l(3-3\alpha+3m\alpha)}{(m+2)}\right\}\frac{1}{(slt+b)}$$

$$H_2 = \left\{\frac{l(3+3m-3\alpha-6m\alpha)}{(m+2)}\right\}\frac{1}{(slt+b)}$$

$$H_3 = \left\{\frac{3l\alpha)}{(slt+b)}\right\}$$

From equation (2.27) the average generalized parameter H is

$$H = \frac{l}{(slt+b)} \tag{3.9}$$

Also from the equations (2.28) and (3.1) we get

$$\sigma^2 = \frac{X_1}{2(m+2)^2(slt+b)^2} \tag{3.10}$$

Where $X_1 = l^2\left\{(3+3m\alpha-3\alpha)^2+(3+3m-6m\alpha-3\alpha)^2+(9\alpha^2-3)(m+2)^2\right\}$

From the above equations (3.9) and (3.10) it is observed that the Hubble parameter and Shear scalar are the decreasing function of the time t. also it is diverge at t_0 where

$$t_0 = \frac{-b}{sl}$$

$$q = s-1 \tag{3.11}$$

For $s>1, q>0$; therefore the model represents a decelerating model of the universe. For $s \le 1$, we get $-1<q\le 0$, which implies an accelerating model of the universe. Also recent observations of type Ia supernovae (Perlmutter et al. [4-6]; Reiss et al. [(2,7)]; Tonry et al.[10] ; Knop et al. [48]) reveal that the present universe is accelerating and the value of decelerating parameter lies somewhere in the range $-1<q\le 0$. It follows that the solutions obtained in this model are consistent with the observations.

In the following sections we deals with two cases i) non interacting model and ii) interacting model

1) *For Non-Interacting Two Fluid Model:* In this section we assume that two fluids do not interact. Therefore, the general form of conservation equation (2.18) leads us for barotropic and dark fluid separately as

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right) = 0 \tag{3.12}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right) = 0 \tag{3.13}$$

Here is , of course, a structural difference between equations (3.12) and (3,13), because equation (3.12) is in the form of ω_m which is constant and hence equation (3.12) is integrable . But equation (3.13) is a function of ω_D. Accordingly, p_D and ρ_D are also function of ω_D. Therefore, we can not integrate equation (3.13) as it is a function ω_D which is an unknown time dependent parameter. Integrate of equation (3.12) and using equation (2.21), we get density and pressure of barotropic fluid as

$$\rho_m = \rho_0(slt+b)^{\frac{-3(1+\omega_m)}{s}} \tag{3.14}$$

$$p_m = \rho_0\omega_m(slt+b)^{\frac{-3(1+\omega_m)}{s}} \tag{3.15}$$

Where ρ_0 be the constant of integration.

From equations (2.19) and (2.20), we have obtained effective pressure and energy density as

$$K\bar{p} = \frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 \tag{3.16}$$

$$K\rho = \frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 \tag{3.17}$$

Also we obtained pressure and density for dark fluid as

$$K\rho_D = \left\{ \frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - \rho_0 K(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 \right\} \tag{3.18}$$

$$K\bar{p}_D = \left\{ \frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_0\omega_m(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 \right\} \tag{3.19}$$

The equation of state (E0S) parameter for dark fluid is

$$\omega_D = \frac{\left\{ \frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_0\omega_m(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 \right\}}{\left\{ \frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - \rho_0 K(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 \right\}} \tag{3.20}$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$\omega_D^{eff} = \frac{\left\{\frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_0\omega_m(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 - 3\xi K\frac{l}{(slt+b)}\right\}}{\left\{\frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - \rho_0 K(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\}} \tag{3.21}$$

The expressions for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{\rho_0}{3l^2}(slt+b)^{2-\frac{3(1+\omega_m)}{s}} \tag{3.22}$$

$$\Omega_D = \frac{1}{3l^2}\left\{3l^2 - \frac{X_1}{2(m+2)^2} - \frac{(1+m+m^2)}{(slt+b)^{\frac{(6+6m\alpha-6\alpha)}{s(m+2)}-2}} - \rho_0(slt+b)^{2-\frac{3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2(slt+b)^2\right\} \tag{3.23}$$

2) *For Interacting Two Fluid Model:* In this section we consider the interaction between dark viscous and barotropic fluids. For this purpose we can write the continuity equations as

$$(\rho_m)_4 + (\rho_m + p_m)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = Q \tag{3.24}$$

$$(\rho_D)_4 + (\rho_D + p_D)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -Q \tag{3.25}$$

Where the quantity Q expresses the interaction between the fluids. Since we are interested in an energy transfer from the dark energy to dark matter, we consider Q>0 this ensures that the second law of thermodynamics is fulfilling. Here we emphasize that the continuity equations (3.24) and (3.25) imply that the interaction term Q should be proportional to a quantity with units of universe of time i.e. $Q\alpha 1/t$. Therefore, a first and natural candidate can be the Hubble factor H multiplied with the energy density. Following Amendola et al. [49], Gou et al. [50] And Hassan Amirhashchi et al. [51], we consider

$$Q = 3H\sigma\rho_m \tag{3.26}$$

Where σ is coupling constant.

Solving equation (3.24) with the help of equation (3.26), we get

$$\rho_m = \rho_{00}(slt+b)^{\frac{-3(1+\omega_m-\sigma_C)}{s}} \tag{3.27}$$

$$p_m = \rho_{00}(slt+b)^{\frac{-3(1+\omega_m-\sigma_C)}{s}} \tag{3.28}$$

Where ρ_{00} be the constant of integration.

The density for dark fluid as

$$\rho_D = \frac{1}{K}\left\{\frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}\omega_m(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\} \quad (3.29)$$

The effective pressure as

$$\overline{p}_D = \frac{1}{K}\left\{\frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}\omega_m(slt+b)^{\frac{-3(1+\omega_m-\sigma_C)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\} \quad (3.30)$$

The equation of state (EoS) parameter for dark fluid is

$$\omega_D = \frac{\left\{\frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}\omega_m(slt+b)^{\frac{-3(1+\omega_m-\sigma_C)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\}}{\left\{\frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\}} \quad (3.31)$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$\omega_D^{eff} = \frac{\left\{\frac{l^2(2q-1)}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} + \frac{(m^2+m+1)}{3(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}\omega_m(slt+b)^{\frac{-3(1+\omega_m-\sigma_C)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2 - \frac{3l\xi}{(slt+b)}\right\}}{\left\{\frac{3l^2}{(slt+b)^2} - \frac{X_1}{2(m+2)^2(slt+b)^2} - \frac{(m^2+m+1)}{(slt+b)^{\left(\frac{6+6m\alpha-6\alpha}{s(m+2)}\right)}} - K\rho_{00}(slt+b)^{\frac{-3(1+\omega_m)}{s}} + \frac{\omega}{2}\phi^n\phi_4^2\right\}} \quad (3.32)$$

The physical quantities i.e. Average scale factor ad spatial volume, expansion scalar and Anisotropic parameter are

a) Average scale factor $=(a)=(slt+b)^{\frac{1}{s}}$

b) Spatial volume $(V)=(slt+b)^{\frac{3}{s}}$

c) Expansion scalar $(\theta) = \dfrac{3l}{(slt+b)}$

d) Anisotropic parameter $(A) = constant$

It is observed that the spatial volume is zero at $t = \dfrac{-b}{sl} = t_0$ and expansion scalar is finite, which shows that the universe starts evolving with zero volume at $t = t_0$

With an infinite rate of expansion. The scale factors also vanish at $t = t_0$ and hence the model has a point singularity at the initial epoch. Hubble's parameter and shear scalar diverge at the initial singularity. The anisotropy parameter is constant. As t increases the scale factors and spatial volume increase but the expansion scalar and shear scalar decreases. Thus the rate of expansion slows down with increases in time.

B. CASE II

1) In this case we find value of metric potentials for $k = 0, s = 0$ and $C = V^{\alpha}$, then from equations (2.6), (2.16) and (3.3), we get

$$A = d^{\frac{3+3m\alpha-3\alpha}{m+2}} e^{\frac{(3+3m\alpha-3\alpha)lt}{(m+2)}} \tag{3.33}$$

$$B = d^{\frac{3+3m-6m\alpha-3\alpha}{m+2}} e^{\frac{(3+3m-6m\alpha-3\alpha)lt}{(m+2)}} \tag{3.34}$$

$$C = d^{3\alpha} e^{3\alpha lt} \tag{3.35}$$

$$\phi = \left\{ -\left(\frac{n+2}{6ld^3} \right) e^{-3lt} \right\}^{\frac{2}{n+2}} \tag{3.36}$$

The directional Hubble parameter H_1, H_2 and H_3 having the values

$$H_1 = \left\{ \frac{l(3-3\alpha+3m\alpha)}{(m+2)} \right\}$$

$$H_2 = \left\{ \frac{l(3+3m-3\alpha-6m\alpha)}{(m+2)} \right\}$$

$$H_3 = 3l\alpha$$

From equation (2.26) the average generalized Hubble parameter is

$$H = l \tag{3.37}$$

Also from equations (2.28) and (3.1) we get

$$\sigma^2 = \frac{l^2}{2} \left\{ \frac{(3-3\alpha+3m\alpha)^2 + (3+3m-3\alpha-6m\alpha)^2}{(m+2)^2} \right\} + \frac{3l^2(3\alpha^2-1)}{2} \tag{3.38}$$

From above equation (3.37) and (3.38), it is observed that the Hubble parameter and shear scalar are constant.

$$q = -1 \tag{3.39}$$

For $s = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $\dfrac{dH}{dt} = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore the solutions presented in this model are consistent with the observations.

In the following section we deals with two cases i) non interacting model and ii) interacting model

2) *For Non-Interacting Two Fluid Model:* Proceeding just as in section 3 case-I (i) we again obtain the expression for the density and pressure of barotropic fluid

$$\rho_m = \rho_0 (de^{lt})^{-3(1+\omega_m)} \tag{3.40}$$

$$p_m = \rho_0 \omega_m (de^{lt})^{-3(1+\omega_m)} \tag{3.41}$$

Where ρ_0 be the constant of integration.

From equation (2.19) and (2.20), we get energy density and effective pressure as

$$K\rho = X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 \tag{3.42}$$

$$K\bar{p} = X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 \tag{3.43}$$

Where $$X_2 = 3l^2 - \frac{l^2}{2}\left\{\frac{(3+3m\alpha-3\alpha)^2 + (3+3m-6m\alpha-3\alpha)^2}{(m+2)^2}\right\} + \frac{3l^2(3\alpha^2-1)}{2}$$

$$X_3 = l^2(2q-1) - \frac{l^2}{2}\left\{\frac{(3+3m\alpha-3\alpha)^2 + (3+3m-6m\alpha-3\alpha)^2}{(m+2)^2}\right\} + \frac{3l^2(3\alpha^2-1)}{2}$$

The expression for density and pressure for dark fluid as

$$K\rho_D = X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0 (de^{lt})^{-3(1+\omega_m)} \tag{3.44}$$

$$K\bar{p}_D = X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0\omega_m (de^{lt})^{-3(1+\omega_m)} \tag{3.45}$$

The equation of state (EoS) parameter for viscous dark fluid is

$$w_D = \frac{\left\{X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0\omega_m (de^{lt})^{-3(1+\omega_m)}\right\}}{\left\{X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0 (de^{lt})^{-3(1+\omega_m)}\right\}} \tag{3.46}$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0\omega_m (de^{lt})^{-3(1+\omega_m)} - 3K\xi l\right\}}{\left\{X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0 (de^{lt})^{-3(1+\omega_m)}\right\}} \tag{3.47}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.177
Volume 7 Issue V, May 2019- Available at www.ijraset.com

The expression for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{\rho_0}{3l^2}(de^{lt})^{-3(1+\omega_m)} \tag{3.48}$$

$$\Omega_D = \frac{1}{3l^2 K}\left\{X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\rho_0 (de^{lt})^{-3(1+\omega_m)}\right\} \tag{3.49}$$

3) *For Interacting Two Fluid Model:* Proceeding just as in section 3 Case-I (ii), we again obtain the expression for the density and pressure of barotropic fluid

$$\rho_m = \delta_1 e^{-3(1+\omega_m-\sigma_C)lt} \tag{3.50}$$

$$p_m = \delta_1 \omega_m e^{-3(1+\omega_m-\sigma_C)lt} \tag{3.51}$$

Where $\delta_1 = \rho_{00} d^{-3(1+\omega_m-\sigma_C)}$.

The expression for density and pressure for dark fluid as

$$K\rho_D = X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 e^{-3(1+\omega_m-\sigma_C)lt} \tag{3.52}$$

$$K\bar{p}_D = X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 \omega_m e^{-3(1+\omega_m-\sigma_C)lt} \tag{3.53}$$

The equation of state (EoS) parameter for viscous dark fluid is

$$w_D = \frac{\left\{X_3 + \frac{(1+m+m^2)}{3d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 \omega_m e^{-3(1+\omega_m-\sigma_C)lt}\right\}}{\left\{X_2 - \frac{(1+m+m^2)}{d^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 e^{-3(1+\omega_m-\sigma_C)lt}\right\}} \tag{3.54}$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{X_3 + \frac{(1+m+m^2)}{3d^{\frac{(6+6m\alpha-6\alpha)}{(m+2)}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 \omega_m e^{-3(1+\omega_m-\sigma_C)lt} - 3K\xi l\right\}}{\left\{X_2 - \frac{(1+m+m^2)}{d^{\frac{(6+6m\alpha-6\alpha)}{(m+2)}} e^{\frac{(6+6m\alpha-6\alpha)lt}{(m+2)}}} + \frac{\omega}{2}\phi^n \phi_4^2 - K\delta_1 e^{-3(1+\omega_m-\sigma_C)lt}\right\}} \tag{3.55}$$

The physical quantities i.e. Average scale factor ad spatial volume, expansion scalar and Anisotropic parameter are

a) Average scale factor $= (a) = de^{lt}$

b) Spatial volume $(V) = (de^{lt})^3$

c) Expansion scalar $(\theta) = 3l$

d) Anisotropic parameter $(A) = cons\tan t$

The model has no initial singularity. The special volume, scale factors for t=0 and the other cosmological parameters are constants. Thus the universe starts evolving with a constant volume and expands with exponential rate. As t increases, the scale factors and the spatial volume increases become infinitely large. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model.

C. CASE III

F In this case we find value of metric potentials for $k>0, s=3$ and $C=V^{\alpha}$ [63] where α be the any constant number, then from equations (2.7), (2.16) and (3.4), we get

$$A = f^{\frac{(3+3m\alpha-3\alpha)}{(m+2)}} e^{\frac{2(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)} \tag{3.56}$$

$$B = f^{\frac{(3+3m-6m\alpha-3\alpha)}{(m+2)}} e^{\frac{2(3+3m-6m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)} \tag{3.57}$$

$$C = f^{3\alpha} e^{2\alpha\tanh^{-1}\left(\frac{kt}{3}-1\right)} \tag{3.58}$$

$$\phi = \left\{\frac{(n+2)}{2f^3}\left[\frac{6}{k}\log t - t\right]\right\}^{\frac{2}{n+2}} \tag{3.59}$$

The directional Hubble parameter H_1, H_2 and H_3 having the values

$$H_1 = \left\{\frac{2(3-3\alpha+3m\alpha)}{t(m+2)(6-kt)}\right\}$$

$$H_2 = \left\{\frac{2(3+3m-3\alpha-6m\alpha)}{t(m+2)(6-kt)}\right\}$$

$$H_3 = \frac{6\alpha}{t(6-kt)}$$

From equation (2.26) the average generalized Hubble parameter is

$$H = \frac{2}{t(6-kt)} \tag{3.60}$$

Also from equations (2.28) and (3.1) we get

$$\sigma^2 = 12\left\{\frac{1+m+m^2-6\alpha-6m\alpha+9m\alpha^2-6m^2\alpha+9m^2\alpha^2+9\alpha^2}{t^2(m+2)^2(6-kt)^2}\right\} \tag{3.61}$$

From above equation (3.57) and (3.58), it is observed that the Hubble parameter and shear scalar are constant.

$$q = 2-kt \tag{3.62}$$

For $kt<2$, we get $q>0$; therefore the model represents a decelerating model of universe. For $kt\geq 2$, we get $q\leq 0$; therefore the model represents a accelerating model of universe.

In the following section we deals with two cases i) non interacting model and ii) interacting model

1) *For Non-Interacting Two Fluid Model:* Proceeding just as in section 3 case-I (i) we again obtain the expression for the density and pressure of barotropic fluid

$$\rho_m = \lambda_1 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} \tag{3.63}$$

$$p_m = \lambda_1\omega_m e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} \tag{3.64}$$

Where $\lambda_1 = \rho_0 f^{-3(1+\omega_m)}$.

From equation (2.19) and (2.20), we get effective pressure and energy density as

$$K\bar{p} = X_5 + \frac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 \quad (3.65)$$

$$K\rho = X_4 - \frac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 \quad (3.66)$$

Where $X_4 = 12\left\{\frac{(3+3m+6\alpha+6m\alpha-9m\alpha^2-6m^2\alpha)+9m^2\alpha^2+9\alpha^2}{t^2(m+2)^2(6-kt)^2}\right\}$

$$X_5 = \frac{4(2q-1)}{t^2(2s-kt)^2} - 12\left[\frac{(1+m+m^2-6\alpha-6m\alpha+9m\alpha^2-6m^2\alpha)+9m^2\alpha^2+9\alpha^2}{t^2(m+2)^2(6-kt)^2}\right]$$

The expression for density and pressure for dark fluid as

$$K\rho_D = X_4 - \frac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} \quad (3.67)$$

$$K\bar{p}_D = X_5 + \frac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1\omega_m e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} \quad (3.68)$$

The equation of state (EoS) parameter for viscous dark fluid is

$$w_D = \frac{\left\{X_5 + \frac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1\omega_m e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)}\right\}}{\left\{X_4 - \frac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)}\right\}} \quad (3.69)$$

The effective equation of state (EoS) parameter for viscous dark energy is

$$w_D^{eff} = \frac{\left\{X_5 + \frac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1\omega_m e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} - \frac{6K\xi}{t(6-kt)}\right\}}{\left\{X_4 - \frac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\lambda_1 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)}\right\}} \quad (3.70)$$

The expression for the matter energy density Ω_m and dark energy density Ω_D are given by

$$\Omega_m = \frac{\lambda_1 t^2 (6-kt)^2 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)}}{12} \tag{3.71}$$

$$\Omega_D = \frac{t^2(2s-kt)^2}{12K}\left\{\begin{matrix} X_4 - \dfrac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}\left(\frac{kt}{3}-1\right)}} + \dfrac{\omega}{2}\phi^n\phi_4^2 \\ -K\lambda_1 e^{-2(1+\omega_m)\tanh^{-1}\left(\frac{kt}{3}-1\right)} \end{matrix}\right\} \tag{3.72}$$

2) *For Interacting Two Fluid Model:* Proceeding just as in section 3 Case-I (ii), we again obtain the expression for the density and pressure of barotropic fluid

$$\rho_m = \delta_2 e^{-2(1+\omega_m-\sigma_C)\tanh h^{-1}(\frac{kt}{3}-1)} \tag{3.73}$$

$$p_m = \delta_2\omega_m e^{-2(1+\omega_m-\sigma_C)\tanh h^{-1}(\frac{kt}{3}-1)} \tag{3.74}$$

Where $\delta_2 = \rho_{00} f^{-3(1+\omega_m-\sigma_C)}$.

The expression for density and pressure for dark fluid as

$$K\rho_D = X_4 - \frac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{(m+2)}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\delta_2 e^{-2(1+\omega_m-\sigma_C)\tanh^{-1}(\frac{kt}{3}-1)} \tag{3.75}$$

$$K\bar{p}_D = X_5 + \frac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \frac{\omega}{2}\phi^n\phi_4^2 - K\delta_2\omega_m e^{-2(1+\omega_m-\sigma_C)\tanh^{-1}(\frac{kt}{3}-1)} \tag{3.76}$$

The equation of state (EoS) parameter for viscous dark fluid is

$$w_D = \frac{\left\{X_5 + \dfrac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \dfrac{\omega}{2}\phi^n\phi_4^2 - K\delta_2\omega_m e^{-2(1+\omega_m-\sigma_C)\tanh^{-1}(\frac{kt}{3}-1)}\right\}}{\left\{X_4 - \dfrac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{(m+2)}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \dfrac{\omega}{2}\phi^n\phi_4^2 - K\delta_2 e^{-2(1+\omega_m-\sigma_C)\tanh^{-1}(\frac{kt}{3}-1)}\right\}} \tag{3.77}$$

The effective equation of state (EoS) parameter for viscous dark energy

$$w_D^{eff} = \frac{\left\{\begin{matrix} X_5 + \dfrac{(1+m+m^2)}{3f^{\frac{6+6m\alpha-6\alpha}{m+2}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \dfrac{\omega}{2}\phi^n\phi_4^2 - K\delta_2\omega_m e^{-2(1+\omega_m-\sigma_C)\tanh^{-1}(\frac{kt}{3}-1)} \\ -\dfrac{6K\xi}{t(6-kt)} \end{matrix}\right\}}{\left\{X_4 - \dfrac{(1+m+m^2)}{f^{\frac{6+6m\alpha-6\alpha}{(m+2)}} e^{\frac{4(3+3m\alpha-3\alpha)}{3(m+2)}\tanh^{-1}(\frac{kt}{3}-1)}} + \dfrac{\omega}{2}\phi^n\phi_4^2 - K\delta_2 e^{\frac{-6(1+\omega_m-\sigma_C)}{s}\tanh^{-1}(\frac{Kt}{s}-1)}\right\}} \tag{3.78}$$

The physical quantities i.e. Average scale factor ad spatial volume, expansion scalar and Anisotropic parameter are

a) Average scale factor $=(a)= fe^{\frac{2}{3}\tanh^{-1}\left(\frac{kt}{3}-1\right)}$

b) Spatial volume $(V)=\left\{ fe^{\frac{2}{3}\tanh^{-1}\left(\frac{kt}{3}-1\right)} \right\}^3$

c) Expansion scalar $(\theta)=\dfrac{6}{t(6-kt)}$

d) Anisotropic parameter $(A_m)= cons\tan t$

The model has no initial singularity. The special volume, scale factors for t=0 and the other cosmological parameters are constants. Thus the universe starts evolving with a constant volume and expands with exponential rate. As t increases, the scale factors and the spatial volume increases become infinitely large. It is interesting to note that the expansion scalar is constant throughout the evolution of universe and therefore the universe exhibits uniform exponential expansion in this model.

IV. CONCLUSION

we study a class of solution of Saez-Ballester scalar-tensor theory with barotropic fluid and bulk viscous fluid for the new class of Bianchi model. The law of generalized linearly varying deceleration parameter defined by (3.1) for Bianchi type- III , Bianchi type- V , Bianchi type-VI_0 , Bianchi type-VI_h space time Corresponding to $m=0, m=1, m=-1$ and all other values of m respectively, gives three types of cosmologies, i) first for $k=0$ and $s>0$ gives solution for positive value of deceleration parameter , therefore the models represents a decelerating models. ii) for $k=0$ and $s=0$ gives solution for negative value of deceleration parameter , therefore the models represents a accelerating model of universe. iii) for $k>0$ and $s=3$, for $kt<2$, the model represents a decelerating model of universe. For $kt \geq 2$, the model represents a accelerating model of universe. In all three cases, it has also been observed that the ratio $\dfrac{\theta}{\sigma}$ becomes constant at $t \to \infty$. Hence the model approaches anisotropically and matter is dynamically negligible near the origin; this result match with the result already given by Collins [52].

REFERENCES

[1] Reddy D.R.K. ,Rao,M.V.S.,Rao G.K.:2006, Astrophys. Space Sci.306,171.
[2] Saez,D.,Ballester,V.J., (1986),Phys. Lett,113,467.
[3] Riess, A.G., et al.,1998, Astronomical Journal,116, 1009.
[4] Perlmutter,S.,et al., 1999,Astrophysical Journal,517,5
[5] Perlmutter,S.,et al., 1997,Astrophysical Journal,483,565.
[6] Perlmutter,S.,et al., 1998,Nature,391,51.
[7] Riess, A.G., et al., ,2000, Publications of the Astronomical Society of the Pacific,112, 1284.
[8] Garnavich,P.M.,et al., 1998, Astrophysical Journal,493,L53.
[9] Schmidt,B.P.,et al.,1998, Astrophysical Journal,507,46.
[10] Tonry,J.L.,et al.,2003, Astrophysical Journal,594 ,1.
[11] Clocchiatti,A.,et al., 2006, Astrophysical Journal,642,1.
[12] De Bernardis,P.,et al.,1998,Nature,391,5.
[13] Spergel,D.N.,et al.,2000, Astrophysical Journal Supplement Series,148,175.
[14] Tegmark,M.,at al., 2004, Phys.Rev.D,69 103501.
[15] Seljak,U.,et al.,2005,Phys Rev.D, 71,103515.
[16] Adelman-McCarthy,J.K.,et al.,2006, Astrophysical Journal Supplement 162,38.
[17] Bennett,C.L., et al. ,2003, Astrophysical Journal Supplement 148,1.
[18] P.J.E.Peebles and Bharat Ratra,2003, Reviews of Modern Physics 75(2), 559arXiv:astr0- ph/0207347
[19] Carroll,S.M.,Hoffman,M.: , 2003,Phys.Rev.D,68,023509.
[20] Gron.O,,1990, Astrophysics and Space Science,173,191.
[21] Padmanabhan,T.,Chitre,S.M.,1987,Phys. Lett.A,120,433.
[22] Barrow,J.D.,1986, Physics Letters B,180,335.
[23] Zimdahl,W.,1996, Phys.Rev.D,53,53483.
[24] Farzin,A.,et al.,2012,RAA,12,26.
[25] Misner,C.W., 1966, Astrophysical Journal,151,431.

[26] Misner,C.W., 1967, Phys.Rev.Letters,19,533.
[27] Zel'dovich and Novikov ,1971,
[28] Jaffe,T.R.,et al.,2005, Astrophysical Journal,629,L1-L4.
[29] Brevik,I.,&Gorbunovac,O., 2005,General relativity & Gravitation,37,2039.
[30] Brevik,I.,Gorbunova.O.,& Shaido,Y.A., 2005,Int. Jou.Theo. Phys D,14,1899.
[31] Cataldo,M.,Cruz,N.,&Lepe,S., 2005,Physics Letters B,619,5.
[32] Caldwell,R.R.,Kamiionkowski,M.,& Weinberg,N.N., 2003, Physical Rev.Letters,91,07301.
[33] Nojiri,S.,& Odintsov,S.D., 2005, Phys.Rev. D,72,023003.
[34] Nojiri,S.,& Odintsov,S.D., 2004, Physics Letter B,595,1.
[35] Elizalde,E.,Nojiri,S.& Odintsov,S.D., 2004,Phys.Rev.D,70,0343539.
[36] Meng,X.H., Ren,J.,& Hu,M., 2007,Communication in thoer. Phys. 27,379.
[37] Barrow,J.D., 1987,Physics Letter B,180.335
[38] Barrow,J.D., 1988,Nuclear Letter B,310.743
[39] Folomeev,V.,& Gurovich,V., 2008,Physics Letters B,661,75.
[40] Ren,J.,& Meng,X.H. , 2006,Pysics Letters B,633,1.
[41] Nojiri,S.,& Odintsov,S.D.,& Tsujikawa,S., 2005, Phys.Rev. D,71,063004.
[42] Setare,M.R., 2007, Pysics Letters B,644,99.
[43] Setare,M.R., 2007,European Physical J. C,50,991.
[44] Setare,M.R, 2007, Pysics Letters B,654,1.
[45] Setare,M.R.& Saridakis,E.N., 2008, Pysics Letters B,668,177
[46] Sheykhi,A,&Setare,M.R., 2010,International Journal of Theor. Physics,49,2777.
[47] Akarsu,O.,Dereli,T., (2012), Int. J. Theor. Phys.51,612(2012)
[48] Knop, R.A., et al.:, 2003, Astrophys.J. 598,102.
[49] Amendola L.G.,Camargo campus and R. Rosenfeld.,2007, Phys.Rev.D 75,083506
[50] Guo, Z.-K., Cai, R.-G., Zhang, Y.-Z.: J. Cosmol. Astropart. Phys. 05, 002 (2005)
[51] Amirhashchi,H.,Pradhan,A.& Saha,B.:2011, arXiv:1011.3940v2.
[52] Collins, C.B., 1977, :J. Math.phys. 18,2116.