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Planar Serial type Medical Robot in Minimally Invasive Surgery

K. Swathi Vaidehi[1], Dr. G. Satish Babu[2]
[1, 2]Department of Mechanical Engineering, Jawaharlal Nehru Technological University, Hyderabad

Abstract: ***Minimally invasive surgery refers to an array of techniques that aim to minimize the incisions made in the body during a surgical operation. There has been a growing interest in the field owing to numerous benefits like faster recovery times of patient, lower blood loss, less painful post-surgical period etc. We present a framework incorporating an n-link robotic chain with an embedded sensor at the end effector which provides feedback to the surgeon and facilitates correction of digressions from intended path. A numerical validation has been presented which takes the piezometric signal from the end effector and provides real-time corrective signals to minimize tremors and digressions of end effector.***
Keywords: Planar serial type robot, Lagrangian Mechanics, Medical robot, Minimally Invasive Surgery, Minimally Invasive Procedures

I. INTRODUCTION

Surgery involves making incisions in the body to gain access to the interior organs. However, it causes immense pain, discomfort to the patient and takes a long time for the wounds to heal. There is also the risk of infections and complications if the wounds are not properly taken care of. “Minimally Invasive Surgery” (MIS), also known as “Minimally Invasive Procedure” is a term coined by John E.A. Wickham in 1984. MIS refers to the surgeries carried out using various techniques to reduce the incisions and wounds made to reach to the surgical site in the body.
In lieu of the above, there is a huge interest and necessity to come up with novel techniques which reduce or eliminate surgical incisions. Advancements in computational power, recent encouraging results in the field of Micro and Nano devices has breathed fresh life into this field as outlined in [1].
While performing a surgery. the surgeons rely heavily on haptic feedback and dexterity. In this work we propose a simple model of incorporating an *n-link* robotic chain with a sensor embedded in the end effector. The sensor feeds the workstation with the spatial location data, the piezometric response i.e. forces due to the tissue around the link. Effectively, the surgeon is thus provided with spatial awareness and haptic feedback without actually reaching the site of surgery.
The model we present incorporates a control system which gives a feedback of corrective motion to the end effector. We numerically validate the robustness of our model. A discussion on forward and inverse kinematics involved is included detailing how the desired motion of the end effector is achieved.

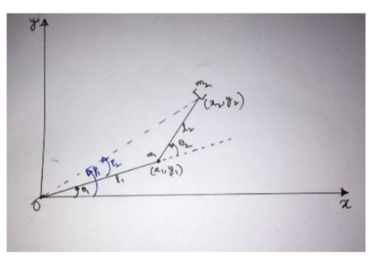

Fig. 1 Plant Schematic

II. PLANT DESCRIPTION AND DYNAMICS

In this section, we shall derive the governing equations of motion of our system. Then, Jacobian is found to obtain transformation from Task Space to Configuration Space and vice-versa.

A. Plant Description and Dynamics

Consider a two-link manipulator as shown in Fig. 1.
We shall use the Lagrangian Formulation [2,4] to obtain the equations of motion.

B. Governing Equations

We will derive the governing equations using Lagrangian Mechanics.
Friction and other dissipation mechanisms are ignored.
End-effector coordinates: (x_2, y_2)

Generalised Coordinates: $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$

1) Angular displacement (rotation about *z-axis*) is

$\theta_{z1} = \theta_1$

$\theta_{z2} = \theta_1 + \theta_2$

$x_1 = l_1 \cos(\theta_1)$

$y_1 = l_1 \sin(\theta_1)$

$x_2 = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$

$y_2 = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$

$\dot{x_1} = -l_1 \sin(\theta_1)\,\dot{\theta_1}$

$\dot{y_1} = l_1 \cos(\theta_1)\,\dot{\theta_1}$

$\dot{x_2} = -l_1 \sin(\theta_1)\,\dot{\theta_1} - l_2 \sin(\theta_1 + \theta_2)\,(\dot{\theta_1} + \dot{\theta_2})$

$\dot{y_2} = l_1 \cos(\theta_1)\,\dot{\theta_1} + l_2 \cos(\theta_1 + \theta_2)\,(\dot{\theta_1} + \dot{\theta_2})$

$\dot{x_1^2} = l_1^2 \sin^2(\theta_1)\,\dot{\theta_1^2}$

$\dot{y_1^2} = l_1^2 \cos^2(\theta_1)\,\dot{\theta_1^2}$

$\dot{x_2^2} = l_1^2 \sin^2(\theta_1)\,\dot{\theta_1^2} + l_2^2 \sin^2(\theta_1 + \theta_2)\,(\dot{\theta_1} + \dot{\theta_2})^2 + 2l_1 l_2 \sin\theta_1 \sin(\theta_1 + \theta_2)\dot{\theta_1}(\dot{\theta_1} + \dot{\theta_2})$

$\dot{y_2^2} = l_1^2 \cos^2(\theta_1)\,\dot{\theta_1^2} + l_2^2 \cos^2(\theta_1 + \theta_2)\,(\dot{\theta_1} + \dot{\theta_2})^2 + 2l_1 l_2 \cos\theta_1 \cos(\theta_1 + \theta_2)\dot{\theta_1}(\dot{\theta_1} + \dot{\theta_2})$

$\dot{x_1^2} + \dot{y_1^2} = l_1^2 \dot{\theta_1^2}\,[\sin^2(\theta_1) + \cos^2(\theta_1)] = l_1^2 \dot{\theta_1}^2$

$\dot{x_2^2} + \dot{y_2^2} = l_1^2\dot{\theta_1^2} + l_2^2(\dot{\theta_1} + \dot{\theta_2})^2 + 2l_1 l_2\, \dot{\theta_1}(\dot{\theta_1} + \dot{\theta_2})\ [\sin\theta_1 \sin(\theta_1 + \theta_2) + \cos\theta_1 \cos(\theta_1 + \theta_2)]$

$= l_1^2\dot{\theta_1^2} + l_2^2(\dot{\theta_1} + \dot{\theta_2})^2 + 2l_1 l_2\, \dot{\theta_1}(\dot{\theta_1} + \dot{\theta_2})\ \cos\theta_2$ Type equation here.

2) Kinetic Energy of the system

$$KE = \frac{1}{2}m_1 \dot{x_1}^2 + \frac{1}{2}m_1 \dot{y_1}^2 + \frac{1}{2}m_2 \dot{x_2}^2 + \frac{1}{2}m_2 \dot{y_2}^2$$
$$= \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_2}^2 + m_2 l_2^2\dot{\theta_1}\dot{\theta_2} + m_2 l_1 l_2 cos\theta_2\left(\dot{\theta_1}\dot{\theta_2} + \dot{\theta_1}^2\right)cos\theta_2$$

3) Potential Energy of the system

$$PE = m_1 g l_1 sin\theta_1 + m_2 g[l_1 sin\theta_1 + l_2 sin(\theta_1 + \theta_2)]$$

Lagrangian, $L = KE - PE$

$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_2}^2 + m_2 l_2^2\dot{\theta_1}\dot{\theta_2} + m_2 l_1 l_2 cos\theta_2\left(\dot{\theta_1}\dot{\theta_2} + \dot{\theta_1}^2\right)cos\theta_2 - m_1 g l_1 sin\theta_1 - m_2 g[l_1 sin\theta_1 + l_2 sin(\theta_1 + \theta_2)]$$

4) Our dynamics equations are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta_i}}\right) - \frac{\partial L}{\partial \theta_i} = F_{\theta_i} \quad ; i = 1,2$$

5) We thus obtain dynamic equations after simplification as

$$F_{\theta_1} = [(m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 cos]\ddot{\theta_1} + [m_2 l_2^2 + m_2 l_1 l_2 cos\theta_2]\ddot{\theta_2} - m_2 l_1 l_2 \sin\theta_2[2\,\dot{\theta_1}\dot{\theta_2} + \dot{\theta_2^2}] + (m_1 + m_2)g l_1 \cos\theta_1 + m_2 g l_2 \cos(\theta_1 + \theta_2)$$

$$F_{\theta_2} = [m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_1]\,\ddot{\theta_1} + [m_2 l_2^2]\ddot{\theta_2} + m_2 l_1 l_2 \sin\theta_2\dot{\theta_1^2} + m_2 g l_2 \cos(\theta_1 + \theta_2)$$

6) We can express it in matrix form as

$$\underline{\underline{M}}\,\underline{\ddot{q}} + \underline{\underline{C}}\,\underline{\dot{q}} + \underline{\underline{K}}\,\underline{q} = \underline{F}$$

where

$$\underline{\underline{M}} = \begin{bmatrix} (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_1 l_1 l_2 cos\theta_2 & m_2 l_2^2 + m_2 l_1 l_2 cos\theta_2 \\ m_2 l_2^2 + m\_2 l_1 l_2 cos\theta_2 & m_2 l_2^2 \end{bmatrix}$$

$$\underline{\underline{K}}\,\underline{q} = \begin{bmatrix} (m_1 + m_2) g l_1 cos\theta_1 + m_2 g l_2 \cos(\theta_1 + \theta_2) \\ m_2 g l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

$$\underline{F} = \begin{bmatrix} F_{\theta_1} \\ F_{\theta_2} \end{bmatrix}$$

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\underline{\dot{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\underline{\ddot{q}} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$ Type equation here.

C. Velocities and Jacobian

1) From Fig 1, we have:

$x_1 = l_1 \cos(\theta_1)$

$y_1 = l_1 \sin(\theta_1)$

$x_2 = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$

$y_2 = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$

$$\underline{v_1} \coloneqq \begin{bmatrix} v_{x1} \\ v_{y1} \\ \omega_{z1} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_{z1} \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} x_1 \\ y_1 \\ \theta_{z1} \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial \theta_2} \\ \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial \theta_2} \\ \frac{\partial \theta_{z1}}{\partial \theta_1} & \frac{\partial \theta_{z1}}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\underline{v_2} \coloneqq \begin{bmatrix} v_{x2} \\ v_{y2} \\ \omega_{z2} \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \\ \dot{\theta}_{z2} \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} x_2 \\ y_2 \\ \theta_{z2} \end{bmatrix} = \begin{bmatrix} \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} \\ \frac{\partial \theta_{z2}}{\partial \theta_1} & \frac{\partial \theta_{z2}}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

2) Linear Velocity part of the Jacobian, J_v is

$$J_v = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial \theta_2} \\ \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial \theta_2} \\ \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} \end{bmatrix}$$

3) Angular Velocity part of the Jacobian, J_ω is

$$J_\omega = \begin{bmatrix} \frac{\partial \theta_{z1}}{\partial \theta_1} & \frac{\partial \theta_{z1}}{\partial \theta_2} \\ \frac{\partial \theta_{z2}}{\partial \theta_1} & \frac{\partial \theta_{z2}}{\partial \theta_2} \end{bmatrix}$$

4) Full Jacobian for end-effector, J is

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \frac{\partial x_1}{\partial \theta_2} \\ \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial \theta_2} \\ \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} \\ \frac{\partial \theta_{z1}}{\partial \theta_1} & \frac{\partial \theta_{z1}}{\partial \theta_2} \\ \frac{\partial \theta_{z2}}{\partial \theta_1} & \frac{\partial \theta_{z2}}{\partial \theta_2} \end{bmatrix}$$

We will now obtain the end-effector velocities as a function of joint velocities, and vice-versa. We shall refer to the end-effector as '*ee*' in further discussions for convenience.

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = J_{ee} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

5) Where J_{ee} is the end-effector Jacobian and is given by

$$J_{ee} = \begin{bmatrix} \frac{\partial x_2}{\partial \theta_1} & \frac{\partial x_2}{\partial \theta_2} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} \end{bmatrix}$$
$$= \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \cos(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

6) Joint velocities as a function of the end-effector velocities can be expressed as

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J_{ee}^{-1} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

Where J_{ee}^{-1} is the inverse of the end-effector Jacobian J_{ee}, provided the inverse exists. We shall not include a discussion on singularities i.e., the cases when $\det(J_{ee}) = 0$.

D. Inverse Kinematics of the Plant

We have,

$$x_2 = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$y_2 = l_1 sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

Thus,

$$x_2^2 + y_2^2 = l_1^2 \cos^2\theta_1 + l_2^2 \cos^2(\theta_1 + \theta_2) + 2l_1 l_2 \cos\theta_1 \cos(\theta_1 + \theta_2) + l_1^2 \sin^2\theta_1 + l_2^2 \sin^2(\theta_1 + \theta_2) + 2l_1 l_2 sin\theta_1 \sin(\theta_1 + \theta_2)$$
$$= l_1^2 + l_2^2 + 2l_1 l_2 [sin\theta_1 \sin(\theta_1 + \theta_2) + cos\theta_1 \cos(\theta_1 + \theta_2)]$$

1) Using the well-known trigonometric identity $\cos a \cos b + \sin a \sin b = \cos(a - b)$, we have

$$x_2^2 + y_2^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_2$$
$$\Rightarrow q_2 = \theta_2 = \cos^{-1}\left[\frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2l_1 l_2}\right]$$

We must find similar expression for θ_1 in terms of (x_2, y_2, l_1, l_2) to complete the inverse kinematics.
For that, consider Fig 1.

2) We can see that

$$\tan p_1 = \frac{y_2}{x_2}$$
$$\Rightarrow p_1 = \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

Consider Fig.1 again.
We have:

$$\cos p_2 = \frac{l_1 + l_2 \cos\theta_2}{\sqrt{x_2^2 + y_2^2}}$$
$$= \frac{l_1 + l_2\left[\frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2\, l_1 l_2}\right]}{\sqrt{x_2^2 + y_2^2}}$$
$$= \frac{l_1^2 - l_2^2 + x_2^2 + y_2^2}{2l_1\sqrt{x_2^2 + y_2^2}}$$
$$= \frac{x_2^2 + y_2^2 + l_1^2 - l_2^2}{2l_1\sqrt{x_2^2 + y_2^2}}$$
$$\Rightarrow p_2 = \cos^{-1}\left(\frac{x_2^2 + y_2^2 + l_1^2 - l_2^2}{2l_1\sqrt{x_2^2 + y_2^2}}\right)$$

We have two cases for $q_1\ (=\ \theta_1)$

a) *Case 1:* $\theta_2 > 0$ $i.e., q_2 > 0$

Consider Fig. 2:

$$\theta_1 = q_1 = p_1 - p_2$$

b) *Case 2:* $\theta_2 < 0$ $ie., q_2 < 0$

Consider Fig.3:

$$\theta_1 = q_1 = p_1 + p_2$$

Consolidating the above two cases, we have:

$$q_1 = \begin{cases} p_1 - p2; & for\ q_2 > 0 \\ p_1 + p_2; & for\ q_2 < 0 \end{cases}$$

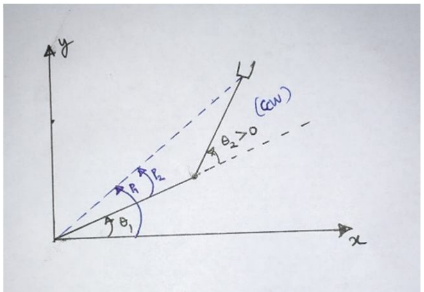

Fig. 2 Link Orientation: Case 1

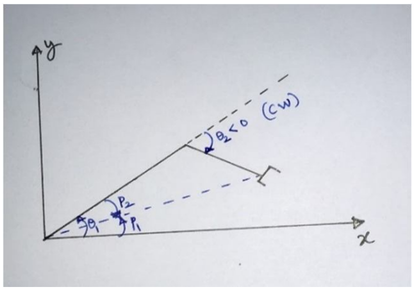

Fig. 3: Link Orientation: Case 2

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III. NUMERICAL VALIDATION

In the following section shall discuss the validation of the dynamics discussed in the earlier sections using a controller model described in [5].

The masses and geometry of the medical robot vary widely depending on the type of medical procedure being carried out, robot type and other factors. To check our model, we take hint form [3], and choose following values for the parameters involved.

Mass of the links: $m_1 = m_2 = 0.1kg$

Length of the links: $l_1 = l_2 = 0.05m$

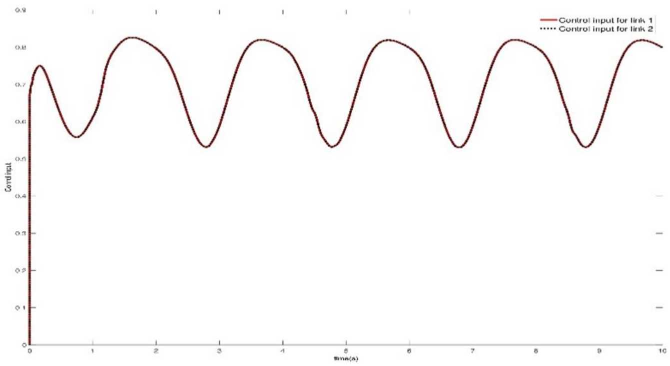

Fig. 4: Control Input to the link.

IV. RESULTS AND DISCUSSION

In this section we shall consider our plant and choose an appropriate controller model for the same. We shall discuss the results of numerical simulation using the chosen controller model.

To show the effectiveness of applied methodology, we have applied the controller model discussed in [5].

Fig. 4 shows the control input to the links. This can be thought of as torque input to actuators located at the joints as a function of time. We can see from Fig. 4 that control inputs for both the links is smooth and there are no sudden changes in the given input signal.

To validate our framework, we have done the following.

First, we have defined a given path/curve. In our case, we have chosen a circle. In principle, it can be any smooth and continuous curve in space. Ideally, we wish/expect our tool point to trace the given path.

We start our simulation with end-effector located at a certain position in space. We intentionally keep the initial position of tool point away from the desired path. This helps us check the following two things:

1) Is the tool point coming on to the desired path?
2) If it does, how fast and smooth is the transition?

We know that the circle can be described in Cartesian Coordinate system as curve C such that:

$$\text{C}: (x_1 - x_{10})^2 + (x_2 - x_{20})^2 = a^2$$

where a is the radius of the circle and (x_{10}, x_{20}) are the coordinates of the centre of the circle.

The equation simplifies to $C: x_1^2 + x_2^2 = a^2$ in case the centre of the circle coincides with the origin of the coordinate system.

We can also represent C in the Polar Coordinate System by using the well-known substitution:

$$x_1 = a\cos\theta$$
$$x_2 = a\sin\theta$$

which readily gives:

$$a = \sqrt{x_1^2 + x_2^2}$$
$$\theta = \tan^{-1}\left(\frac{x_2}{x_1}\right)$$

As we can see, the curve C is a summation of sine and cosine components.

As can be seen from Fig. 5, we have started with initial conditions as:

a) x_1 not coinciding with desired location.
b) x_2 coinciding with the desired location.

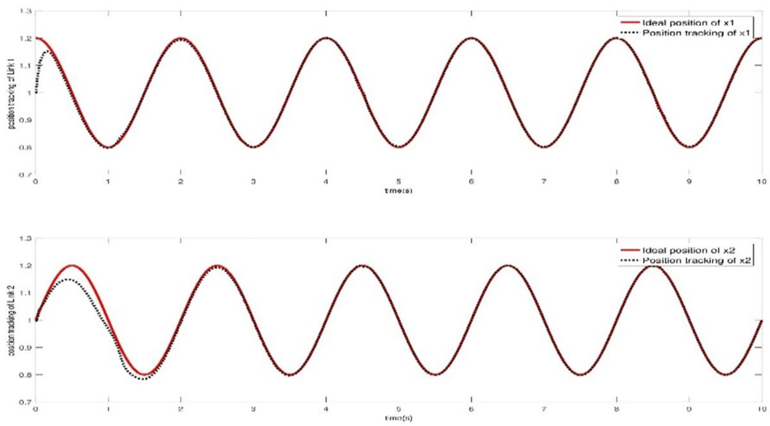

Fig. 5: Components of the desired and actual trajectory.

It can be seen that the position tracking of x_1 as well as x_2 coincide with the desired positions in a very short time after the simulation starts. Now, putting it all together, we can see our model in action by referring to Fig. 6. The desired curve *C* is shown in solid red line. As shown by the black coloured dash-dotted lines, we start with an initial location of the tool point far away from the desired path. It aligns with the desired path quickly and traces it.

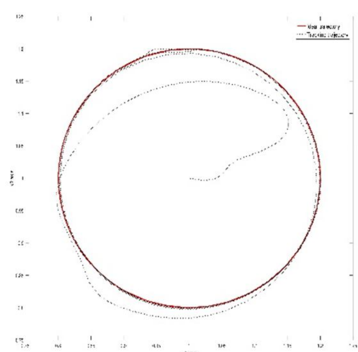

Fig. 6: Complete Trajectory: Ideal and tracked.

For the sake of completeness, we also include the speed tracking of tool point. Barring a few minor digressions, the speed tracking also follows and nicely aligns with the desired speed curve as shown in Fig. 7

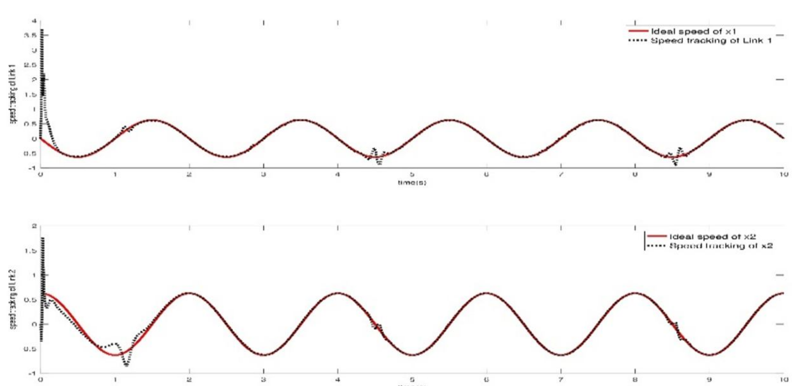

Fig. 7: Speed tracking of end-effector.

V. CONCLUSIONS

The main objective of the study was to provide a simple model of a robotic manipulator and describe its application in minimally invasive surgeries. In conclusion, we have seen

A. A two-link robotic manipulator has been chosen as our plant and its dynamic governing equations have been derived using the Lagrangian Formulation.
B. A novel framework of incorporating a sensor in the end effector has been described.
C. The feedback obtained from the sensors and its consequent use has been detailed.
D. Controller relevant to the sensor at the end effector has been chosen and its suitability for our plant has been shown.
E. Trajectory tracking and speed-tracking of the end-effector has been discussed.

VI. FUTURE WORK

A. The model can be extended for robotic chain consisting of a sufficiently large chain length to which gives extra degrees of freedom i.e., dexterity to effectively and efficiently gain access to required regions of the body during the surgery.
B. Haptic feedback model for the sense of touch can be incorporated.
C. Better control systems can be designed which make use of the progress being made in the fields of Neural Networks and Artificial Intelligence etc.

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