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Comparison of Roman Domination Number with Domination Number, Independent Domination Number and Chromatic Number of a Graph

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***Abstract:** Domination is a rapidly developing area of research in graph theory, and its various applications to ad-hock net work, distributed computing, social net works and web graph, partly explain the increased interest. Domination in graphs has been an extensively researched branch of graph theory. Different types of dominations like roman domination, independent domination and coloring is rapidly developing areas of research in graph theory. Here in this paper we compare roman domination number with domination number, independent domination number and chromatic number of a graph.*
***Keywords:** Dominating set, Domination number, Independent set, Independent dominating set, Independent domination number, Roman dominating set, Roman domination number, Graph coloring and Chromatic number.*

I. INTRODUCTION

Graph theory is rapidly moving into the mainstream of mathematics mainly due to its applications in diverse fields which include biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields.

Few subjects in mathematics have as specified an origin as graph theory. Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler (1707-1783) solved in 1736. Over the past sixty years, there has been a great deal of exploration in the area of graph theory. Its popularity has increased due to its many modern day applications and it has become the source of interest to many researchers. High-speed digital computer is one of the main reasons for the recent growth of interest in graph theory and its applications.

Nowadays graphs are really important in different fields. Probably, more important than we think. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in Physical, Biological and Social systems.

Many problems of practical interest can be represented by graphs and can be solved using graph theory. In Architecture, bipartite graphs play an important role in finding minimum number of cross beams required to make a grid of beams rigid, when the joints provide no rigidity to the structure.

The structure is rigid if and only if the corresponding bipartite graph is connected. But the smallest connected graph is a tree and the largest possible tree in the bipartite graph with m, n vertices has m+n-1 edges. Hence an mxn graph is rigid iff the corresponding bipartite graph is connected. The rigid bracing will have minimum cross beams iff the bipartite graph is a tree with m+n-1 cross beams. So, one can confidently put forward that a mere act of thinking about a problem in terms of a graph will certainly suggest insights and probable solution methods.

Graph theory is branch of mathematics, which has becomes quite rich and interesting for several regions. In last three decades thousand of research articles have been published in graph theory. There are several areas in graph theory which have reserved good attention from mathematicians. Graphs are very convenient tool for representing the relationship among objects, which are represented by vertices. In their term relationships among vertices are represented by connections. In general, any mathematical objects involving points and connection among them can be called a graph. For a great diversity of problems such pictorial representations may lead to a solution. For example data basics map coloring web graph, physical net work and organic molecular as well as less tangible interactions occurring in social net works in a flow of a computer program.

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Domination is a rapidly developing area of research in graph theory, and its various applications to ad-hock net work, distributed computing, social net works and web graph, partly explain the increased interest. Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of graph theory due to its wide application to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of application to many fields like engineering, physical, social and biological sciences, linguistics ect., the theory of domination has been the nucleus of research activity in graph theory in recent time.

Domination in graphs has applications to several fields. Domination arises in facility location problem, where the number of facilities like hospitals, fire stations is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility.

A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that ever one is serviced. Concepts from domination also appear in problems involving finding sets of representatives in monitoring communication or electrical networking, and in land surveying like minimizing the number of places a surveyor must stand in order to take high measure mints for an entire region.

Roman Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army, which would stay and protect the home. Thus, there are two types of armies, stationary and traveling. Each vertex with no army must have a neighboring vertex with a traveling army. Stationary armies then dominate their own vertices, and its stationary army dominates a vertex with two armies, and the traveling army dominates its open neighborhood, which motivates to our roman domination number of a graph.

A set S of vertices in a graph G is an independent dominating set of G if S is an independent set and every vertex not in S is adjacent to a vertex in S. An independent dominating set in a graph is a set that is both dominating and independent. Equivalently, an independent dominating set is a maximal independent set. Independent dominating sets have been studied extensively in the literature. Dominating and independent dominating sets. A dominating set of a graph G is a set S of vertices of G such that every vertex not in S is adjacent to a vertex in S.

The domination number of G, denoted by $\gamma(G)$, is the minimum size of a dominating set. A set is independent (or stable) if no two vertices in it are adjacent. An independent dominating set of G is a set that is both dominating and independent in G. The independent domination number of G, denoted by i(G), is the minimum size of an independent dominating set. The independence number of G, denoted $\alpha(G)$, is the maximum size of an independent set in G. It follows immediately that $\gamma(G)\leq i(G)\leq\alpha(G)$. A dominating set of G of size $\gamma(G)$ is called a γ-set, while an independent dominating set of G of size i(G) is called an i-set.

Independent sets play an important role in Graph Theory and other areas like discrete optimization. They appear in matching theory, coloring of graphs and in trees. A set of vertices in a graph G is said to be an independent set or an internally stable set if no two vertices in the set are adjacent.

An independent set S is said to be maximal independent set if S U {v} is not an independent set for every vertex v not in S. The independence number is the, maximum cardinality of an independent set in G. It is denoted by $\beta 0(G)$. A set with minimum cardinality among all the maximal independent set of G is called minimum independent dominating set of G or just i set of G. The cardinality of a minimum independent dominating set is called independent domination number of the graph G and it is denoted by i(G). Note: A maximal independent set is a dominating set of G.

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. Many variants of domination models are available in existing literature - Independent domination, Total domination, Global domination, Edge domination, just to name a few. Independent sets play an important role in graph theory and other area like discrete optimization. They appear in matching theory, coloring of graphs and in theory of trees.

A proper coloring of a graph G = (V,E) is a function from the vertices of the graph to a set of colors such that any two adjacent vertices have different colors.

The chromatic number $\chi(G)$ of G is the minimum number of colors needed in a proper coloring of a graph. In a proper coloring of a graph a color class is the set of all same colored vertices of the graph. Graph coloring is used as a model for a vast number of practical problems involving allocation of scarce resources (e.g., scheduling problems), and has played a key role in the development of graph theory and, more generally, discrete mathematics and combinatorial optimization.

Graph coloring has played a major role in the development of Graph Theory. A coloring of a graph G using at most n colors is called a n-coloring. A proper coloring of a graph G is a function c: $V\rightarrow \{1, 2, 3\ldots.n\}$ such that any two adjacent vertices have

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different colors. The minimum number of colors needed to color the vertices of the graph such that no two adjacent vertices receive the same color is called the chromatic number. It is denoted by $\chi(G)$.A graph G is set to be n colorable if $\chi(G) \leq n$.

Applications of Graph Coloring Graph coloring is one of the most important concepts in graph theory. It is used in many real-time applications of computer science such as –Clustering, Data mining, Image capturing, Image segmentation, Networking, Resource allocation and Processes scheduling.

A. Preliminaries

Let G be a graph. An edge which joins the same vertex is called a loop. An edge that joins two distinct vertices is called a link. If two or more edges of G have the same end vertices, then these edges are called parallel edges or multiple edges. A graph is said to be a simple graph if it has no loops and no parallel edges. A graph G is said to be finite if both its vertex set and edge sets are finite, otherwise it is called an infinite graph. The number of edges incident with a vertex v of a graph is called the degree of v and is denoted by d(v). A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph. Neighborhood of a vertex $v \in V$ is a set consisting all vertices adjacent to v (including v), it is denoted by nbd [v]. i.e., nbd [v] = {the set of all vertices adjacent to v} $\cup$ {v}. A subset S of V is called an independent set of G if no two vertices in S are adjacent. A maximum independent set of G is an independent set whose cardinality is largest among all independent sets of G. A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G. A dominating set D of a graph G is a connected dominating set if the induced subgraph $< V - D >$ is connected. The connected domination number $\gamma_c(G)$ of the graph G is the minimum cardinality of the connected dominating set. A dominating set D of a graph G is called a minimum dominating set if no proper subset of D is a dominating set. A total dominating set D of a graph G is a dominating set in which every vertex is adjacent to some vertex in it. The total domination number $\gamma_t(G)$ of the graph G is the minimum cardinality of the total dominating set. The distance between two vertices u and v of a graph is the length of the shortest path (path of minimum length) between them and is denoted by d_G of (u,v).

A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G.

A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in $V - D$ as well as adjacent to some vertex in T . The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.

An independent dominating set of G is a set that is both dominating and independent in G. The independent domination number of G, denoted by i(G), is the minimum size of an independent dominating set.

A proper coloring of a graph G = (V,E) is a function from the vertices of the graph to a set of colors such that any two adjacent vertices have different colors. The chromatic number $\chi(G)$ of G is the minimum number of colors needed in a proper coloring of a graph.

B. Main Theorems

1) *Theorem 1:* Comparison of Roman domination number with domination number of a graph
2) *Proof:* Roman Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army, which would stay and protect the home. Thus, there are two types of armies, stationary and traveling. Each vertex with no army must have a neighboring vertex with a traveling army. Stationary armies then dominate their own vertices, and its stationary army dominates a vertex with two armies, and the traveling army dominates its open neighborhood, which motivates to our roman domination number of a graph.

A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in $V - D$ as well as adjacent to some vertex in T. The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.

A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G.

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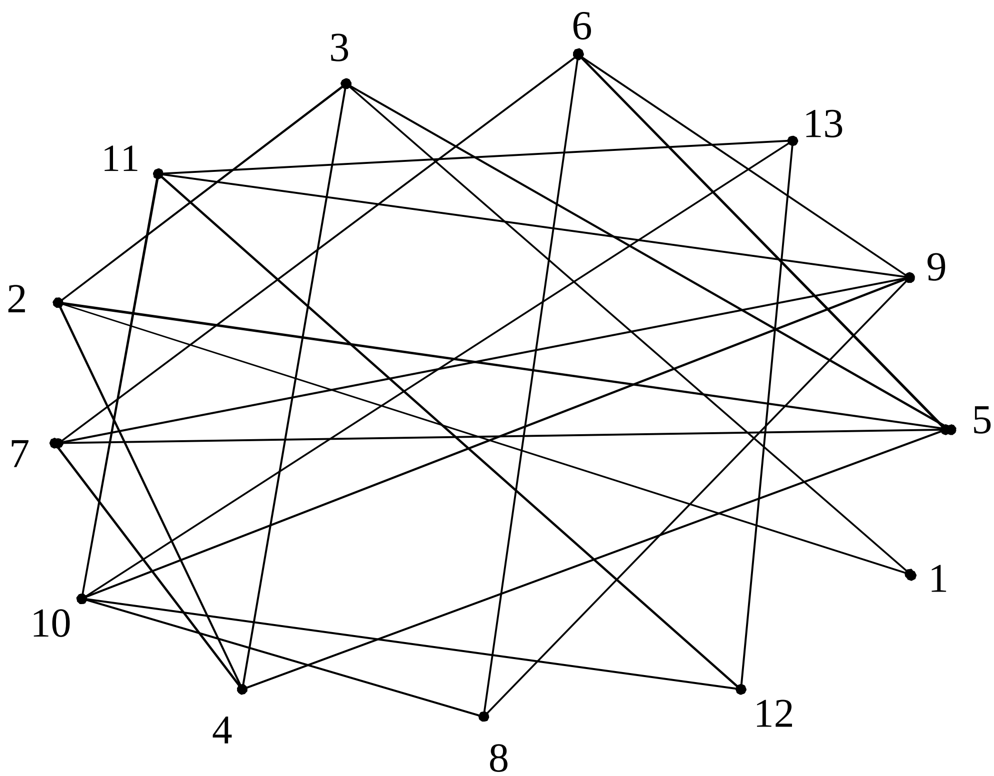

C. Find the Roman Domination Number of G

D is a dominating set v_1 dominates v_2, v_3 dominates v_4 and v_5, v_9 dominates v_6, v_7, v_8, v_{10}, v_{11}, v_{11} dominates v_{12}, v_{13}

S and T are two subsets of D

S= { v_1, v_8} and T={v_3, v_6, v_9, v_{11}}

Vertex v_1 in S is adjacent to exactly one vertex v_2 in V-D as well as adjacent to vertex v_3 in T.

Vertex v_8 in S is adjacent to exactly one vertex v_{10} in V-D as well as adjacent to vertex v_6 and v_8 in T.

S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T.

Roman dominating set of G is {v_1, v_3, v_6, v_8, v_9, v_{11}}

Roman domination number of G is 6.

D. Find the Domination Number of G

Vertex v_3 dominates v_1, v_2, v_4 and v_5

Vertex v_9 dominates v_6, v_7, v_8, v_{10}, and v_{11}

Vertex v_{11} dominates v_{12} and v_{13}

The dominating set of G is {v_3, v_9, v_{11}}

Domination number of G is 3

Roman domination number and domination number of G are 6 and 3 respectively

Roman domination number of G is greater than that of domination number of G 6>3

We proved that Roman domination number of a graph G is greater than domination number of G.

1) *Theorem 2:* Comparison of Roman domination number with independent domination number of a graph
2) *Proof:* Roman Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army, which would stay and protect the home. Thus, there are two types of armies, stationary and traveling. Each vertex with no army must have a neighboring vertex with a traveling army. Stationary armies then dominate their own vertices, and its stationary army dominates a vertex with two armies, and the traveling army dominates its open neighborhood, which motivates to our roman domination number of a graph.

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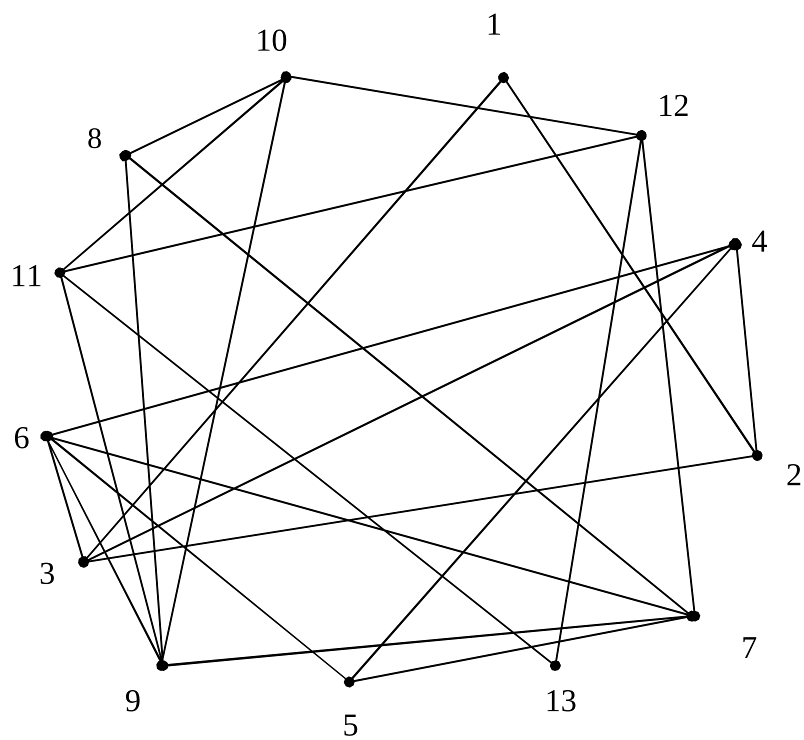
A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T . The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.

A set S of vertices in a graph G is an independent dominating set of G if S is an independent set and every vertex not in S is adjacent to a vertex in S. An independent dominating set in a graph is a set that is both dominating and independent. Equivalently, an independent dominating set is a maximal independent set. Independent dominating sets have been studied extensively in the literature. Dominating and independent dominating sets. A dominating set of a graph G is a set S of vertices of G such that every vertex not in S is adjacent to a vertex in S. The domination number of G, denoted by γ(G), is the minimum size of a dominating set. A set is independent (or stable) if no two vertices in it are adjacent. An independent dominating set of G is a set that is both dominating and independent in G. The independent domination number of G, denoted by i(G), is the minimum size of an independent dominating set. The independence number of G, denoted α(G), is the maximum size of an independent set in G. It follows immediately that γ(G)≤i(G)≤α(G). A dominating set of G of size γ(G) is called a γ-set, while an independent dominating set of G of size i(G) is called an i-set. Independent sets play an important role in Graph Theory and other areas like discrete optimization. They appear in matching theory, coloring of graphs and in trees. A set of vertices in a graph G is said to be an independent set or an internally stable set if no two vertices in the set are adjacent. An independent set S is said to be maximal independent set if S U {v} is not an independent set for every vertex v not in S. The independence number is the, maximum cardinality of an independent set in G. It is denoted by β0(G). A set with minimum cardinality among all the maximal independent set of G is called minimum independent dominating set of G or just i set of G. The cardinality of a minimum independent dominating set is called independent domination number of the graph G and it is denoted by i(G). Note: A maximal independent set is a dominating set of G.



E. Find the Roman Domination Number of G

D is a dominating set v_1 dominates v_2 and v_3, v_3 dominates v_4 and v_6, v_6 dominates v_5, v_7, v_9, v_9 dominates v_{10} and $v_{11,}$ v_{11} dominates v_{12}, v_{13}

S and T are two subsets of D

S= { v_1} and T={v_3, v_6, v_9, v_{11}}

Vertex v_1 in S is adjacent to exactly one vertex v_2 in V-D as well as adjacent to vertex v_3 in T.

S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T.

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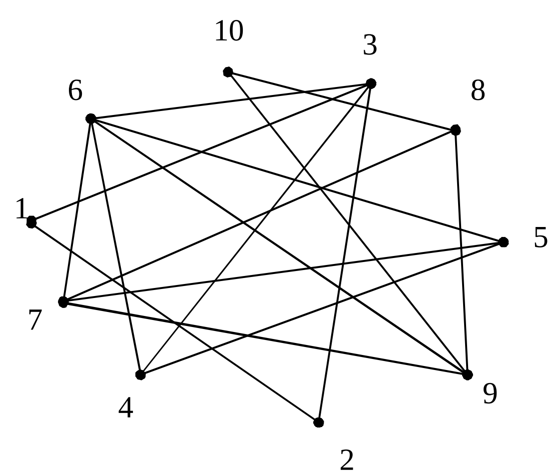
Roman dominating set of G is $\{v_1, v_3, v_6, v_9, v_{11}\}$
Roman domination number of G is 5

F. Find the Independent Domination Number of G

A set I of vertices in a graph G is an independent dominating set of G if I is an independent set and every vertex not in I is adjacent to a vertex in I. An independent dominating set in a graph is a set that is both dominating and independent. A set is independent (or stable) if no two vertices in it are adjacent.
Independent set of G is $\{v_3, v_5, v_9, v_{13}\}$
I= $\{v_3, v_5, v_9, v_{13}\}$ I is independent set because no two vertices in it are adjacent and I is dominating set also, v_3 dominates v_1, v_2, v_4, v_6, v_5 dominates v_4, v_6, v_7, v_9 dominates v_6, v_7, v_8, v_{10}, v_{11} and v_{13} dominates v_{11}, v_{12}.
Independent dominating set of G is $\{v_3, v_5, v_9, v_{13}\}$
Independent domination number of G is 4
Roman domination number and Independent domination number of G are 5 and 4 respectively. Roman domination number of G is greater than that of Independent domination number of G 5>4. We proved that Roman domination number of a graph G is greater than Independent domination number of G.

1) *Theorem 3:* Comparison of Roman domination number with Chromatic number of a graph
2) *Proof:* Roman Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army, which would stay and protect the home. Thus, there are two types of armies, stationary and traveling. Each vertex with no army must have a neighboring vertex with a traveling army. Stationary armies then dominate their own vertices, and its stationary army dominates a vertex with two armies, and the traveling army dominates its open neighborhood, which motivates to our roman domination number of a graph.

A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T . The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.
A proper coloring of a graph G = (V, E) is a function from the vertices of the graph to a set of colors such that any two adjacent vertices have different colors. The chromatic number $\chi(G)$ of G is the minimum number of colors needed in a proper coloring of a graph. In a proper coloring of a graph a color class is the set of all same colored vertices of the graph. Graph coloring is used as a model for a vast number of practical problems involving allocation of scarce resources (e.g., scheduling problems), and has played a key role in the development of graph theory and, more generally, discrete mathematics and combinatorial optimization.
Graph coloring has played a major role in the development of Graph Theory. A coloring of a graph G using at most n colors is called a n-coloring. A proper coloring of a graph G is a function c: V→ {1, 2, 3….n} such that any two adjacent vertices have different colors. The minimum number of colors needed to color the vertices of the graph such that no two adjacent vertices receive the same color is called the chromatic number. It is denoted by $\chi(G)$.A graph G is set to be n colorable if $\chi(G) \leq n$.



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G. Find the Roman Domination Number of G

D is a dominating set v_2 dominates v_1 and v_3, v_3 dominates v_4 and v_6, v_5 dominates v_7, v_9 dominates v_7, v_8 and $v_{10,}$ v_{10} dominates v_8.
S and T are two subsets of D
S= { $v_{2,}$ v_{10}} and T={v_3, v_5, v_9}
Vertex v_2 in S is adjacent to exactly one vertex v_1 in V-D as well as adjacent to vertex v_3 in T.
Vertex v_{10} in S is adjacent to exactly one vertex v_8 in V-D as well as adjacent to vertex v_9 in T.
S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T.
Roman dominating set of G is {v_2, v_3, v_5, v_9, v_{10}}
Roman domination number of G is 5.

H. Find the Chromatic Number of G

Give the colors 1, 2 and 3 to v_1, v_2 and v_3 respectively. Vertices v_4 and v_6 are adjacent to v_3 but not adjacent to v_1 and v_2 so we can repeat the colors 1 and 2 and give them to v_4 and v_6 respectively. Vertex v_5 is adjacent to v_4 and v_6 but not adjacent to v_3 so we can repeat the color 3 and give it to v_5. Vertex v_7 is adjacent to v_5 and v_6 but not adjacent to v_4 so we can repeat the color 1 and give it to v_7. Vertex v_8 is adjacent to v_7 but not adjacent to v_5 and v_6 so we can repeat the colors 2 and 3 and give 2 to v_8. Vertex v_9 is adjacent to v_6, v_7 and v_8 but not adjacent to v_3 or v_5 so we can repeat the color 3 and give it to v_9. Vertex v_{10} is adjacent to v_8 and v_9 but not adjacent to any other vertex so we can repeat the color 1 and give it to v_{10}.
A proper coloring of a graph G = (V, E) is a function from the vertices of the graph to a set of colors such that any two adjacent vertices have different colors. The chromatic number $\chi(G)$ of G is the minimum number of colors needed in a proper coloring of a graph.
We used 3 colors to color the vertices such that any two adjacent vertices have different colors. The minimum number of colors needed in a proper coloring of a graph G is 3. The chromatic number of G is $\chi(G)=3$
Roman domination number and chromatic number of G are 5 and 3 respectively. Roman domination number of G is greater than chromatic number of G 5>3. We proved that roman domination number of a graph G is greater than chromatic number of G.

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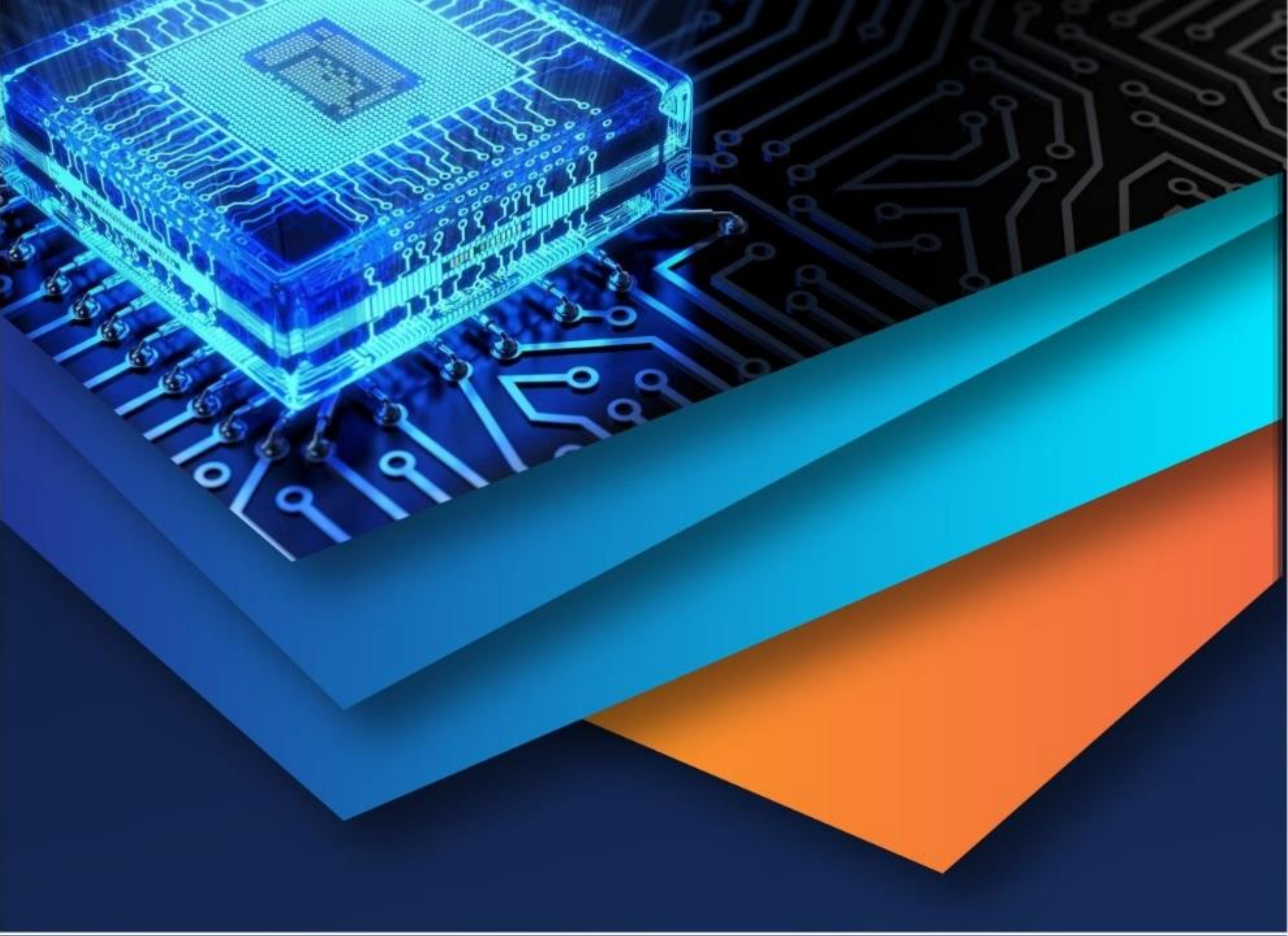

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