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On Edge Regular Fuzzy Soft Graphs

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Abstract: In this paper, degree of an egde and total degree of an edge in a fuzzy soft graph is introduced, edge regular fuzzy soft graphs, and totally edge regular fuzzy soft graphs are also introduced . Theorems for edge regular fuzzy soft graphs and totally edge regular fuzzy soft graphs are introduced. A necessary condition under which they are equivalent is provided. Some properties of edge regular fuzzy soft graphs and totally edge regular fuzzy soft graphs are studied.
***Keywords**: Degree of an edge in a fuzzy soft graph, total degree of an edge in a fuzzy soft graph, edge regular fuzzy soft graph , totally edge regular fuzzy soft graph.*

I. INTRODUCTION

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well-known Konigsberg bridge problem is considered to be the first theorem of graph theory. The graph theory is a very useful tool for solving combinatorial problems in different areas such as operations research, optimization, topology, geometry, number theory, algebra and computer science. Fuzzy set theory, introduced by Zadeh in 1965 is a mathematical tool for handling uncertainties like vagueness, ambiguity and imprecision in linguistic variables [12]. Research on theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its application. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The first definition of fuzzy graph was introduced by Haufmann in 1973, based on Zadeh’s fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs [9]. Nagoor Gani and Latha[26] introduced irregular fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs using fuzzy relations, obtaining analogs of several graph theoretical concepts. During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph [11]. Now, fuzzy graphs have been witnessing a tremendous growth and finds application in many branches of engineering and technology. A. NagoorGani and K. Radha introduced the concept of regular fuzzy graphs in 2008 [5]. In 1999, D.Molodtsov[12] introduced the notion of soft set theory to solve imprecise problems in economics, engineering and environment. He has shown several applications of this theory in solving many practical problems. There are many theories like theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of rough sets, etc. which can be considered as mathematical tools to deal with uncertainties. But all these theories have their own inherent difficulties. The theory of probabilities can deal only with possibilities. The most appropriate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh[9] in 1965. But it has an inherent difficulty to set the membership function in each particular r cases. Also the theory of intuitionistic fuzzy set is more generalized concept than the theory of fuzzy set, but also there have same difficulties. The soft set theory is free from above difficulties. In 2001, P.K.Maji, A.R.Roy,R.Biswas [20, 21] initiated the concept of fuzzy soft sets which is a combination of fuzzy set and soft set. In fact, the notion of fuzzy soft set is more generalized than that of fuzzy set and soft set. Muhammad Akram and Saira Nawaz [25] introduced more concepts on fuzzy soft graphs. K. Radha and N. Kumaravel [26] introduced new concepts based on edge regular fuzzy soft graph.

II. PRELIMINARIES

1) *Definition 2.1:* A graph G is called regular if every vertex is adjacent only to vertices having the same degree.
2) *Definition 2.2:* A graph G is called edge regular if every edge is adjacent only to edges having the same degree.
3) *Definition 2.3:* A fuzzy graph G is a pair of functions $G:(\sigma,\mu)$ where σ is a fuzzy Sub set of a non empty set V and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*(V,E)$ where $E \subseteq V\times V$. A fuzzy graph G is complete if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ where uv denotes the edge between u and v.
4) *Definition 2.4:* A fuzzy graph H : (t, u) is called a partial fuzzy sub graph of G : (s, μ) if $t(u) \leq s(u)$ for every u and $u(u, v) \leq \mu(u, v)$ for every u and v . In particular we call a partial fuzzy sub graph H : (t, u) a fuzzy sub graph of G : (s, μ) if $t(u) = s(u)$ for every u in t * and $u(u, v) = \mu(u, v)$ for every arc (u, v) in u*.

5) *Definition 2.5:* Let U be a nonempty finite set of objects called Universe and let E be a nonempty set called parameters. An ordered pair (F,E) is said to be a Soft set over U, where F is a mapping from E into the set of all subsets of the set U. That is F: E $\rightarrow$ P (U). The set of all Soft sets over U is denoted by S (U)
6) *Definition 2.6:* Let (F, A) be a soft set over V. Then (F, A) is said to be a Soft graph of G if the sub graph induced by F(x) in G, F%(x) is a connected sub graph of G for all x belongs to A. The set of all soft graph of G is denoted by SG(G).
7) *Definition 2.7:* Let V be a non empty set of vertices , E be the set of parameters and A$\subseteq$ E . Also let

a) $\rho : A \rightarrow F(V)$ (collection of all fuzzy subsets in V)
$e \mapsto \rho(e) = \rho_e$.(say) and
$\rho_e : V \rightarrow [0,1]$
$x_i \mapsto \rho_e(x_i)$
(A, ρ) is a fuzzy soft vertex.

b) $\mu : A \rightarrow F(V \times V)$ (collection of all fuzzy subsets in E)
$e \mapsto \mu(e) = \mu_e$.(say) and
$\mu_e : V \times V \rightarrow [0,1]$
$(x_i, x_j) \mapsto \mu_e(x_i, x_j)$
(A, μ) is a fuzzy soft edge.

Then ((A, ρ),(A,μ)) is called fuzzy soft graph if and only if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for all e belongs to A.
Which is also equivalent to the definition that,
A fuzzy soft graph $G_{A,V} = (G^*, F, K, A)$ is a 4- tuple such that,

i) $G^* = (V, E)$ is a simple graph
ii) A is tha set of parameters
iii) (F,A) is a fuzzy soft set over V
iv) (F(e), K(e)) is a fuzzy soft set over E
v) (F(e),K(e)) is a fuzzy subgraph of G^* for all e belongs to A i.e)

$K(e)(xy) \leq \min \{ F(e)(x), K(e)(y) \}$ for all e$\in$A x,y $\in$ V. the fuzzy graph (F(e) ,K(e)) is denoted by $H_{A,V}(e)$.

8) *Definition 2.8:* Let $G_{A,V} = ((A, \rho),(A,\mu))$ be a fuzzy soft graph, then the order of $G_{A,V}$ is defined as: $O(G_{A,V}) = \sum_{e\in A} (\sum_{x_i\in A} \rho_e(x_i))$.
9) *Definition 2.9:* Let $G_{A,V} = ((A, \rho),(A,\mu))$ be a fuzzy soft graph. Then the size of $G_{A,V}$ is defined as: $S(G_{A,V}) = \sum_{e\in A} (\sum_{u\neq v} \mu_e(x_i, x_j))$
10) *Definition 2.10:* Let $G_{A,V} = ((A, \rho),(A,\mu))$be a fuzzy soft graph. The degree of a vertex u is defined as $dG_{A,V}(u) = \sum_{e\in A} (\sum_{u\neq v} \mu_e(u, v))$
11) *Definition: 2.11* Let $G^* = (V, E)$ be a graph and let e = uv $\in$ E . Then the degree of an edge uv is defined by $d_{G^*}(uv) = d_{G^*}(u) + d_{G^*}(v) - 2$.
12) *Definition: 2.12* Let G:(σ,μ) be a graph on $G^* = (V, E)$ and let e = uv $\in$ E . Then the degree of an edge uv is defined by $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$. This is also equivalent to $d_G(uv) = \{\sum_{w\neq v} \mu(uw)\} + \{\sum_{w\neq u} \mu(wv)\}$.
13) *Definition: 2.13* Let G:(σ,μ) be a graph on $G^* = (V, E)$ and let e = uv $\in$ E . Then the total degree of an edge uv is defined by $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$. This is also equivalent to $td_G(uv) = d_G(u)v + \mu(uv)$.
14) *Theorem : 2.14* Let G: (σ, μ) be a fuzzy graph on G*: (V, E). If G is both edge regular and totally edge regular, then G is a regular fuzzy graph if and only if G* is a regular graph.

III. DEGREE OF AN EDGE AND TOTAL DEGREE OF AN EDGE IN FUZZY SOFT GRAPH

1) *Definition 3.1:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph and let e = uv $\in$ E . Then the degree of an edge uv is defined by $d_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) - 2(\sum_{e\in A} \mu_e(uv))$. This is also equivalent to $d_{GA,V}(uv) = \{ \sum_{e\in A} \sum_{w\neq v} \mu_e(uw) \} + \{\sum_{e\in A} \sum_{w\neq u} \mu_e(wv)\}$.

Minimum edge degree of $G_{A,V} = \delta_E G_{A,V} = \wedge \{ d_{GA,V}(uv)$, for all uv $\in$ E }.

Maximum edge degree of $G_{A,V} = \Delta_E G_{A,V} = \vee\{ d_{GA,V}(uv)$, for all $uv \in E$ }.

2) *Definition 3.2:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph and let $e = uv \in E$. Then the total degree of an edge uv is defined by $td_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) - (\sum_{e\in A} \mu_e(uv))$. This is also equivalent to $td_{GA,V}(uv) = \{ \sum_{e\in A} \sum_{w\neq v} \mu_e(uw) \} + \{ \sum_{e\in A} \sum_{w\neq u} \mu_e(wv) \}.+ \{\sum_{e\in A} \mu_e(uv)\}$.

Which implies $td_{GA,V}(uv) = d_{GA,V}(uv) + \sum_{e\in A} \mu_e(uv)$.

Minimum total edge degree of $G_{A,V} = \delta_E G_{A,V} = \wedge \{ td_{GA,V}(uv)$, for all $uv \in E$ }.

Maximum total edge degree of $G_{A,V} = \Delta_E G_{A,V} = \vee\{ td_{GA,V}(uv)$, for all $uv \in E$ }.

3) *Example 3.3*

Consider the following fuzzy soft graph

$F(e_1) = \{ X_1 | 0.2 , X_2 | 0.3 , X_3 | 0.4 \}$

$F(e_2) = \{ X_1 | 0.3 , X_2 | 0.4 , X_3 | 0.5 \}$

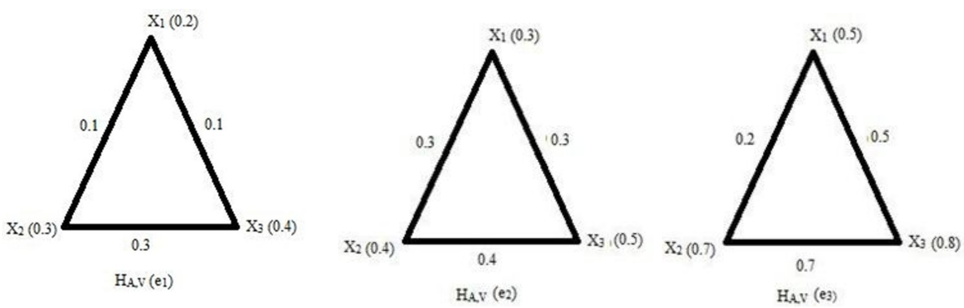
$F(e_3) = \{ X_1 | 0.5 , X_2 | 0.7 , X_3 | 0.8 \}$

and

$K(e_1) = \{ X_1 X_2 | 0.1 , X_1 X_3 | 0.1 , X_2 X_3 | 0.3 \}$

$K(e_2) = \{ X_1 X_2 | 0.3 , X_1 X_3 | 0.3 , X_2 X_3 | 0.4 \}$

$K(e_3) = \{ X_1 X_2 | 0.2 , X_1 X_3 | 0.5 , X_2 X_3 | 0.7 \}$



$d_{GA,V}(x_1) = 0.2 + 0.6 + 0.7 = 1.5.$

$d_{GA,V}(x_2) = 0.4 + 0.7 + 0.9 = 2.0.$

Now,

$d_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) - 2 (\sum_{e\in A} \mu_e(uv)) = 1.5 + 2.0 - 2 (0.1 + 0.3 + 0.2) = 2.3.$

Also,

$d_{GA,V}(uv) = \{ \sum_{e\in A} \sum_{w\neq v} \mu_e(uw) \} + \{\sum_{e\in A} \sum_{w\neq u} \mu_e(wv) \} = (0.3 + 0.4 + 0.7) + (0.1 + 0.3 + 0.5) = 2.3.$

Now,

$td_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) - (\sum_{e\in A} \mu_e(uv)) = 1.5 + 2.0 - (0.1 + 0.3 + 0.2) = 2.9.$

Also,

$td_{GA,V}(uv) = d_{GA,V}(uv) + \sum_{e\in A} \mu_e(uv) = 2.3 + 0.6 = 2.9.$

4) *Theorem : 3.4* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^*: (V , E)$. Then $\sum_{uv\in E} d_{GA,V}(uv) = \sum_{e\in A} \sum_{uv\in E} d_{G^*}(uv) \mu_e(uv)$.

5) *Theorem : 3.5* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^*: (V , E)$. Then $\sum_{uv\in E} td_{GA,V}(uv) = \sum_{e\in A} \sum_{uv\in E} d_{G^*}(uv) \mu_e(uv) + S(G_{A,V})$.

Proof:

The Size of $G_{A,V} = S(G_{A,V}) = \sum_{uv\in E} \sum_{e\in A} \mu_e(uv)$.

Now,

$td_{GA,V}(uv) = d_{GA,V}(uv) + \sum_{e\in A} \mu_e(uv)$.

$\Rightarrow \sum uv\in E\ td_{GA,V}(uv) = d_{GA,V}(uv) + \sum uv\in E \sum e\in A\ \mu_e(uv)$.

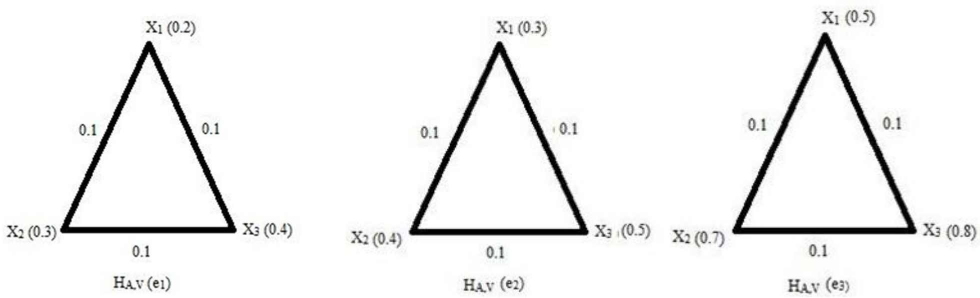
$\Rightarrow \sum uv\in E\ td_{GA,V}(uv) = d_{GA,V}(uv) + S(G_{A,V})$.

$\Rightarrow \sum uv\in E\ td_{GA,V}(uv) = \sum e\in A \sum uv\in E\ d_{G*}(uv)\ \mu_e(uv) + S(G_{A,V})$. (By theorem 3.4).

IV. EDGE REGULAR FUZZY SOFT GRAPH AND TOTALLY EDGE REGULAR FUZZY SOFT GRAPH

1) *Definition: 4.1:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$ .Then $G_{A,V}$ is said to be an edge regular fuzzy soft graph if $H_{A,V}(e)$ is an edge regular fuzzy graph for all e ∈A. If $H_{A,V}(e)$is an edge regular fuzzy graph of degree r for all e∈A; then $G_{A,V}$ is a r - edge regular fuzzy soft graph.
2) *Definition: 4.2 :* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$ .Then $G_{A,V}$ is said to be a totally edge regular fuzzy soft graph if $H_{A,V}(e)$ is an totally edge regular fuzzy graph for all e ∈A. If $H_{A,V}(e)$ is an totally edge regular fuzzy graph of degree r for all e∈A; then $G_{A,V}$ is a r - totally edge regular fuzzy soft graph.

Example : 2
Consider tha following Example
$F(e_1) = \{ X_1 | 0.2 , X_2 | 0.3 , X_3 | 0.4 \}$
$F(e_2) = \{ X_1 | 0.3 , X_2 | 0.4 , X_3 | 0.5 \}$
$F(e_3) = \{ X_1 | 0.5 , X_2 | 0.7 , X_3 | 0.8 \}$
and
$K(e_1) = \{ X_1 X_2 | 0.1 , X_1 X_3 | 0.1 , X_2 X_3 | 0.1 \}$
$K(e_2) = \{ X_1 X_2 | 0.1 , X_1 X_3 | 0.1 , X_2 X_3 | 0.1 \}$
$K(e_3) = \{ X_1 X_2 | 0.1 , X_1 X_3 | 0.1 , X_2 X_3 | 0.1 \}$



Which is an example of both edge regular and totally edge regualar fuzzy soft graph.
Note:

a) $G_{A,V}$ is r- edge regular fuzzy soft graph iff its minimum edge degree and maximum edge degree are equal to r.
b) $G_{A,V}$ is r- totally edge regular fuzzy soft graph iff its minimum total edge degree and maximum total edge degree are equal to r.

3) *Theorem : 4.3* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$. K is a constant function iff the following conditions are equivalent.
   i) $G_{A,V}$ is an edge regular fuzzy soft graph.
   ii) $G_{A,V}$ is a totally edge regular fuzzy soft graph.

Proof:

Suppose K is a constant function , K(e)(uv) = c, a constant for all uv∈E and e∈A.
$\Rightarrow \sum e\in A\ \mu_e(uv) = c$ for all uv∈E and e∈A.
Assume that $G_{A,V}$ is a k- edge regular fuzzy soft graph.
$\Rightarrow d_{GA,V}(uv) = k$, for all uv∈E.
Now,
$td_{GA,V}(uv) = d_{GA,V}(uv) + \sum e\in A\ \mu_e(uv)$.

$\Rightarrow td_{GA,V}(uv) = k + \sum e \in A\ \mu_e(uv)$ for all $uv \in E$.
$\Rightarrow td_{GA,V}(uv) = k + c$ for all $uv \in E$.
$\Rightarrow G_{A,V}$ is a (k+c) totally edge regular fuzzy soft graph.
Thus (i) $\Rightarrow$ (ii) is proved.
Now,
Assume that $G_{A,V}$ is a r- totally edge regular fuzzy soft graph.
$\Rightarrow td_{GA,V}(uv) = r$, for all $uv \in E$.
$\Rightarrow d_{GA,V}(uv) + \sum e \in A\ \mu_e(uv) .= r$, for all $uv \in E$.
$\Rightarrow d_{GA,V}(uv) + c .= r$, for all $uv \in E$.
$\Rightarrow d_{GA,V}(uv) .= (r - c)$, for all $uv \in E$.
$\Rightarrow G_{A,V}$ is (r-c) edge regular fuzzy soft graph.
Thus (ii) $\Rightarrow$ (i) is proved.
Conversely,
Suppose (i) and (ii) are equivalent.
1) *Claim:* K is a constant function.
Suppose K is not a constant function, $K(e)(uv) \neq c$, a constant for all $uv \in E$ and $e \in A$.
$\Rightarrow \sum e \in A\ \mu_e(uv) \neq c$ for all $uv \in E$ and $e \in A$..
$\Rightarrow \sum e \in A\ \mu_e(uv) \neq \sum e \in A\ \mu_e(xy)$ for some uv , $xy \in E$ and $e \in A$..
$\Rightarrow \mu_e(uv) \neq \mu_e(xy)$ for atleast one pair of edges uv , $xy \in E$.
Let $G_{A,V}$ be a k- edge regular fuzzy soft graph.
$\Rightarrow d_{GA,V}(uv) = d_{GA,V}(xy) = k$, for all $uv \in E$.
Now,
$td_{GA,V}(uv) = d_{GA,V}(uv) + \sum e \in A\ \mu_e(uv)$.
$\Rightarrow td_{GA,V}(uv) = k + \sum e \in A\ \mu_e(uv)$.
and
$td_{GA,V}(xy) = d_{GA,V}(xy) + \sum e \in A\ \mu_e(xy)$.
$\Rightarrow td_{GA,V}(xy) = k + \sum e \in A\ \mu_e(xy)$.
Since $\sum e \in A\ \mu_e(uv) \neq c$ for all $uv \in E$ and $e \in A$, We have
$td_{GA,V}(uv) \neq td_{GA,V}(xy)$, Which is a contradiction.
Now,
Let $G_{A,V}$ be a k- totally edge regular fuzzy soft graph.
$\Rightarrow td_{GA,V}(uv) = td_{GA,V}(xy) = k$, for all $uv \in E$.
$\Rightarrow d_{GA,V}(uv) + \sum e \in A\ \mu_e(uv) = d_{GA,V}(xy) + \sum e \in A\ \mu_e(xy)$
Since $\sum e \in A\ \mu_e(uv) \neq c$ for all $uv \in E$ and $e \in A$,We have,
$d_{GA,V}(uv) - d_{GA,V}(xy) = \sum e \in A\ \mu_e(xy) - \sum e \in A\ \mu_e(uv) \neq 0$.
$\Rightarrow d_{GA,V}(uv) \neq td_{GA,V}(xy)$, Which is a contradiction.
Hence K is a constant function.

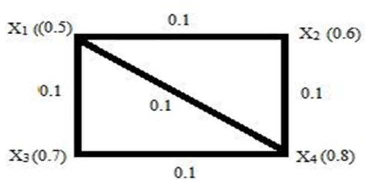
4) *Theorem : 4.4:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$. If $G_{A,V}$ is both edge regular and totally edge regular , then K is a constant function.

Proof:
Let $G_{A,V}$ be a k-edge regular and r totally edge regular fuzzy soft graph.
$\Rightarrow td_{GA,V}(uv) = k$, for all $uv \in E$ and $td_{GA,V}(uv) = r$, for all $uv \in E$.
Now,
$td_{GA,V}(uv) = d_{GA,V}(uv) + \sum e \in A\ \mu_e(uv)$.
$\Rightarrow r = k + \sum e \in A\ \mu_e(uv)$ for all $uv \in E$.
$\Rightarrow \sum e \in A\ \mu_e(uv) = r - k = c$, a constant for all $uv \in E$.
Hence K is constant fuction.

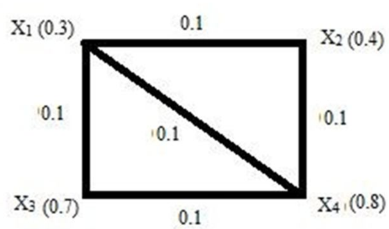
5) *Note:4.5*

Converse of the above theorem is not true.
Consider the following example,

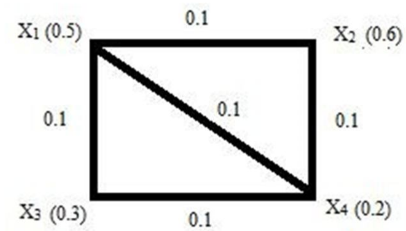
$F(e_1) = \{ X_1| 0.5 , X_2| 0.6 , X_3| 0.7, X_4| 0.8 \}$
$F(e_2) = \{ X_1| 0.3 , X_2| 0.4 , X_3| 0.7, X_4| 0.8 \}$
$F(e_3) = \{ X_1| 0.5 , X_2| 0.6 , X_3| 0.3, X_4| 0.2 \}$
$F(e_4) = \{ X_1| 0.1 , X_2| 0.2 , X_3| 0.3, X_4| 0.4 \}$
and
$K(e_1) =\{ X_1X_2| 0.1 , X_2X_4| 0.1 , X_3X_4| 0.1, X_1X_3| 0.1 , X_1X_4| 0.1 \}$
$K(e_2) = =\{ X_1X_2| 0.1 , X_2X_4| 0.1 , X_3X_4| 0.1, X_1X_3| 0.1 , X_1X_4| 0.1 \}$
$K(e_3) = =\{ X_1X_2| 0.1 , X_2X_4| 0.1 , X_3X_4| 0.1, X_1X_3| 0.1 , X_1X_4| 0.1 \}$
$K(e_4) = =\{ X_1X_2| 0.1 , X_2X_4| 0.1 , X_3X_4| 0.1, X_1X_3| 0.1 , X_1X_4| 0.1 \}$



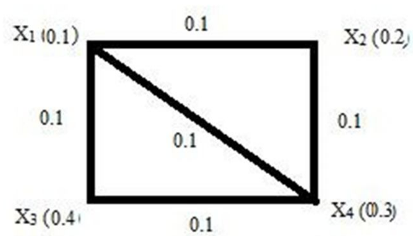
HA,V (e1)



HA,V (e2)



HA,V (e3)



HA,V (e4)

Here,
K is a constant function for all e belongs to A. But it is not both edge regular and totally edge regular fuzzy soft graph.

6) *Theorem: 4.6* Let $G_{A,V} = (G^*, F, K ,A)$ be a fuzzy soft graph on $G^* = (V ,E)$. Let $G_{A,V}$ is both edge regular and totally edge regular . Then $G_{A,V}$ is a regular fuzzy soft graph iff G^* is a regular graph.

Proof
Let $G_{A,V} = (G^*, F, K ,A)$ be a fuzzy soft graph on $G^* = (V ,E)$.
Let $G_{A,V}$ is both edge regular and totally edge regular fuzzy soft graph.
Assume that $G_{A,V}$ is a regular fuzzy soft graph.
$\Rightarrow H_{A,V}(e)$ is a regular fuzzy graph.
By theorem 2.14 , We have $H_{A,V}(e)$ is a regular fuzzy graph iff G^* is a regular graph for all $e \in A$.
$\Rightarrow G_{A,V}(e)$ is a regular fuzzy soft graph.

7) *Remark:4.7* Converse of the above theorem is nottrue.
Consider the Example : 1
Here , G^* is regular but $G_{A,V}$ is neither edge regular nor totally edge regular fuzzy soft graph.

8) *Theorem : 4.8*
Let $G_{A,V} = (G^*, F, K ,A)$ be a fuzzy soft graph on $G^* = (V ,E)$. Let K is a constant function. . If $G_{A,V}$ is k- regular fuzzy soft graph then $G_{A,V}$ is edge regular fuzzy soft graph.

Proof
Let $G_{A,V} = (G^*, F, K ,A)$ be a fuzzy soft graph on $G^* = (V ,E)$.
Assume that K is a constant function.
Also assume that $G_{A,V}$ is k- regular fuzzy soft graph.
$\Rightarrow d_{GA,V}(u) = k$, for all $u \in V$.
Since K is a constant function, $K(e)(uv) = c$, a constant for all $uv \in E$ and $e \in A$.
$\Rightarrow \sum e \in A\ \mu_e(uv) = c$ for all $uv \in E$ and $e \in A$.

Now,

$d_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) – 2 (\sum e\in A\ \mu_e(uv))$

$\Rightarrow d_{GA,V}(uv) = k + k – 2c$ for all $uv\in E$

$\Rightarrow d_{GA,V}(uv) = 2k – 2c$ for all $uv\in E$

$\Rightarrow d_{GA,V}(uv) = 2(k – c) = r$ (say) for all $uv\in E$

$\Rightarrow G_{A,V}$ is edge regular fuzzy soft graph.

9) *Theorem : 4.9* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$. Let K is a constant function. . If $G_{A,V}$ is k- regular fuzzy soft graph then $G_{A,V}$ is totally edge regular fuzzy soft graph.

Proof

Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$.

Assume that K is a constant function.

Also assume that $G_{A,V}$ is k- regular fuzzy soft graph.

$\Rightarrow d_{GA,V}(u) = k$, for all $u\in V$.

Since K is a constant function, $K(e)(uv) = c$, a constant for all $uv\in E$ and $e\in A$.

$\Rightarrow \sum e\in A\ \mu_e(uv) = c$ for all $uv\in E$ and $e\in A$.

Now,

$td_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) – (\sum e\in A\ \mu_e(uv))$

$\Rightarrow t\,d_{GA,V}(uv) = k + k – c$ for all $uv\in E$

$\Rightarrow t\,d_{GA,V}(uv) = 2k – c$ for all $uv\in E$

$\Rightarrow td_{GA,V}(uv) = (2k – c) = r$ (say) for all $uv\in E$

$\Rightarrow G_{A,V}$ is a totally edge regular fuzzy soft graph.

10) *Theorem: 4.10:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$. K is a constant function iff the following conditions are equivalent.

a) $G_{A,V}$ is a regular fuzzy soft graph.

b) $G_{A,V}$ is an edge regular fuzzy soft graph.

c) $G_{A,V}$ is a totally edge regular fuzzy soft graph.

11) *Theorem : 4.11:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$ and K is a constant function. If $G_{A,V}$ is an edge regular fuzzy soft graph iff G^* is a edge regular graph.

a) *Proof:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$.

let K be a constant function.

Assume that $G_{A,V}$ is an edge regular fuzzy soft graph.

Since K is a constant function, $K(e)(uv) = c$, a constant for all $uv\in E$ and $e\in A$.

$\Rightarrow \sum e\in A\ \mu_e(uv) = c$ for all $uv\in E$ and $e\in A$.

b) Claim : G^* is a edge regular graph..

Suppose G^* is a not an edge regular graph, $d_{G^*}(uv) \neq d_{G^*}(xy)$ for some uv , $xy \in E$.

Now,

$d_{GA,V}(uv) = \{ \sum e\in A \sum w\neq v\ \mu_e(uw) \} + \{\sum e\in A \sum w\neq u\ \mu_e(wv) \}$

$\Rightarrow d_{GA,V}(uv) = \{ \sum w\neq v \sum e\in A\ \mu_e(uw) \} + \{ \sum w\neq u \sum e\in A\ \mu_e(wv)\}$

$\Rightarrow d_{GA,V}(uv) = \{ \sum w\neq v\ c \} + \{ \sum w\neq u \sum e\in A\ c\}$

$\Rightarrow d_{GA,V}(uv) = c\,(d_{G^*}(u) – 1) + c\,(d_{G^*}(v) – 1)$

$\Rightarrow d_{GA,V}(uv) = c\,(d_{G^*}(u) + d_{G^*}(v) – 2)$

$\Rightarrow d_{GA,V}(uv) = c\,(d_{G^*}(uv))$

Similarly , $d_{GA,V}(xy) = c\,(d_{G^*}(xy))$

Since $d_{G^*}(uv) \neq d_{G^*}(xy)$, we have $d_{GA,V}(uv) \neq d_{GA,V}(xy)$ which is a contradiction.

Hence G^* is a edge regular graph.

Conversely,

Assume that G^* is a edge regular graph.

c) *Claim :* $G_{A,V}$ is an edge regular fuzzy soft graph.

Suppose $G_{A,V}$ is a not an edge regular graph, $d_{GA,V}(uv) \neq d_{GA,V}(xy)$ for some uv , $xy \in E$.

$\Rightarrow \{\sum e\in A \sum w\neq v\ \mu_e(uw)\}+\{\sum e\in A \sum w\neq u\ \mu_e(wv)\}\neq\{\sum e\in A \sum x\neq z\ \mu_e(xz)\}+\{\sum e\in A \sum z\neq y\ \mu_e(zy)\}$

$\Rightarrow \{\sum w\neq v \sum e\in A\ \mu_e(uw)\}+\{\sum w\neq u \sum e\in A\ \mu_e(wv)\}\neq\{\sum x\neq z \sum e\in A\ \mu_e(xz)\}+\{\sum z\neq y \sum e\in A\ \mu_e(zy)\}$
$\Rightarrow \{\sum w\neq v\ c\}+\{\sum w\neq u\ c\}\neq\{\sum x\neq z\ c\}+\{\sum z\neq y\ c\}$
$\Rightarrow c\,(d_{G^*}(u) - 1) + c\,(d_{G^*}(v) - 1) \neq c\,(d_{G^*}(x) - 1) + c\,(d_{G^*}(y) - 1)$
$\Rightarrow c\,(d_{G^*}(u) + d_{G^*}(v) - 2) \neq c\,(d_{G^*}(x) + d_{G^*}(y) - 2)$
$\Rightarrow c\,(d_{G^*}(uv)) \neq c\,(d_{G^*}(xy))$
$\Rightarrow d_{G^*}(uv) \neq (d_{G^*}(xy)$
$\Rightarrow G^*$ is not an edge regular graph , which is a contradiction.
Hence $G_{A,V}$ is an edge regular graph.

12) *Theorem : 4.12:* Let $G_{A,V} = (G^*, F, K, A)$ be a k- regular fuzzy soft graph on $G^* = (V, E)$. Then $G_{A,V}$ is an edge regular fuzzy soft graph iff K is a constant function.

a) *Proof:* Let $G_{A,V} = (G^*, F, K, A)$ be a fuzzy soft graph on $G^* = (V, E)$.
Assume that $G_{A,V}$ is k- regular fuzzy soft graph.
$\Rightarrow d_{GA,V}(u) = k$, for all $u\in V$.
Also assume that K is a constant function.
$\Rightarrow K(e)(uv) = c$, a constant for all $uv \in E$ and $e\in A$.
$\Rightarrow \sum e\in A\ \mu_e(uv) = c$ for all $uv \in E$ and $e\in A$.

b) *Claim :* $G_{A,V}$ is edge regular fuzzy soft graph.
W.K.T , $d_{GA,V}(uv) = d_{GA,V}(u) + d_{GA,V}(v) - 2\,(\sum e\in A\ \mu_e(uv))$
$\Rightarrow d_{GA,V}(uv) = k + k - 2\,c$ for all $uv\in E$
$\Rightarrow d_{GA,V}(uv) = 2k - 2\,c$ for all $uv\in E$
$\Rightarrow d_{GA,V}(uv) = 2(k - c) = r$ (say) for all $uv\in E$
$\Rightarrow G_{A,V}$ is edge regular fuzzy soft graph.
Conversely ,
Assume that $G_{A,V}$ is edge regular fuzzy soft graph.

c) *Claim:* K is a constant function.
i.e) To Prove : $\sum e\in A\ \mu_e(uv) = c$ for all $uv \in E$ and $e\in A$.
Since $G_{A,V}$ is edge regular fuzzy soft graph , $d_{GA,V}(uv) = r$ for all $uv\in E$
$\Rightarrow d_{GA,V}(u) + d_{GA,V}(v) - 2\,(\sum e\in A\ \mu_e(uv)) = r$ for all $uv\in E$
$\Rightarrow k + k - 2\,(\sum e\in A\ \mu_e(uv)) = r$ for all $uv\in E$
$\Rightarrow (\sum e\in A\ \mu_e(uv)) = (r - 2k) / 2$ for all $uv\in E$
Hence K is a constant function.

V. PROPERTIES OF AN EDGE REGULAR FUZZY SOFT GRAPH

A. *Theorem : 5.1*

Let $G_{A,V} = (G^*, F, K, A)$ be a k – edge regular fuzzy soft graph on $G^*: (V, E)$. Then $S(G_{A,V}) = qc$, where $q = |E|$ and c is a constant.

1) Proof: Let $G_{A,V} = (G^*, F, K, A)$ be a k – edge regular fuzzy soft graph on $G^*: (V, E)$.
The Size of $G_{A,V} = S(G_{A,V}) = \sum uv\in E \sum e\in A\ \mu_e(uv)$.
Since $G_{A,V} = (G^*, F, K, A)$ is a k – edge regular fuzzy soft graph , K is a constant function.
$\Rightarrow (\sum e\in A\ \mu_e(uv)) = c$, for all $uv\in E$.
$\Rightarrow S(G_{A,V}) = \sum uv\in E\ c$, Where c is a constant.
$\Rightarrow S(G_{A,V}) = qc$, where $q = |E|$ and c is a constant.

B. *Theorem : 5.2*

Let $G_{A,V} = (G^*, F, K, A)$ be a k – edge regular and r – totally edge fuzzy soft graph on $G^*: (V, E)$. Then $S(G_{A,V}) = q\,(r - k)$, where $q = |E|$.

1) Proof : Let $G_{A,V} = (G^*, F, K, A)$ be a k – edge regular and r – totally edge fuzzy soft graph on $G^*: (V, E)$.
Since $G_{A,V} = (G^*, F, K, A)$ is a k – edge regular edge fuzzy soft graph , $d_{GA,V}(uv) = k$, for all $uv\in V$.
Since $G_{A,V} = (G^*, F, K, A)$ is a r – edge regular edge fuzzy soft graph , $td_{GA,V}(uv) = r$, for all $uv\in V$.
Now ,
$\sum uv\in E\ td_{GA,V}(uv) = \sum uv\in E\ d_{GA,V}(uv) + \sum uv\in E \sum e\in A\ \mu_e(uv)$.

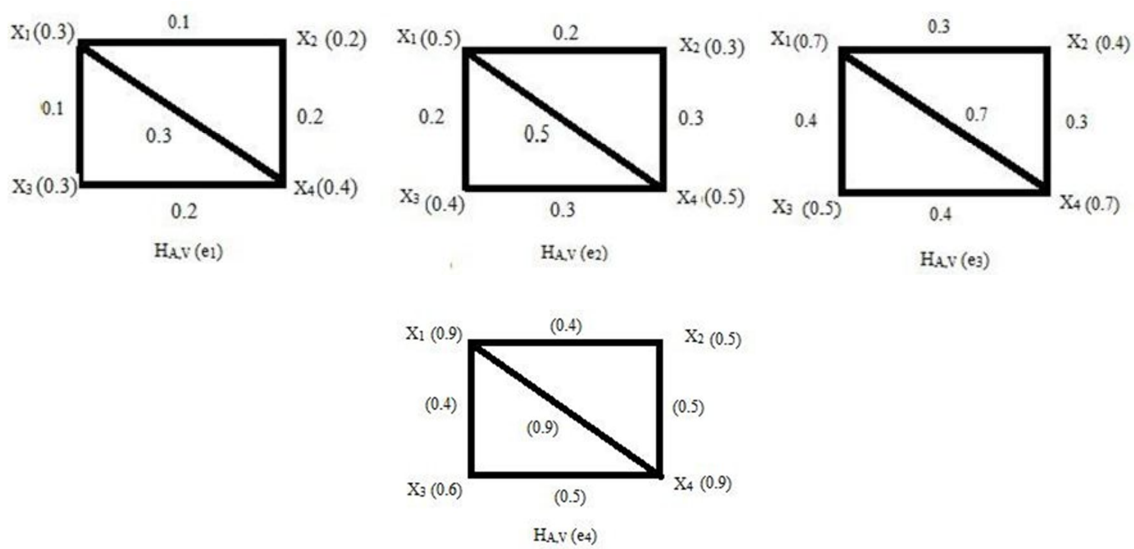
$\Rightarrow \sum uv \in E\ r = \sum uv \in E\ k + S(G_{A,V})$
$\Rightarrow$ q r = q k + $S(G_{A,V})$ where q = |E| .
$\Rightarrow S(G_{A,V})$ = q r - q k where q = |E| .
$\Rightarrow S(G_{A,V})$ = q (r - k) where q = |E| .

a) Proposition : 1

An Edge regular fuzzy soft graph need not be a totally edge regular fuzzy soft graph.

Consider the following example ,

$F(e_1) = \{ X_1| 0.3 , X_2| 0.2 , X_3| 0.3, X_4| 0.4 \}$
$F(e_2) = \{ X_1| 0.5 , X_2| 0.3 , X_3| 0.4, X_4| 0.5 \}$
$F(e_3) = \{ X_1| 0.7 , X_2| 0.4 , X_3| 0.5, X_4| 0.6 \}$
$F(e_4) = \{ X_1| 0.9 , X_2| 0.5 , X_3| 0.6, X_4| 0.7 \}$
and
$K(e_1) =\{ X_1X_2| 0.1 , X_2X_4| 0.2 , X_3X_4| 0.2, X_1X_3| 0.1 , X_1X_4| 0.3 \}$
$K(e_2) = =\{ X_1X_2| 0.2 , X_2X_4| 0.3 , X_3X_4| 0.3, X_1X_3| 0.2 , X_1X_4| 0.5 \}$
$K(e_3) = =\{ X_1X_2| 0.3 , X_2X_4| 0.4 , X_3X_4| 0.4, X_1X_3| 0.3 , X_1X_4| 0.7 \}$
$K(e_4) = =\{ X_1X_2| 0.4 , X_2X_4| 0.5 , X_3X_4| 0.5, X_1X_3| 0.4 , X_1X_4| 0.9 \}$



$d_{GA,V}(x_1)$ = 0.2 + 0.4 + 0.6 + 0.8 + 0.3 + 0.5 + 0.7 +0.9 = 4.4.
$d_{GA,V}(x_2)$ = 0.3 + 0.5 +0.7 +0.9 = 2.4.
$d_{GA,V}(x_3)$ = 0.3 + 0.5 +0.7 +0.9 = 2.4.
$d_{GA,V}(x_4)$ = 0.4 + 0.6 + 0.8 + 1.0 + 0.3 + 0.5 +0.7 +0.9 = 5.2.
Now ,
$d_{GA,V}(x_1x_2)$ = 4.4 + 2.4 – 2 (0.1 + 0.2 + 0.3 + 0.4) =4.8.
$d_{GA,V}(x_1x_3)$ = 4.4 + 2.4 – 2 (0.1 + 0.2 + 0.3 + 0.4) =4.8.
$d_{GA,V}(x_1x_4)$ = 4.4 + 5.2 – 2 (0.3 + 0.5 + 0.7 + 0.9) =4.8.
$d_{GA,V}(x_2x_4)$ = 2.4 + 5.2 – 2 (0.2 + 0.3 + 0.4 + 0.5) =4.8.
$d_{GA,V}(x_3x_4)$ = 2.4 + 5.2 – 2 (0.2 + 0.3 + 0.4 + 0.5) =4.8.
Hence it an edge regular fuzzy soft graph.
But ,
$td_{GA,V}(x_1x_2) = d_{GA,V}(x_1) + d_{GA,V}(x_2) – (\sum e \in A\ \mu_e(x_1x_2))$.
$\Rightarrow td_{GA,V}(x_1x_2)$ = 4.8 + (0.1 + 0.2 + 0.3 + 0.4) = 5.8.
$td_{GA,V}(x_1x_4) = d_{GA,V}(x_1) + d_{GA,V}(x_4) – (\sum e \in A\ \mu_e(x_1x_4))$.
$\Rightarrow td_{GA,V}(x_1x_4)$ = 4.8 + (0.3 + 0.5 + 0.7 + 0.9) = 7.2.

Hence it is not a totally edge regular fuzzy soft graph.

b) Proposition : 2

A totally edge regular fuzzy soft graph need not be an edge regular fuzzy soft graph.

Consider the following example ,

$F(e_1) = \{ X_1 | 0.5 , X_2 | 0.1 , X_3 | 0.2 \}$

$F(e_2) = \{ X_1 | 0.6 , X_2 | 0.2 , X_3 | 0.3 \}$

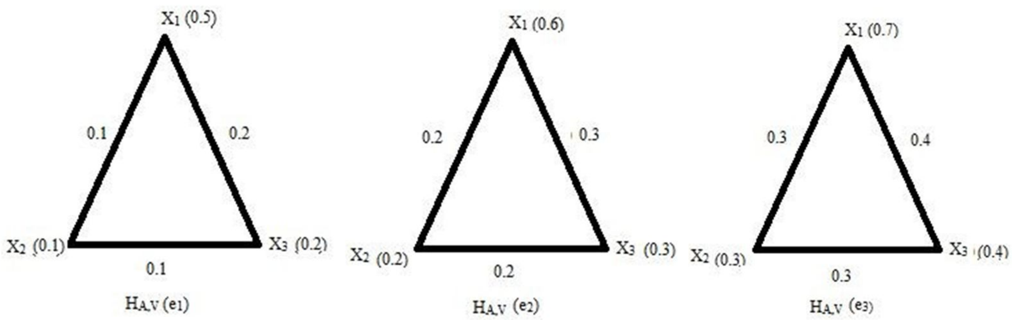
$F(e_3) = \{ X_1 | 0.7 , X_2 | 0.3 , X_3 | 0.4 \}$

and

$K(e_1) = \{ X_1 X_2 | 0.1 , X_1 X_3 | 0.2 , X_2X_3 | 0.1 \}$

$K(e_2) = \{ X_1 X_2 | 0.2 , X_1 X_3 | 0.3 , X_2X_3 | 0.2 \}$

$K(e_3) = \{ X_1 X_2 | 0.3 , X_1 X_3 | 0.4 , X_2X_3 | 0.3\}$



$d_{GA,V}(x_1) = 0.3 + 0.5 + 0.7 = 1.5.$

$d_{GA,V}(x_2) = 0.2 + 0.4 + 0.6 = 1.2.$

$d_{GA,V}(x_3) = 0.3 + 0.5 + 0.7 = 1.5.$

Now ,

$td_{GA,V}(x_1x_2) = 1.5 + 1.2 – (0.1 + 0.2 + 0.3) = 2.1.$

$td_{GA,V}(x_1x_3) = 1.5 + 1.5 – (0.2 + 0.3 + 0.4) = 2.1.$

$td_{GA,V}(x_2x_3) = 1.2 + 1.5 – 2 (0.1 + 0.2 + 0.3) = 2.1.$

Hence it totally edge regular fuzzy soft graph.

But ,

$d_{GA,V}(x_1x_2) = d_{GA,V}(x_1) + d_{GA,V}(x_2) – 2(\sum e\in A\ \mu_e(x_1x_2)).$

$\Rightarrow d_{GA,V}(x_1x_2) = 1.5 + 1.2 – 2(0.1 + 0.2 + 0.3) = 1.5.$

$d_{GA,V}(x_1x_4) = d_{GA,V}(x_1) + d_{GA,V}(x_4) – 2(\sum e\in A\ \mu_e(x_1x_4)).$

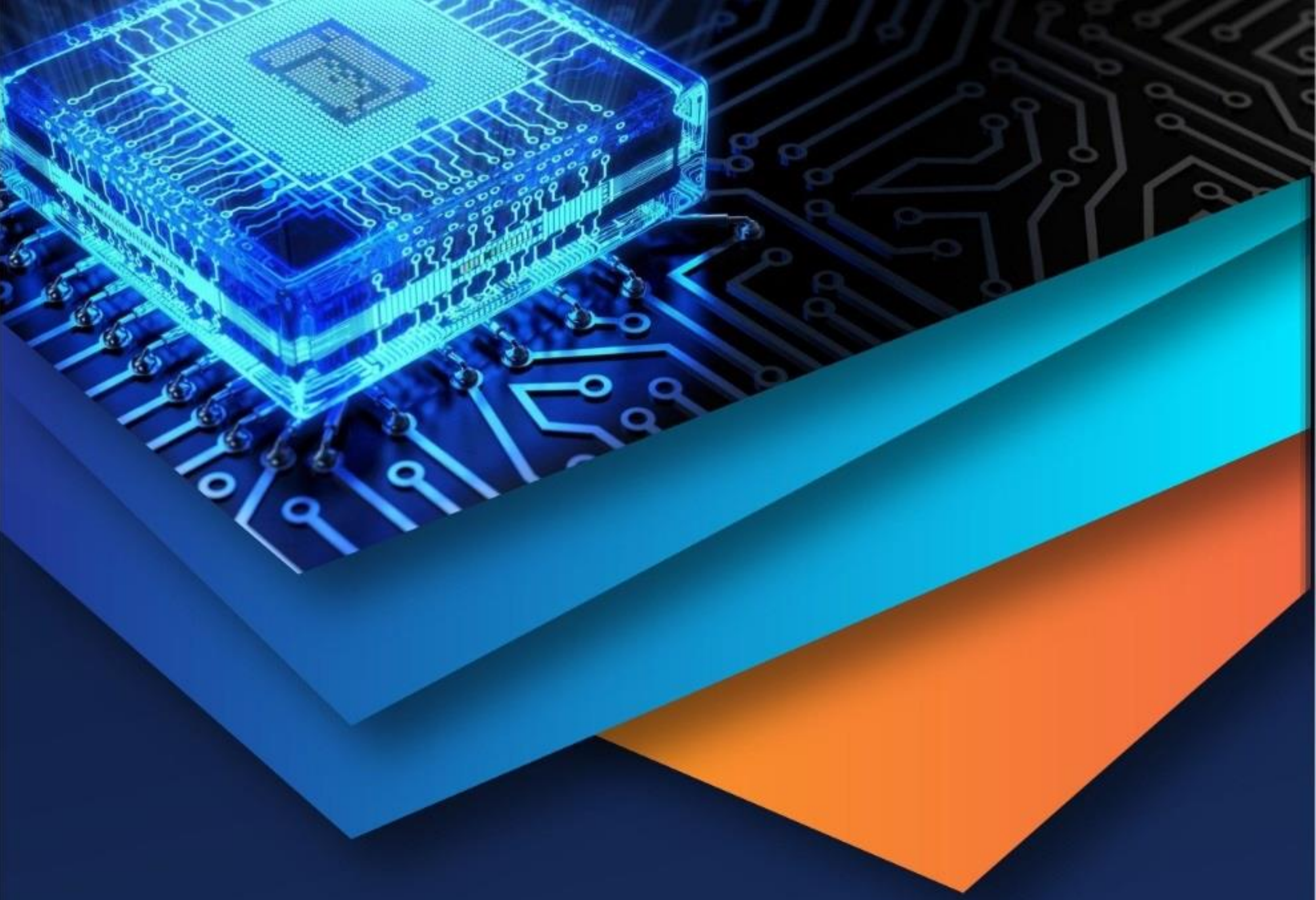
$\Rightarrow d_{GA,V}(x_1x_4) = 1.5 + 1.5 – (0.2 + 0.3 + 0.4) = 1.2.$

Hence it is not an edge regular fuzzy soft graph.

REFERENCES

[1] Bhattacharya, P., Some Remarks on Fuzzy Graphs, Pattern Recognition Lett. 6(1987) 297-302.
[2] Bhutani, K.R., On Automorphism of fuzzy Graphs, Pattern Recognition Letters 12:413-420, 1991.
[3] Frank Harary, Graph Thoery, Narosa /Addison Wesley, Indian Student Edition,1988.
[4] John N. Mordeson and Premchand S.Nair, Fuzzy Graphs and Fuzzy Hypergrphs, Physica-verlag, Heidelberg2000.
[5] Nagoor Gani, A., and Basheer Ahamed, M., Order and Size in Fuzzy Graph, Bulletin of Pure and Appplied Sciences, Vol.22E (No.1) 2003, 145-148.
[6] Rosenfeld, A., Fuzzy Graphs, In: L.A.Zadeh, K.S.Fu, M.Shimura, Eds., Fuzzy Sets and Their Applications, Academic Press (1975), 77-95.
[7] Sunitha,M.S., and Vijayakumar, A.A., Characterization of Fuzzy Trees, Information Sciences, 113(1999), 293-300.
[8] A. Aygünoglu, H. Aygün, Introduction to fuzzy soft groups, Computers and Mathematics with Applications 58 (2009), 12791286.
[9] L. A. Zadeh, Fuzzy sets, Information and control 8, (1965), 338353.
[10] A. Rosenfeld, Fuzzy graphs, Fuzzy sets and their Applications ,(L. A. Zadeh, K. S. Fu, M. Shimura, Eds) Academic Press, New York (1975) 7795
[11] B. Dinda, T. K. Samanta, Relations on intuitionistic fuzzy soft sets, General Mathematics Notes 1(2) (2010), 7483.
[12] D. Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications 37(4-5) (1999) 19-31.
[13] J. Ghosh, T. K. Samanta, S. K. Roy, A Note on Operations of Intuitionistic Fuzzy Soft Sets, Journal of Hyperstructures 2 (2) (2013), 163-184.
[14] J. Ghosh, B. Dinda, T. K. Samanta, Fuzzy soft rings and fuzzy soft ideals, Int. J. Pure Appl. Sci. Technol., 2(2) (2011), 66-74.

[15] J. N. Mordeson and C. S. Peng, Operations on fuzzy graphs, Information science, 79, September 1994, 159-170.
[16] K. R. Bhutani and A. Rosenfeld, Fuzzy end nods in fuzzy graphs, Information science, 152, June 2003, 323-326.
[17] L. A. Zadeh, Fuzzy sets, Information and control 8, (1965), 338353.
[18] M. S. Sunitha and A. Vijayakumar, Complement of a fuzzy graph, Indian J. Pure and Applied Math. 33, No.9, 2002, 1451-1464.
[19] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letters 6, (1987) 297302.
[20] P. K. Maji, A. R. Roy, R. Biswas, Fuzzy soft sets, The Journal of fuzzy mathematics 9(3) (2001), 589602.
[21] P. K. Maji, A. R. Roy, R. Biswas, An application of soft sets in a decision making problem, Computers and Mathematics with applications 44(89) (2002), 10771083.
[22] S. Roy, T.K. Samanta, A note on a Soft Topological Space, Punjab University Journal of Mathematics, 46(1) (2014) 19 - 24.
[23] S. Roy, T.K. Samanta, A note on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics 3(2) (2012) 305-311. 48 An Introduction to Fuzzy Soft Graph
[24] S. Roy, T.K. Samanta, An introduction of open and closed sets on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics 6(2) (2013) 425-431.
[25] Muhammad Akram and Saira Nawaz , On fuzzy soft graphs, italian journal of pure and applied mathematics { n. 34¡2015 (497¡5
[26] K. Radha and N. Kumaravel, on edge regular fuzzy graph. International Journal of Mathematical Archive,100 – 112.
[27] B. Akilandeswari, on regular fuzzy soft graph. International journal of scientific research and engineering trends,831 – 834..



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