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Sufficient Condition for Wavelet Frame on Positive Half-Line

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Abstract: In this paper, we present p-wavelet frames on positive half-line using Walsh-Fourier transform and also prove a sufficient condition for the system $\{\psi_{j,k}: j \in \mathbb{Z},\ k \in \mathbb{Z}^+\}$ to be a wavelet frame in $L^2(\mathbb{R}^+)$.

I. INTRODUCTION

In recent years, wavelets have been generalized in many different settings, for example locally compact abelian group, abstract Hilbert spaces, locally compact Cantor dyadic group, Vilenkin group, local fields and positive half-line. In this paper our interest is in positive half-line. Farkov [12] has given general construction of compactly supported orthogonal p-wavelets in $L^2(\mathbb{R}^+)$. Farkov et al. [13] gave an algorithm for biorthogonal wavelets related to Walsh functions on positive half line. Shah and Debnath [19], constructed wavelet frame packets on the positive half-line $\mathbb{R}^+$ using the splitting trick for frames. Abdullah [3] has given characterization of nonuniform wavelet sets on positive half-line. The characterization of wavelets on positive half line by means of two basic equations in the Fourier domain established in [1]. A constructive procedure for constructing tight wavelet frames on positive half-line using extension principles was recently considered by Shah in [17], in which he has pointed out a method for constructing affine frames in $L^2(\mathbb{R}^+)$. Moreover, the author has established sufficient conditions for a finite number of functions to form a tight affine frames for $L^2(\mathbb{R}^+)$.

In this paper, we investigate wavelet frames on positive half-line $\mathbb{R}^+$. The main result presented in this paper is sufficient condition for the system $\{\psi_{j,k}(x): j \in \mathbb{Z},\ k \in \mathbb{Z}^+\}$ to be a wavelet frame in $L^2(\mathbb{R}^+)$.

II. NOTATIONS AND PRELIMINARIES

Let p be a fixed natural number greater than 1. As usual, let $\mathbb{R}^+ = [0,\infty)$, $\mathbb{N} = \{1,2,\dots\}$ and $\mathbb{Z}^+ = \{0,1,\dots\}$. Denote by $[x]$ the integer part of x. For $x \in \mathbb{R}^+$ and for any positive integer j, we set

$$x_j = [p^j x](\mathrm{mod}\, p), \qquad x_{-j} = [p^{1-j}x](\mathrm{mod}\, p), \tag{2.1}$$

where $x_j,\ x_{-j} \in \{0,1,\dots,p-1\}$.

Consider the addition defined on $\mathbb{R}^+$ as follows:

$$x \oplus y = \sum_{j<0} \xi_j p^{-j-1} + \sum_{j>0} \xi_j p^{-j} \tag{2.2}$$

with

$$\xi_j = x_j + y_j(\mathrm{mod}\, p), \qquad j \in \mathbb{Z}\setminus\{0\}, \tag{2.3}$$

where $\xi_j \in \{0,1,\dots,p-1\}$ and $x_j,\ y_j$ are calculated by (2.1). Moreover, we write $z = x \ominus y$ if $z \oplus y = x$, where $\ominus$ denotes subtraction modulo p in $\mathbb{R}^+$.

For $x \in [0,1)$, let $r_0(x)$ be given by

$$r_0(x) = \begin{cases} 1, & x \in \left[0,\frac{1}{p}\right) \\ \epsilon_p^j, & x \in [jp^{-1},(j+1)p^{-1}),\ \ j = 1,2,\dots,p-1, \end{cases} \tag{2.4}$$

where $\epsilon_p = \exp\left(\frac{2\pi i}{p}\right)$. The extension of the function r_0 to $\mathbb{R}^+$ is defined by the equality $r_0(x+1) = r_0(x)$, $x \in \mathbb{R}^+$. Then the generalized Walsh functions $\{\omega_m(x)\}_{m\in\mathbb{Z}^+}$ are defined by

$$\omega_0(x) = 1, \qquad \omega_m(x) = \prod_{j=0}^{k}\left(r_0(p^j x)\right)^{\mu_j},$$

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where $m = \sum_{j=0}^{k} \mu_j p^j, \ \mu_j \in \{0,1,2,\dots,p-1\}, \mu_k \neq 0$.
For $x, \ \omega \in \mathbb{R}^+$, let

$$\chi(x,\omega) = \exp\left(\frac{2\pi i}{p}\sum_{j=1}^{\infty}(x_j\omega_{-j} + x_{-j}\omega_j)\right), \tag{2.5}$$

where x_j and ω_j are calculated by (2.1).
We observe that

$$\chi\left(x,\frac{m}{p^{n-1}}\right) = \chi\left(\frac{x}{p^{n-1}}, m\right) = \omega_m\left(\frac{x}{p^{n-1}}\right) \qquad \forall x \in [0,p^{n-1}), \qquad m \in \mathbb{Z}^+.$$

The Walsh-Fourier transform of a function $f \in L^1(\mathbb{R}^+)$ is defined by

$$\tilde{f}(\omega) = \int_{\mathbb{R}^+} f(x)\,\overline{\chi(x,\omega)}\,dx, \tag{2.6}$$

where $\chi(x,\omega)$ is given by (2.5).
If $f \in L^2(\mathbb{R}^+)$ and

$$J_a f(\omega) = \int_0^a f(x)\,\overline{\chi(x,\omega)}\,dx \qquad (a < 0), \tag{2.7}$$

then $\tilde{f}$ is defined as limit of $J_a f$ in $L^2(\mathbb{R}^+)$ as $a \longrightarrow \infty$.
The properties of Walsh-Fourier transform are quite similar to the classical Fourier transform. It is known that systems $\{\chi(\alpha,.)\}_{\alpha=0}^{\infty}$ and $\{\chi(.,\alpha)\}_{\alpha=0}^{\infty}$ are orthonormal bases in $L^2(0,1)$. Let us denote by $\{\omega\}$ the fractional part of ω. For $l \in \mathbb{Z}^+$, we have $\chi(l,\omega) = \chi(l,\{\omega\})$.
If $x, y, \omega \in \mathbb{R}^+$ and $x \oplus y$ is p-adic irrational, then

$$\chi(x \oplus y, \omega) = \chi(x,\omega)\,\chi(y,\omega), \quad \chi(x \ominus y, \omega) = \chi(x,\omega)\,\overline{\chi(y,\omega)}, \tag{2.8}$$

By a p-adic interval of range n in [0,1), we mean intervals of the form

$$I_{(n)}^k = [k2^{-n}, (k+1)2^{-n}), \qquad k \in \mathbb{Z}^+.$$

It is easy to verify that

$$I_{(n)}^k \cap I_{(n)}^l = \emptyset, \qquad k \neq l \ \ and \ \bigcup_{k=0}^{2^n-1} I_{(n)}^k = [0,1).$$

For each $x \in [0,1)$ and $n \in \mathbb{N}$, we denote the p-adic interval of length p^{-n} which contains x by $I_n(x)$. Thus

$$I_n(x) = I_{(n)}^k(x),$$

where $0 \leq k < p^n$ is uniquely determined by relationship $x \in I_n(x)$.

III. MAIN RESULTS

Let

$$\psi \in L^2(\mathbb{R}^+), \quad \psi_{j,k}(x) = p^{j/2}\,\psi(p^j x \ominus k), \qquad j \in \mathbb{Z}, k \in \mathbb{Z}^+. \tag{3.1}$$

By taking Walsh-Fourier transform, we obtain

$$\hat{\psi}_{j,k}(\xi) = p^{-j/2}\,\overline{\chi(k,p^{-j}\xi)}\,\hat{\psi}(p^{-j}\xi). \tag{3.2}$$

Then, we call the function system $\{\psi_{j,k}(x) : j \in \mathbb{Z}, \ k \in \mathbb{Z}^+\}$ a wavelet frame for $L^2(\mathbb{R}^+)$ if there are two constants A and B, $0 < A \leq B < \infty$ such that

$$A\|f\|^2 \leq \sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}^+}\left|\langle f,\psi_{j,k}\rangle\right|^2 \leq B\|f\|^2, \tag{3.3}$$

for all $f \in L^2(\mathbb{R}^+)$. The largest A and the smallest B for which (3.3) holds are called frame bounds. A frame is a tight frame if A and B are chosen so that $A = B$ and is a normalized tight frame if $A = B = 1$.
For $j \ \in \mathbb{Z}, \ l \in \mathbb{Z}^+$, we have

$$\int_{p^j l}^{p^j(l+1)} \omega_k(p^{-j}\xi)\,d\xi = \int_0^{p^j} \omega_k(p^{-j}(\xi + p^j l))\,d\xi = \int_0^{p^j} \omega_k(p^{-j}\xi)\,d\xi.$$

Let $f \in L^2(\mathbb{R}^+)$ and $\psi \in L^2(\mathbb{R}^+)$, then

$$\langle f, \psi_{j,k}\rangle = p^{-j/2}\int_0^{p^j}\left[\sum_{l\in\mathbb{Z}^+}\hat{f}(\xi\oplus p^j l)\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right]\omega_k(p^{-j}\xi)\, d\xi.$$

Applying Parseval's formula and the fact that $\{\omega_n : n\geq 0\}$ forms an orthonormal basis for $L^2[0,1)$, we obtain

$$\sum_{k\in\mathbb{Z}^+}\left|f,\psi_{j,k}\right|^2 = \int_{\mathbb{R}^+}\overline{\hat{f}(\xi)}\,\hat{\psi}(p^{-j}\xi)\left\{\sum_{l\in\mathbb{Z}^+}\hat{f}(\xi\oplus p^j l)\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right\} d\xi. \qquad (3.4)$$

Now, we first prove a lemma, which will be used in the proofs of the main results.

1) *Lemma3.1.* Let $\mathcal{D} = \{f\in L^2(\mathbb{R}^+): \operatorname{supp}\hat{f}\subset\mathbb{R}^+\backslash\{0\}\}$ and let $f\in\mathcal{D}$ and $\psi\in L^2(\mathbb{R}^+)$. If $\operatorname{ess\,sup}_{\xi\in\mathbb{R}^+}\sum_{j\in\mathbb{Z}}\left|\hat{\psi}(p^{-j}\xi)\right|^2 < +\infty$, then

$$\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}^+}\left|f,\psi_{j,k}\right|^2 = \int_{\mathbb{R}^+}\left|\hat{f}(\xi)\right|^2\sum_{j\in\mathbb{Z}}\left|\hat{\psi}(p^{-j}\xi)\right|^2 d\xi + R_\psi(f), \qquad (3.5)$$

where

$$R_\psi(f) = \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^+}\overline{\hat{f}(\xi)}\,\hat{\psi}(p^{-j}\xi)\,\hat{f}(\xi\oplus p^j l)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\, d\xi. \qquad (3.6)$$

Furthermore, the iterated series in (3.6) is absolutely convergent.

Proof. From (3.4), we have

$$\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}^+}\left|f,\psi_{j,k}\right|^2 = \sum_{j\in\mathbb{Z}}\int_{\mathbb{R}^+}\left\{\left|\hat{f}(\xi)\right|^2\left|\hat{\psi}(p^{-j}\xi)\right|^2 + \overline{\hat{f}(\xi)}\,\hat{\psi}(p^{-j}\xi)\right.$$
$$\left.\times\sum_{l\in\mathbb{Z}^+}\hat{f}(\xi\oplus p^j l)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right\} d\xi$$
$$= \sum_{j\in\mathbb{Z}}\int_{\mathbb{R}^+}\left|\hat{f}(\xi)\right|^2\left|\hat{\psi}(p^{-j}\xi)\right|^2 d\xi + R_\psi(f).$$

Since $\operatorname{ess\,sup}_{\xi\in\mathbb{R}^+}\sum_{j\in\mathbb{Z}}\left|\hat{\psi}(p^{-j}\xi)\right|^2 < +\infty$, and therefore, by the Levi lemma, we obtain

$$\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}^+}\left|f,\psi_{j,k}\right|^2 = \int_{\mathbb{R}^+}\left|\hat{f}(\xi)\right|^2\sum_{j\in\mathbb{Z}}\left|\hat{\psi}(p^{-j}\xi)\right|^2 d\xi + R_\psi(f).$$

Now we claim that the iterated series in (3.6) is absolutely convergent. To do this, let

$$I = \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^+}\left|\hat{f}(\xi)\,\hat{\psi}(p^{-j}\xi)\,\hat{f}(\xi\oplus p^j l)\hat{\psi}(p^{-j}\xi\oplus l)\right| d\xi$$
$$= \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}p^j\int_{\mathbb{R}^+}\left|\hat{f}(p^j\xi)\,\hat{\psi}(\xi)\,\hat{f}(p^j(\xi\oplus l))\hat{\psi}(\xi\oplus l)\right| d\xi.$$

Note that

$$\left|\hat{\psi}(\xi)\,\hat{\psi}(\xi\oplus l)\right| \leq \frac{1}{2}\left(\left|\hat{\psi}(\xi)\right|^2 + \left|\hat{\psi}(\xi\oplus l)\right|^2\right).$$

Therefore, it suffices to prove that

$$\sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}p^j\int_{\mathbb{R}^+}\left|\hat{f}(p^j\xi)\ \hat{f}(p^j\xi\oplus p^j l)\right|\left|\hat{\psi}(\xi)\right|^2 d\xi < \infty. \qquad (3.7)$$

Since $l\neq 0$ $(l\in\mathbb{N})$ and $f\in\mathcal{D}$, there exists $J>0$ such that for all $|j|>J$,

$$\hat{f}(p^j\xi)\ \hat{f}(p^j\xi\oplus p^j l) = 0.$$

On the other hand, for each fixed $|j|\leq J$ and $\xi\in\mathbb{R}^+$, there exists a constant L such that for all $l>L$,

$$\hat{f}(p^j\xi\oplus p^j l) = 0.$$

Thus, it follows that only a finite number of terms of the iterated series in (3.7) are non-zero. Consequently, there exists a constant C such thsat

$$I \leq C\,\|\hat{f}\|_{\infty}^{2}\|\hat{\psi}\|_{2}^{2}.$$

This shows that the iterated series in (3.6) is absolutely convergent.

The following necessary condition for the system $\{\psi_{j,k}: j\in\mathbb{Z},\ k\in\mathbb{Z}^{+}\}$ to be a frame was proved by Abdullah [2].

2) *Theorem 3.2.* If $\{\psi_{j,k}(x): j\in\mathbb{Z},\ k\in\mathbb{Z}^{+}\}$ is a wavelet frame for $L^{2}(\mathbb{R}^{+})$ with bounds A and B, then

$$A \leq \sum_{j\in\mathbb{Z}} \left|\hat{\psi}(p^{j}\xi)\right|^{2} \leq B, \quad a.e.\ \xi\in\mathbb{R}^{+}.$$

Now, in the following theorem we establish a sufficient condition of the system (3.1) to be a frame in $L^{2}(\mathbb{R}^{+})$

3) *Theorem 3.3.* Let $\psi\in L^{2}(\mathbb{R}^{+})$ such that

$$A(\psi) = \operatorname{ess\,inf}\left\{\sum_{j\in\mathbb{Z}}\left|\hat{\psi}(p^{-j}\xi)\right|^{2} - \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|^{2} : \xi\in\mathbb{R}^{+}\right\} > 0,$$

$$B(\psi) = \operatorname{ess\,sup}\left\{\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}^{+}}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus k)}\right|^{2} : \xi\in\mathbb{R}^{+}\right\} < +\infty.$$

Then $\{\psi_{j,k}(x): j\in\mathbb{Z}, k\in\mathbb{Z}^{+}\}$ is a wavelet frame for $L^{2}(\mathbb{R}^{+})$ with frame bounds $A(\psi)$ and $B(\psi)$.

Proof. We can estimate $R_{\psi}(f)$ by rearranging the series in (3.6),

$$\begin{aligned}
|R_{\psi}(f)| &= \left|\sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}\overline{\hat{f}(\xi)}\,\hat{\psi}(p^{-j}\xi)\,\hat{f}(\xi\oplus p^{j}l)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\,d\xi\right| \\
&\leq \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\left\{\int_{\mathbb{R}^{+}}|\hat{f}(\xi)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi\right\}^{\frac{1}{2}} \\
&\qquad \left\{\int_{\mathbb{R}^{+}}|\hat{f}(\xi\oplus p^{j}l)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi\right\}^{\frac{1}{2}} \\
&\leq \sum_{j\in\mathbb{Z}}\left\{\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}|\hat{f}(\xi)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi\right\}^{\frac{1}{2}} \\
&\qquad \left\{\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}|\hat{f}(\xi\oplus p^{j}l)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi\right\}^{\frac{1}{2}} \\
&= \sum_{j\in\mathbb{Z}}\left\{\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}|\hat{f}(\xi)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi\right\}^{\frac{1}{2}} \\
&\qquad \left\{\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}|\hat{f}(\omega)|^{2}\left|\hat{\psi}(p^{-j}\omega\ominus l)\,\overline{\hat{\psi}(p^{-j}\omega)}\right|\,d\xi\right\}^{\frac{1}{2}}.
\end{aligned}$$

Therefore

$$|R_{\psi}(f)| \leq \sum_{j\in\mathbb{Z}}\sum_{l\in\mathbb{N}}\int_{\mathbb{R}^{+}}|\hat{f}(\xi)|^{2}\left|\hat{\psi}(p^{-j}\xi)\,\overline{\hat{\psi}(p^{-j}\xi\oplus l)}\right|\,d\xi.$$

By Levi Lemma, we have

$$\left|R_{\psi}(f)\right| \leq \int_{\mathbb{R}^+} \left|\hat{f}(\xi)\right|^2 \left\{ \sum_{j\in\mathbb{Z}} \sum_{l\in\mathbb{N}} \left| \hat{\psi}(p^{-j}\xi)\, \overline{\hat{\psi}(p^{-j}\xi \oplus l)} \right| \right\} d\xi .$$

Applying (3.5), we have

$$\int_{\mathbb{R}^+} \left|\hat{f}(\xi)\right|^2 \left\{ \sum_{j\in\mathbb{Z}} \left|\hat{\psi}(p^{-j}\xi)\right|^2 - \sum_{j\in\mathbb{Z}} \sum_{l\in\mathbb{N}} \left| \hat{\psi}(p^{-j}\xi)\, \overline{\hat{\psi}(p^{-j}\xi \oplus l)} \right| \right\} d\xi \leq \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}^+} \left| f, \psi_{j,k} \right|^2, \quad (3.8)$$

and

$$\sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}^+} \left| f, \psi_{j,k} \right|^2 \leq \int_{\mathbb{R}^+} \left|\hat{f}(\xi)\right|^2 \left\{ \sum_{j\in\mathbb{Z}} \sum_{l\in\mathbb{N}} \left| \hat{\psi}(p^{-j}\xi)\, \hat{\psi}(p^{-j}\xi \oplus l) \right| \right\} d\xi . \quad (3.9)$$

Taking infimum in (3.8) and supremum in (3.9), respectively, we obtain

$$A(\psi)\|f\|_2^2 \leq \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}^+} \left| f, \psi_{j,k} \right|^2 \leq B(\psi)\|f\|_2^2,$$

which holds for all $f \in \mathcal{D}$. This completes the proof of the theorem.

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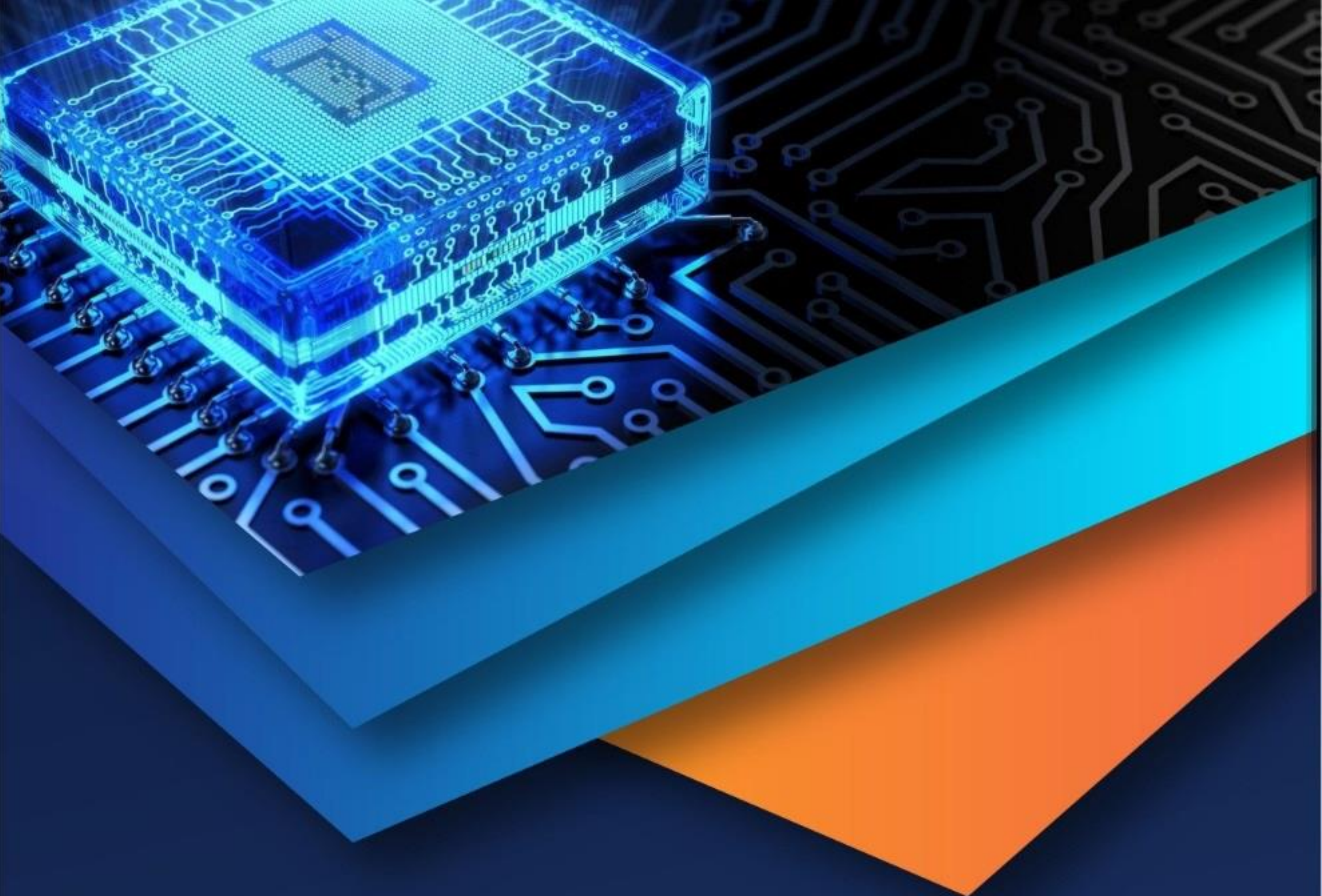

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