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# Investigation of Von - Mises Stress and Stress Concentration Factor for Cylinders with Holes in Different Locations and Varying Pressure using Analytical Method and FEA

Mr. D. L. Pavan Kumar<sup>1</sup>, Mr. M. Rambabu<sup>2</sup>

<sup>1</sup>PG Student, <sup>2</sup>Associate Professor, Viswanadha Institute of Technology and Management, Visakhapatnam, Andhra Pradesh, India

**Abstract:** In this paper, we conducted stress analysis of a thick walled cylinder with and without holes. The main aim of the work is to make a static structural analysis and analytical calculation for the thick walled cylinder with and without holes. In thick walled cylinder without holes the analysis is done for three cylinders with different dimensions and in case of cylinder with holes the analysis performed keeping the holes at different locations. The modelling is done in CATIA and finite element analysis is conducted using ANSYS. By using the properties and dimensions for the three cylinders with three different materials we obtain Maximum principle stress and Von-Mises stress in case of thick walled cylinder without holes. Von-mises stress and stress concentration factors are calculated in thick walled cylinder with holes in different four position with varying hole diameters using analytical and ANSYS. The analytical results are compared with the ANSYS results and shows that the analytical values are approximately coincides with the ANSYS results.

**Keyword:** Thick wall cylinder, lame's equation, von-mises stresses, stress concentration factor, Maximum principle stress.

## I. INTRODUCTION

A Cylinder is solid or hollow tube with long straight sides a two circular ends the same size, or an object shaped like this, often used as a container. There are two types of cylinders' thick wall cylinder and thin wall cylinder. Thick walled cylinders are those which have the thickness to internal diameter ratio more than 1/20. Large internal pressures in thick cylinders produce high tension hoop stress along the inner surface of the cylinder; the later may result in the development of stress in the internal surface of thick wall cylinder. In the current industrial scenario, these cylinders have a lot of applications mainly in the transportation of materials.

### A. Thick Wall Cylinder without Holes

Thick walled cylinders [3] are widely used in chemical, petroleum, military industries as well as in nuclear power plants. In this sector, they function as industrial compressed receiver or storage tank. They are usually subjected to high pressures & temperatures which may be constant or cycling. Industrial problems often witness ductile fracture of materials due to some discontinuity in geometry or material characteristics the conventional static analysis of thick walled cylinders to final radial & hoop stresses is applicable for the internal pressures up to yield strength of material. But the industrial cylinders often undergo pressure about yield strength of material. The analytical calculation was performed in cylinder with three different materials having constant height of 320 mm and 22 mm thickness. Thick walled cylinders with three different internal diameters of 220, 270, and 320 mm were used. End caps at both ends of pressure vessels are attached with 22 mm height and 42 mm thick in the radial direction.

$$\sigma_{\theta} = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} + \frac{(P_i - P_o) r_o^2 r_i^2}{r_o^2 - r_i^2} \quad (1)$$

$$\sigma_r = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} - \frac{(P_i - P_o) r_o^2 r_i^2}{r_o^2 - r_i^2} \quad (2)$$

$$\sigma_z = \frac{P_i}{k^2 - 1} \quad (3)$$

Equations 1-3 were used to get the Hoop, radial and longitudinal stress respectively for analytically results. Where 'r' is the variable radius, 'r<sub>i</sub>' and 'r<sub>o</sub>' are internal and external radii of the cylinder while P<sub>i</sub> and P<sub>o</sub> are the internal and external pressures in the cylinder. The above three stresses represents the three principal stresses acting on the cylinder and with the help of these equations Von Mises stress is also calculated using equation (4).

$$\sigma_{vm}^2 = \frac{(\sigma_{\theta} - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_{\theta})^2}{2} \quad (4)$$

From the above equations we obtained the Maximum principle stress (hoop stress) and Von Mises stress at varying pressure. The maximum von mises stress and maximum principle stress are maximum at the inner surface of the thick walled cylinder i.e.  $r = r_i$ .

### B. Thick Wall Cylinder with Holes

Failure of pressure vessels may cause an economic loss and fatal to human life [1, 2]. Most of the cylinders are incorporated with the openings in the main body for various reasons like fluid transfer, manhole openings, gauges etc. These holes, the geometric discontinuity of the cylinder causes the stress distribution near the vicinity of discontinuity.

### C. Circular Holes in an Infinite Plate under Uni-axial Tension.

The stress distributions around a central hole can be estimated for the simple case of an infinitely wide plate subjected to tensile loading.

$$\sigma_{rr} = \frac{\sigma}{2} \left( 1 - \frac{a^2}{r^2} \right) + \frac{\sigma}{2} \left( \left( 1 - \frac{a^2}{r^2} \right) \left( 1 - \frac{3a^2}{r^2} \right) \cos 2\theta \right) \quad (5)$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2} \left( 1 + \frac{a^2}{r^2} \right) - \frac{\sigma}{2} \left( 1 + \frac{3a^4}{r^4} \right) \cos 2\theta \quad (6)$$

$$\sigma_{r\theta} = -\frac{\sigma}{2} \left( 1 - \frac{a^2}{r^2} \right) \left( 1 + \frac{3a^4}{r^4} \right) \sin 2\theta \quad (7)$$

Where “ $\sigma$ ” is the magnitude of the remotely applied tensile stress,  $\sigma_{xx}^\infty$

From the above equations (5-7) it is obvious that all the stresses depend on the value of  $\theta$  and variable radius ‘ $r$ ’. Any small discontinuity in the form of hole give rise to maximum stress adjacent to the hole in comparison to the nominal stress away from the hole. In order to compare these two stresses stress concentration factor  $k_t$  is used. Using equation stress concentration factor [3] values are calculated.

$$\text{Stress concentration factor} = \frac{\text{maximum equivalent stress in thick wall cylinder with holes}}{\text{normal equivalent stress in thick wall cylinder without holes}} \quad (8)$$

$$k_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

## II. ANALYTICAL CALCULATIONS OF THICK WALLED CYLINDER WITH AND WITHOUT HOLES

Analytical calculations [5] of thick walled cylinder without holes were done by using the above equation (1-3). From these equations we get hoop, radial and longitudinal stress respectively for analytically result for three dimensional cylinders with varying material.

### A. Hoop Stress

Where  $r = 110$  mm,  $r_o = 132$  mm and  $P_i = 110$  MPa

$$k = \frac{r_o}{r_i}$$

$$\sigma_\theta = \frac{110}{1.2^2 - 1} \left[ 1 + \frac{132^2}{r^2} \right]$$

$$\sigma_\theta = \frac{110}{1.2^2 - 1} \left[ 1 + \frac{132^2}{110^2} \right]$$

$$\sigma_\theta = 610 \text{ MPa}$$

When external radius  $r = 132$  mm,  $r_o = 132$  mm

$$\sigma_\theta = \frac{110}{1.2^2 - 1} \left[ 1 + \frac{132^2}{132^2} \right]$$

$$\sigma_\theta = 500 \text{ MPa}$$

### B. Radial Stress

When internal radius  $r = 110$  mm,  $r_o = 132$  mm

Where  $k$  is the ratio of external and internal pressure

$$k = \frac{r_o}{r_i}$$

$$\sigma_r = \frac{110}{1.2^2 - 1} \left[ 1 - \frac{132^2}{r^2} \right]$$

$$\sigma_r = \frac{110}{1.2^2 - 1} \left[ 1 - \frac{132^2}{110^2} \right]$$

$$\sigma_r = -110 \text{ MPa}$$

When external radius  $r=132 \text{ mm}$ ,  $r_o=132 \text{ mm}$

$$\sigma_r = \frac{110}{1.2^2 - 1} \left[ 1 - \frac{132^2}{132^2} \right]$$

$$\sigma_r = 0 \text{ MPa}$$

### C. Axial Stress

$$\sigma_z = \frac{110}{1.2^2 - 1}$$

$$\sigma_z = 250 \text{ MPa}$$

The above three stress represent the principle stress acting on the cylinder and with the help of these equations von mises stress also calculated

$$\sigma_{vm} = \sqrt{\frac{1}{2} \{ (\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 \}}$$

$$\sigma_{vm} = \sqrt{\frac{1}{2} \{ (610 - (-110))^2 + (-110 - 250)^2 + (250 - 610)^2 \}}$$

$$\sigma_{vm} = \sqrt{\frac{1}{2} \{ (720)^2 + (-360)^2 + (-360)^2 \}}$$

$$\sigma_{vm} = 623.5381 \text{ MPa}$$

Analytical calculations of thick walled cylinder with holes in different location were done by the equation 8. By this equation we get stress concentration factor at holes in different location.

$$\text{Stress concentration factor} = \frac{\text{maximum equivalent stress in Thick Walled Cylinder with holes}}{\text{normal equivalent stress in Thick Walled Cylinder without holes}}$$

$$k = \frac{\sigma_{max}}{\sigma_{nom}}$$

$$k = \frac{777.75}{623.5382907}$$

$$k = 1.247317144$$

In the same way three different stresses and stress concentration factor are calculated for the remaining cylinders with different materials. In the present work three different dimensions of cylinders with three different materials are considered. Therefore nine sets of results are Getting from the analytical due to space constraint only one set is presented.

## III. STATIC STRUCTURAL ANALYSIS OF THICK WALLED CYLINDER WITH & WITHOUT HOLES USING ANSYS

The modelling was done by using CATIA R16 for all the thick walled cylinders with and without holes. In the present work we have considered three different dimensions of thick walled cylinders for the modelling and analysis purpose.

Static structural analysis [8, 9, 10, 11] was performed for the three thick walled cylinders without holes consider in this work by varying the internal pressure. From the static structural analysis the maximum principle stress and von mises stress at internal radius and external radius are calculated for the three different materials consider in this work. The ANSYS results were compared with analytical result for all the cases and it is tabulated in table 1 and table 2. The ANSYS results obtain from static structural analysis at pressure 110 MPa is shown in fig 1 and fig 2. The fig 3 shows the graphical representation of maximum principle stress variation for the three different materials. Similarly the fig 4 shows the graphical representation of von-mises stress variation for the three different materials.



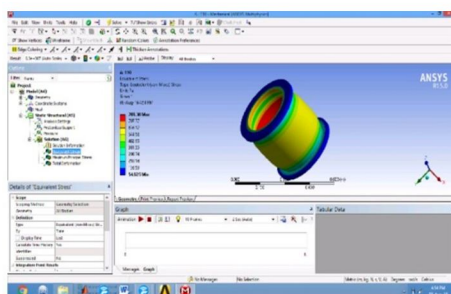


Fig 1 Von Mises Stress

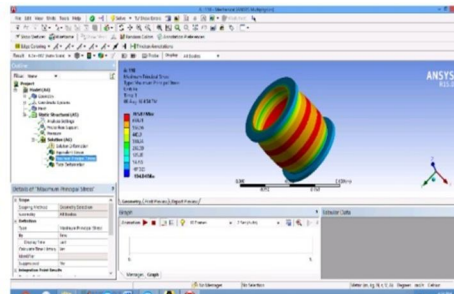


Fig 2 Maximum principle stress

Table 1 Comparison of Maximum Principal Stress

Pressure (MPa)	Analytical		ANSYS	
	Maximum Principle Stress at internal radius (mm) $r_i=110$	Maximum Principle Stress at external radius (mm) $r_o=132$	Maximum Principle Stress at internal radius (mm) $r_i=110$	Maximum Principle Stress at external radius (mm) $r_o=132$
110	610	500	630.1	542.21
120	665.4545455	545.045455	687.38	591.5
130	720.9090909	590.90909	744.66	640.79
140	776.3636364	636.36364	801.94	690.09
150	831.8181818	681.81818	859.22	739.38

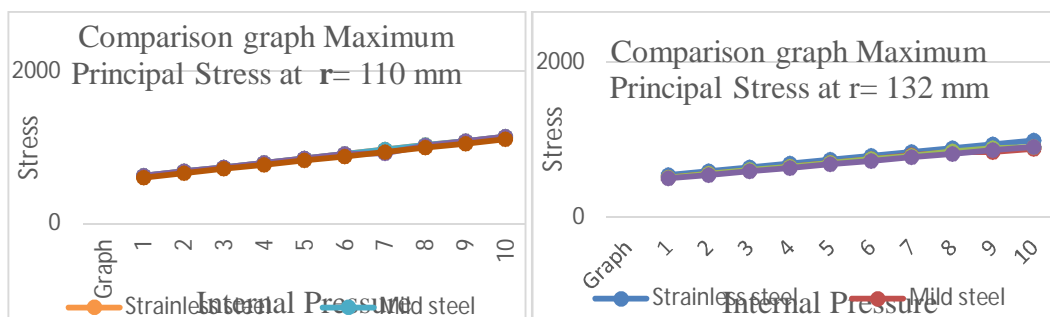


Fig 3 variation of Maximum principle stress

Table 2 Comparisons of Von-Mises Stress

Pressure (MPa)	Analytical		ANSYS	
	Von-mises Stress at internal radius(mm) $r_i=110$	Von-mises Stress at external radius(mm) $r_o=132$	Von-mises Stress at internal radius(mm) $r_i=110$	Von-mises Stress at external radius(mm) $r_o=132$
110	623.5382907	433.0127019	659.75	458.71
120	680.2235899	472.377493	719.72	500.41
130	736.908889	511.7422841	779.7	542.12
140	793.5941882	551.1070751	839.69	583.82
150	850.2794874	590.4718662	899.66	625.52

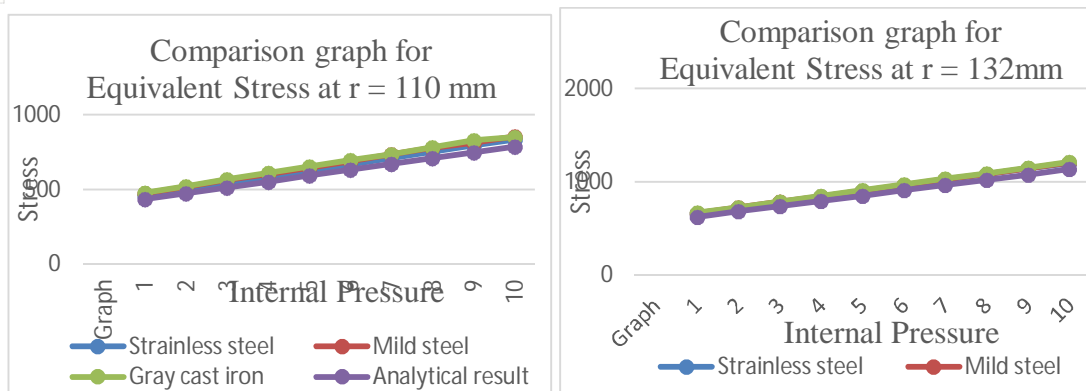


Fig 4 Variation of von mises stress

In the same way maximum principle stress and von mises stress are calculated for the remaining cylinders with different materials. In the present work three different dimensions of cylinders with three different materials are considered. Therefore nine sets of results are getting from the ANSYS due to space constraint only one set is presented.

Similarly Static structural analysis was performed for the three thick walled cylinders with holes at different locations consider in this work by varying the internal pressure. From the static structural analysis the von mises stress and stress concentration factor for different internal radius and external radius are calculated for the three different materials consider in this work.

The von mises stresses results obtained from ANSYS for different hole locations are tabulated in table 3. The fig 5 to fig 8 shows the von mises stress distribution for different cylinders with different hole positions. The stress concentration factor is calculated with the help of ANSYS and analytical results and the values are tabulated in table 4. Fig 9 shows the graphical representations of stress concentration factor variation for different materials.

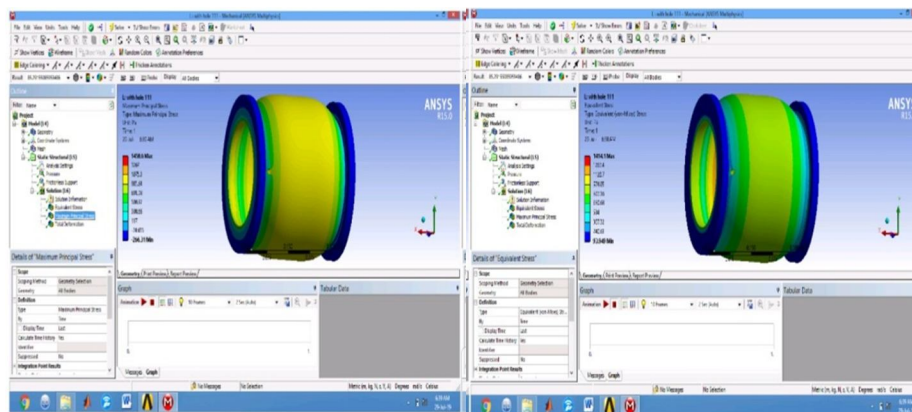


Fig 5 Von mises stress for cylinder with holes

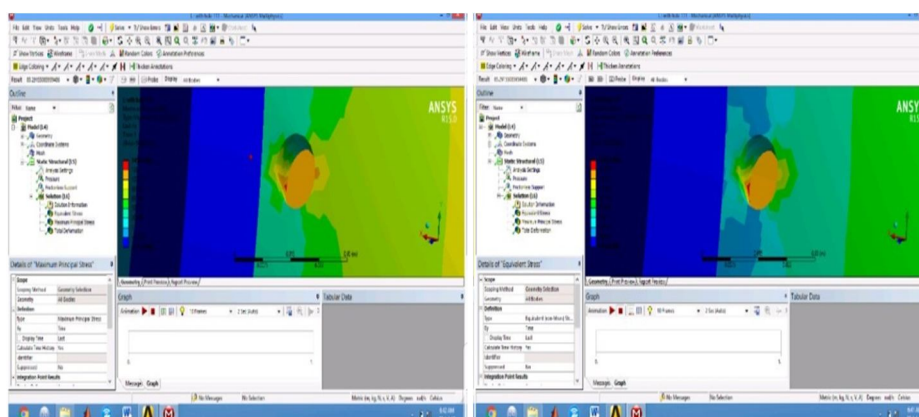


Fig 6 Close view of cylinder with holes

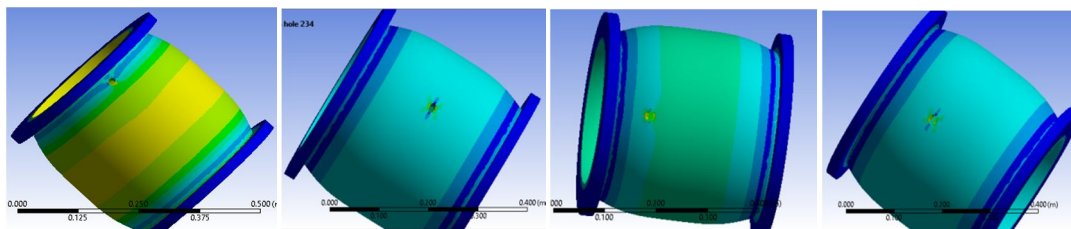


Fig 7 Von-mises stress in cylinder with holes in different location

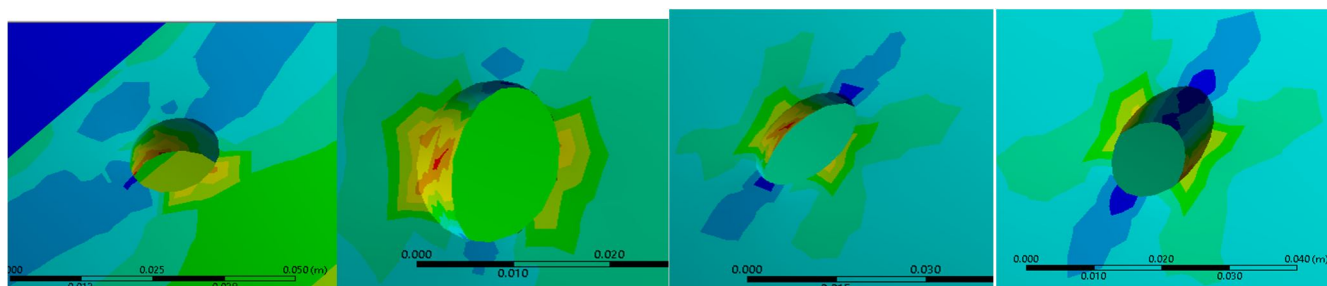


Fig 8 Close view of cylinder with holes in different position

Table 3 Von-Mises Stress results obtained from ANSYS

	ANSYS					
	At Maximum Pressure P =110			At Minimum Pressure P=200		
Hole Diameter	8mm	12mm	14mm	8mm	12mm	14mm
1/8 Location	777.75	768.82	784.88	1414.1	1397.9	1427.1
2/8 Location	1471.8	1343.4	1411.6	2676.1	2442.5	2566.5
3/8 Location	1546.4	1550	1564.2	2811.6	2818.3	2844
4/8 Location	1537.3	1570.7	1563.8	2795.0	2892.1	2843.3

Table 4 Stress Concentration Factor with hole diameter 8 mm

Pressure	At Maximum Pressure P =110		At Minimum Pressure P=200		Stress concentration factor ( $\frac{\sigma_{max}}{\sigma_{nom}}$ )
	Maximum von Mises stress(MPa)	Equivalent stress for without holes	Maximum von Mises stress(MPa)	Equivalent stress for without holes	
1/8 Location	777.75	623.5382	1414.1	1133.70598	1.2473
2/8 Location	1471.8	623.5382	2676.1	1133.70598	2.3605
3/8 Location	1546.4	623.5382	2811.6	1133.70598	2.4807
4/8 Location	1537.3	623.5382	2795.0	1133.70598	2.4654

#### IV. RESULTS AND DISCUSSIONS

The results were obtained for thick walled cylinder with and without holes. In case of thick walled cylinder without holes by the increase of diameter of the cylinder the maximum principle stress and von-mises stress is decrease. The maximum principle stress is less compared with von-mises stress in thick walled cylinder without holes. In case of thick walled cylinder with holes it is found that with the increase in diameter of hole, Von Mises stress initially decreases and then Becomes constant with the hole size. It is observed that the Maximum value of Von Mises stress is obtained at the central position and then it decreases as we move away from the centre. Stress Concentration Factor also determined for the Von Mises stress in different position in cylinders.

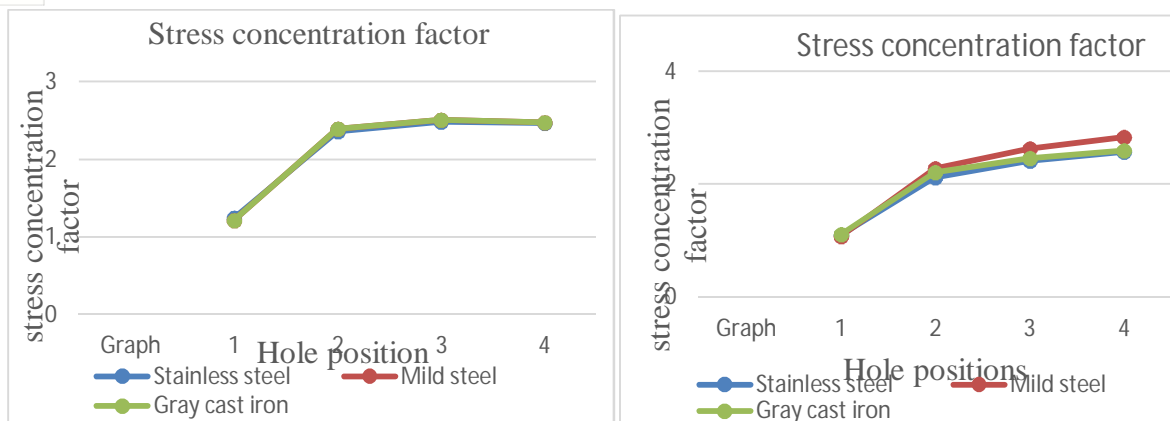


Fig 9 Stress concentration factor

## V. CONCLUSION

An attempt has been made to know the equivalent stress of a cylinder with radial holes. The work is organized under static structural analysis. Classical book work formulae's have been employed to obtain the stress distribution in cylinder with and without holes subjected to internal pressure.

By doing stress analysis for the effect of hole presence in thick wall cylinder, following conclusions can be drawn:

- Size and location of the hole in thick wall cylinder depends on its size of pressure vessel.
- The optimum hole size is the one for which the value of Von Mises is minimum around the vicinity of hole.
- The optimum location of the hole is the one where minimum Von Mises stress is obtained.

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