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Research on Boolean Algebra to Switching Theory

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Abstract: This paper presents some historical remarks about the development and applications of switching theory, and its central part, the Boolean algebra, in computing and engineering practice.

I. INTRODUCTION

Research in general, is supposed to be an entirely pragmatic field within a vast variety of activities intended for solving concrete practical tasks. However, to be efficient, they have to be based on a solid theoretical basis. These theories typically develop via two somewhat different routes.

A. Many particular solutions, or "pieces of theory" have been accumulated, and at a certain point it is possible to connect them and thus create a unified theory.
B. The complexity of an engineering system that is being managed by human experts grows too large to be any more so manageable and it becomes necessary to build a formalism and a theoretical framework.

For instance, the famous Maxwell equations can be viewed as an example for the first case, while the development of the switching theory falls more to the second category. In the following, we will take a closer look at this development.
Digital systems on hardware level basically consists of circuits and networks based upon two stable states. The mathematical models describing the functionality of these basic components are closely related to binary (Aristotelian) logic and mathematical logic. On higher functional levels, digital systems exhibit features that are related to number theory, graph theory, and discrete mathematics in general. Then, it is natural that many parts of these (and other) fields have been tied together to form the theoretical foundations of design and analysis of digital systems and collectively constituting the Switching theory.
In this paper, we present some historical observations about the early development of switching theory, especially the application of Boolean algebra. We want to draw attention to the contributions of scholars from different parts of the World many of which cannot be found in the standard literature.

II. SWITCHING THEORY

Switching theory, and its more practical form logic design, provide mathematical foundations and tools for digital system design, and thus are essential in almost all areas of modern technology. The Boolean algebra occupies a central role in switching theory, and was a vehicle to transfer circuit design from art and skills devised from experience into a scientific discipline. Therefore, its basic concepts will be briefly presented in the following section.

A. Boolean Algebra

Algebra of logic, also called symbolic logic, is a method to express logic in a mathematical context. Instead of dealing with numeric quantities as in ordinary algebra, it is used to represent the truth value of a statement by assigning logic symbols 0 and 1 to two possible truth values false and true. It has been derived by George J. Boole in order to permit an algebraic manipulation of logic statements and, therefore, is often called the Boolean algebra. It is useful in study of information theory, set theory, probability theory, and as it has been noticed, represents the basis of switching theory, which is the aspect that will be primarily discussed in this paper.

1) *Definition 1*: (Boolean algebra) *Consider a set B of at least two distinct elements 0 and 1. Assume that there are defined two binary operations* $\vee$ *and* $\cdot$ *and the unary operation* $-$ *on B, usually called logic disjunction (OR), logic conjunction (AND) and logic negation (NOT), respectively*.

An algebraic system hB, $\vee$, $\cdot$, $-$, 0, 1i is a Boolean algebra if for any a, b, c $\in$ B the axioms in Table 1 are satisfied.
The postulates 2, 5, 7, and 8 are called the Hungtington postulates [35], and are sufficient to specify a Boolean algebra, since the remaining six postulates can be derived from them.

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III. ALGEBRA OF LOGIC

In this section, we discuss the development of the theoretical foundations for switching theory.
Axioms and postulates in Boolean algebra.

1) Idempotence $a \vee a = a,\ a \cdot a = a,$
2) Comutativity $a \vee b = b \vee a,\ a \cdot b = b \cdot a,$
3) Associativity $a \vee (b \vee c) = (a \vee b) \vee c,\ a \cdot (b \cdot c) = (a \cdot b) \cdot c,$
4) Absorption $a \vee (a \cdot b) = a,\ a \cdot (a \vee b) = a,$
5) Distributivity $a \vee (b \cdot c) = (a \vee b) \cdot (a \vee c),\ a \cdot (b \vee c) = (a \cdot b) \vee (a \cdot c),$
6) Involutivity $\bar{a} = a,$
7) Complement $a \vee \bar{a} = 1,\ a \cdot \bar{a} = 0,$
8) Identity $a \vee 0 = a,\ a \cdot 1 = a,$
   $a \vee 1 = 1,\ a \cdot 0 = 0,$
9) De Morgan Laws $(a \vee b) = \bar{a} \cdot \bar{b},\ a \cdot \overline{b = a \vee b}.$

A. Work by G. J. Boole

In the spring of 1847, George J. Boole wrote a pamphlet entitled ”Mathematical Analysis of Logic”, which himself soon after regarded as an imperfect exposition of his logical system, and always rather referred directly to the later much elaborated treatise as a proper expression of his considerations.

Boole emphasized an analogy between the symbols of algebra and the symbols that can be used to represent logical forms and syllogisms. By unity Boole denoted the universe of thinkable objects, while literal symbols were used with elective meaning attaching to common adjectives and substantives. With the use of such symbols, deriving syllogistic conclusion can be expressed in form of equations. Manipulation with logic expressions should be used to

1) Demonstrate the truth value of a statement,
2) Rephrase a complicated statements in a simpler, more convenient, form without changing its meaning.

The latter feature is actually the foundation for engineering applications of the Boolean algebra. Boole also discussed probability theory, however, other logic parts are mainly the same. The underlying idea of his work was to reduce the logical thought to solving of equations. The algebraic operations have been defined to correspond to the basic activities while reasoning. In terms of these operations Boole formulated in an algebraic structure that shares essential properties of both set operations and logic operations. The work was done independently of other works by logicians and mathematicians at that time. For instance, the results by Augustus De Morgan were not used, since Boole did not consider conjunction and disjunction as a pair of dual operations.The approach followed by Boole and his point of view to the subject is possibly best described by himself.

In Boole wrote, That to the existing forms of Analysis a quantitative interpretation is assigned it is the result of the circumstances by which those forms were determined and is not to be construed into a universal condition of Analysis. It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis.

Boole wrote as the introductory statements--

In a work lately published, (meaning) I have exhibited the application of a new and peculiar form of mathematics to the expression of the operations of the mind in reasoning. In the present essay I design to offer such an account of a portion of this treatise as may furnish a correct view of the nature of the system developed.

The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; and to make that method itself the basis of a general method for the application of the mathematical doctrine of Probabilities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind.

As quoted in , it has been pointed out in that Boole had planned to write a non-mathematical sequel to The Laws of Thought in which, as Boole said, he would bring into light and prominence the philosophical elements which in my former exposition were too much hidden beneath the veil of symbolic notation. The analysis in was based on the study of some previously unpublished manuscripts treasured in the Royal Society. In has been pointed out that these manuscripts by Boole include lengthy quotations from material apparently intended for this work. As stated in , see also , most of these quotations pertain the analysis Boole did of the faculty of conception and to his development of the notion that all true axioms are somehow founded upon the laws of the human mind. From these comments, the author in concluded that Boole had planned to include a non-algebraic expression of his logic, but this is not to be found among the present manuscripts.

Considerable contributions to formal formulation of Boolean algebra are due to William S. Jevons and Charles Sanders Pierce.

In particular, Jevons exploited Boolean algebra in constructing a mechanical reasoning machine that was demonstrated to the Royal Society in 1870.

Recall that in, Venn introduced a way for diagramming notation by Boole, which is now called the Venn diagrams.

A systematic presentation of Boolean algebra and distributive lattices has been given in 1890 by Ernst Schroder.

It should be noticed that Ernst Schroder developed his algebraic logic, which is called nowadays symbolic logic, independently on the work by G.J. Boole and A. De Morgan , about whose work he learned in 1873. Schroder published his work in , the third volume that appeared posthumously was edited by Eugen Muller.

Schroder presented Boolean algebraic logic ideas and in this ways supported spreading of these ideas to the continental Europe.

Boolean algebra presented as an axiomatic algebraic structure is due to Edward Vermilye Huntington in 1904.

Marshall Harvey Stone in 1936 , and Garrett Birkhoff in 1940, and Birkhoff and Sanders MacLane raised the Boolean algebra to the level of deep mathematical subjects.

B. Work by Platon S. Poreckii

As it was pointed out in, Platon Sergeevic Poreckii was motivated to study logic by the famous mathematician A.V. Vasilev and father of the founder of imaginary logic N.A. Vasliev.

Poreckii has been primarily interested in logic equations and inequalities, and application of mathematical logic in the probability theory.The method developed by Poreckii in this area have been more universal than approaches by Jevons and Venn , at least as it has been estimated by Couturat . In 1884, I published an article "On methods for solving logic equations", where it has been presented a complete theory of these equations. In this article, I suggest to exploit this theory in solving the following task in the Probability Theory.

Determine probability of a complex event, depending on given simple events, by using probabilities of these simple events as well as probabilities of some other complex events, assuming that given events satisfy an arbitrary number of arbitrary conditions.

A solution of this task has been provided by Boole in his article, which however can hardly be considered as scientific, since it is based upon arbitrary and entirely empirical theory of logic, as well as for the idea itself about the transition from logic equation to algebraic equations has weakly been elaborated by Boole. In this way, the main goal of the present paper is to give a scientific form to the deep, but vague and without proof, idea of Boole about applicability of Mathematical Logic in the Probability Theory.

IV. APPLICATIONS OF BOOLEAN ALGEBRA

In this section, we will discuss history of applications of logic and in particular Boolean algebra in circuit design, and in this way, origins of the switching theory.

A. First Applications Of Logic In Circuit Design

It is reported that in a paper presented at the Seminar on Logic at he Institute of Logic of the Academy of Science of SSSR, Geleii Nikolaevic Povarov discussed combinatory logic, distinguishing technical logic from the algebra of logic and pointing out that the possibility of technical applications of mathematical logic (i.e., the application of Boolean algebra to the analysis of electric relay-contact circuits) was first noted by the Russian physicist P.S. Ehrenfest.

B. Work by V. I. Shestakov As Reported

This proposal by Ehrenfest has been elaborated in detail by Viktor Ivanovic Shestakov in 1934 &1935 and have been published in 1941. It has been pointed out that the ideas suggested Ehrenfest about an algebra of switching circuits have been explored and elaborated by V.I. Shestakov, a student of V.I. Glivenko, and the results were reported in written form in January 1935, however, as Anovskaja states that this paper wan not published at the time, but has been used as foundations for the PhD candidate Thesis by Shestakov . The major part of the thesis has been published in the journal Techniceska fizika ´ , Vol. 11, No. 6, 1941. As it has been pointed out in, credit to Shestakov is given often given by many Soviet authors. The problem has been discussed further by Mikhail Aleksandrovic Gavrilov in a series of papers published between 1943 to 1947. The book by Gavrilov established a basis for further study of switching theory in Soviet Union, and since it has been translated in German, had a considerable influence abroad. Information on publications in this subject by Gavrilov, Povarov, Shestakov and their associates can be found in.

C. Work by C. E. Shannon

Shannon used the following correspondence between the circuits and logic symbols.

1) If a terminal is open, it has an infinite impedance and the logic value 1 is assigned to it.
2) For a closed terminal, the impedance is zero, and the logic value 0 is correspondingly assigned.
3) Negation $\bar{X}$ for a terminal X is defined as the value opposite to the value assigned to X.

The variable X_{ab} connecting two terminals a and b is viewed as a function of time and called hindrance of the two terminal circuit a−b.For simplicity, the symbol X_{ab} Claude Elwood Shannon first page of MSc Thesis 1938 is often replaced by X whenever this does not cause any ambiguity.

For the simple relay locking circuit, Shannon uses the relation X = A(B + X) and interprets it as the relay X operates iff either circuit A is closed or both circuit B is closed and relay X is operating. In a review of the work by Shannon, Alonzo Church suggests to use the notation as $X(t + 1) \equiv A(t) \vee (B(t) \wedge X(t))$ where the time t is taken as an argument explicitly.

The analysis and synthesis method for circuits has been expressed in the following way

Any circuit is represented by a set of equations, the terms of the equations corresponding to the various relays and switches in the circuit. A calculus is developed for manipulating these equations by simple mathematical processes, most of which are similar to ordinary algebraic algorithms.

For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit.

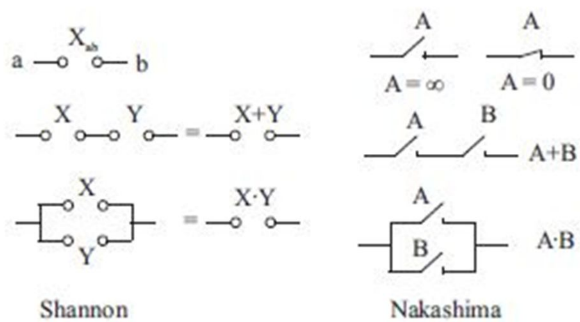

Figure 4. Notation and symbols used by Shannon and Nakashima.

a) *Example 1:* An example of relay circuit realizing the function f(x, y, z, w) = w(x + y(z + x)) in Shannon notation and symbols.
x x _ y z w f x x _ y z w f

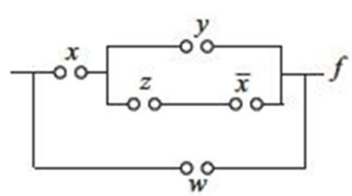

Figure 5. Relay circuit realization of *f* in Example 1.

The first page of the MSc Thesis by Shannon in 1938. For this work, Shannon has been awarded in 1940 by the Alfred Noble American Institute of American Engineers Award used to be given to a young author of a technical paper of exceptional merit, being proposed for this award by his boss Vannevar Bush without his knowledge, in whose Lab Shannon wrote the thesis.

For a simplest introduction into the work by Shannon the shortest way is to read the review by Charles A. Baylis in The Journal of Symbolic Logic

Shannon entered the University of Michigan in 1932, where he attended a course where he learned about the Boolean algebra. Shannon was well informed about the large bibliography on symbolic logic , given as the first reference in his work and in particular Shannon provided English version of subject related.

D. Work by A. Nakashima

Akira Nakashima graduated at the Tokyo University, and worked as an engineer at Nippon Electric Company (NEC) among other task also on the design of relay networks for various purposes.

Nakashima first did an extensive analysis of many case studies of relay networks trying to formulate a unified design theory for such networks. He considered impedances of relay contacts as two-valued variables, and used logic OR and AND operations to represent their series and parallel connections, respectively. Due to that, he formulated a related theory of relay networks by introducing and exploiting some algebraic relations that represent a basis of switching theory. For instance, he defined the rules that are nowadays called De Morgan duality expressions. These results, Nakashima presented without using a symbolic notation in a series of articles in the monthly journal of NEC entitled Theory and Practice of Relay Engineering. The Telegraph and Telephone Society of Japan engaged Nakashima to give an invited talk at the annual meeting of this society early in 1935. This three hours long talk has been afterward published in 1935.

At the time when writing this review, C.A. Baylis was a Professor at the Department of Philosophy of the Brown University, and was a founder of the Association of Symbolic Logic, and among founding editors of The Journal of Symbolic Logic.

In 1936, Nakashima was transferred to transmission engineering, however, being advised to continue this research by Niwa Yasujiro, the Chief Engineer of NEC at that time, he continued the work after official office working time with thew help of Masao Hanzawa, who remained in the exchange engineering team.

In a joint work with Masao Hanzawa, the theory of Nakashima was elaborated by using also symbolic representations and finally evolved into an algebraic structure, for which Nakashima concluded in August 1938 that it is actually equal to the Boolean algebra.

Notice that papers by Nakashima and also these with Hanzawa, have first been published in Japanese in Journal of the Institute of Electrical Communication Engineers, and then latter translated in a reduced from and published in Nippon Electrical Communication Engineering.

which has been published in Japanese in August 1937, the algebra introduced by Nakashima and elaborated in cooperation with Hanzawa was reduced to an algebra of sets by assigning to each partial path a set of times at which its impedance is infinite. In that way, the author was able to state that “theorems and expressions developed in the theory of sets may, therefore, be applied to acting impedance problems of simple partial paths”, see the corresponding remark in [18].

For instance,it is noticed the following correspondence between the algebra of logic and circuits. If A and B are two-terminal circuits, which are called simple partial paths by the authors, then A + B and A B correspond to their serial and parallel connections, respectively. The equation A = B states that acting functions of A and B are equal, meaning that A is open (closed) when B is open (closed). Similarly, A denotes a simple partial path that is closed when A is open and vice versa. Two simple partial paths that are always closed or open are denoted by p and s, respectively. In the terminology used by the authors, such path have and infinite and zero impedance, respectively.

With this notation , that is quite similar to that by Shannon and other authors, Nakashima and Hanzawa defined an algebra which as they realized in 1938 is identical to the Boolean algebra.

In particular, the authors noticed that their expansion theorem is actually the same what the Boole calls the law of development. That has been published in Japanese in August 1937, the author is pointing out the links between his algebra and the algebra of sets, which therefore can be used in discussing problems related to simple partial paths and impedances in them .

The authors for the first time refer explicitly to Boole and Schroder.

The work by Nakashima has been estimated as the first systematic study of logic circuits. It is especially emphasized the expansion theorem of impedance function of relay circuit and the design theory of two terminal relay circuits in 1940. it is explicitly stated that Akira Nakashima published the first paper on switching theory in the World.

In Japan, the work by Nakashima has been continued by Kan-ichi Ohashi, Mochinori Goto, Yasuo Komamiya, T. Kojima, and later by many others, chapter entitled Switching Theory in Japan.

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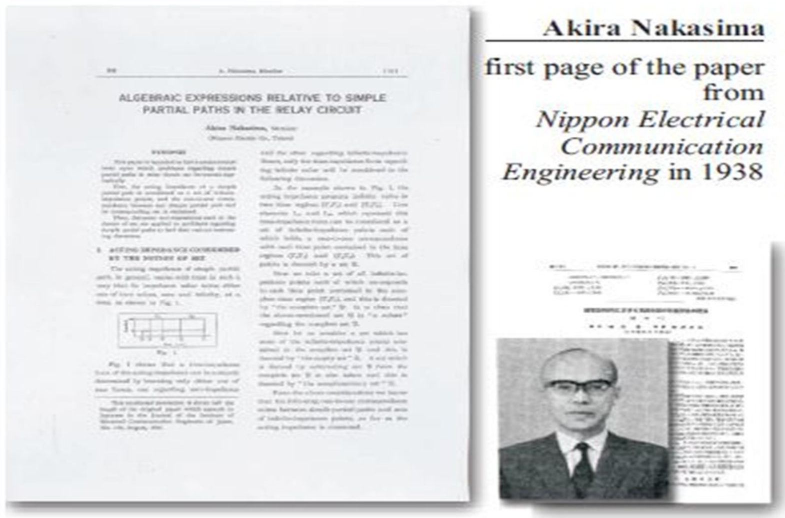

Figure 6. First page of the paper by Nakashima.

Akira Nakasima first page of the paper from 1938
Nippon Electrical Communication Engineering in Figure 6. First page of the paper by Nakashima.

E. Work by J. Piesch

Very considerable work was carried out by the Austrian mathematician Johanna Piesch, whose name is often abbreviated as Hanna, or Hansi, for different reasons. Most probably these were politically caused reasons related to the Jewish origins of this author.
considered are circuits (switches) with any finite number of states (positions) and under the assumption that not all of them must have the same number of states. Switches are denoted by symbols, as a, b, c, etc., with different positions indicated by subscripts. In this way, the symbol a_i will denote that the switch a is in the position i. Capital letters are used to express other propositions as effects caused by certain assignments of states, thus, to describe the outputs of the network. The operations of addition, multiplication, and inverse, corresponding to the disjunction, conjunction, and negation, are used to form expressions of the algebra built on the propositions denoted by symbols. This algebraic structure is actually a propositional algebra, rather than the propositional calculus, since just the equivalences are asserted.
In her work ,Johanna Piesch refers to and an unpublished paper by the Austrian researcher Otto Plechl. It has been remarked that Piesch in publications from 1939, did an extension of the work by Nakashima.
It is interesting to point out that the method by has been used and adapted to the design of n-terminal switching circuits.

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