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Root Finding Methods: Newton Raphson Method

Vishal V .Mehtre[1], Shikhar Sharma[2]
[1]Assistant Proffesor, [2]Department of Electrical Engineering, BhartiVidyapeeth Deemed University, College of Engineering, Pune, India

Abstract: In this study report. I try to represent a brief description of root finding method which is an important part of computational physics. Specially I discussed Newton Raphson's algorithm To find the root of any polynomial Equation. This report also covers the Drawback of this method along with the Graphical representation with examples.
Keywords: Root finding methods, Taylor series, Newton-Raphson method, Polynomial.

I. INTRODUCTION

Finding the solution to the set of nonlinear equations $f(x) = (f1,.......f^n)' = 0$ is been a problem for the past years. Here we consider this nonlinear equation and try find the solution to it and this can be found out by the Newton Raphson method. Root finding is also one of the problems in practical applications. Newton method is very fast and efficient as compared to the others methods.

It uses the idea that a continuous and Differentiable function can be approximated By a straight line tangent to it. Finding roots of the nonlinear equation with the help of Newton Raphson method provides good result with fast convergence speed and Mat lab also adopted this method for finding the roots and tool used for such calculations is scientific calculator.This method cant work With points of inflection, local maxima or Minima around x_0 or the root**.**

II. ROOT FINDING METHODS

A short list of root finding method is given below where I choose some important and popular Algorithms.

A. Bisection Method
B. Newton Raphson Method
C. False position Method
D. Secant Method
E. Bernts Method

III. WHY NUMERICAL ANALYSIS

Why Numerical approach? To give this answer I want to cite an another question asked by overnight mathematician Galois.

"Why is there no formula for the roots of a fifth (or higher) degree polynomial equation in terms of the co-efficient of the polynomial, using only the usual algebraic operation(+,-, _, _) and application of radicals.

The answer is described in Abel-Ruffini's theorem which state that, "There is no general solution in radicals that applies to all equations of a given degree greater than 4."

So for finding roots of fifth or higher degree polynomial equation we have to approach numerically.

That's not enough to describe the importance of numerical analysis. There is more. Say we wish solve a problem which have no equation in our hand but a graph. Then how could we find the solution or root of that problem? In this case Numerical Analysis helps a lot.

Now a question arise how could we find root from a graph? the answer will be clear till you finish reading this.

IV. NEWTON-RAPHSON METHOD

Everybody who are familiar with Taylor Series, They almost know 90% of Newton-Raphson's method unconsciously!

Lets take a look on Taylor series,

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-xo)^2}{2!} f''(x_0)+$$

(1)

To solve a equation f(x) = 0 we substitute into Taylor series.

$$0 = f(x_0) + (x-x_0)f'(x0) + \frac{(x-xo)^2}{2!} f''(x_0) +$$

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(2)
Now, if x0 is close to the root x then we have,
$(x-x_0)$ is small
$(x-x_0)^2$ is more small
$(x-x_0)^3$ is much small
So, we can ignore the quadratic and higher degree terms leaving,

$$f(x_0) + (x-x_0)f'(x_0) \approx 0$$

$$(x-x_0) \approx \frac{-f(x_0)}{f'(x_0)}$$

$$x \approx x_0 \frac{-f(x_0)}{f'(x_0)} \qquad (3)$$

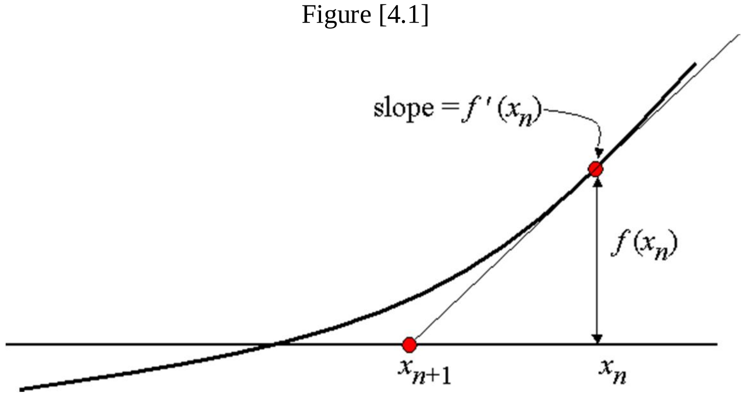
Which we can iterate as,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (4)$$

This means that if we have a starting point (a guess) for x_n we can iterate equation 4 until we find the root.

V. GEOMETRICAL REPRESENTATION

Figure [4.1]



A curve intersect with x-axis

We draw a tangent line to the graph of f(x) at the point $x = x_n$. This line has slope $f'(x_n)$ and goes through the point $(x_n; f(x_n))$ Therefore it has the equation $y = f'(x_n)(x-x_n) + f(x_n)$. Now, we find the root of this tangent line by setting y = 0 and x = xn+1 for our new approximation. Solving this equation gives us our new approximation, which is $x_{n+1} = xn \frac{-f(x_n)}{f'(x_n)}$.

That's how we wish to approach the secant point where the f(x) cut x-axis.

Example

Find the root of the equation x2 - 4x - 7 = 0 near x = 5 to the nearest thousandth.

We have our $x_0 = 5$. In order to use Newton's method, we also need to know the derivative of *f* In this case, f(x) = x2 - 4x - 7, and f'(x) = 2x - 4.

Using Newton's method, we get the following sequence of approximations:

$$x1 = 5 - \frac{5^2 - 4 * 5 - 7}{2 * 5 – 4}$$

$$= 5 – \frac{(-2)}{6}$$

$$= \frac{16}{3}$$

$$\approx 5:33333$$

$$x2 = \frac{16}{3} - \frac{(16 \div 3)^2 - 4(16 \div 3) - 7}{2(16 \div 3) - 4}$$

$$= \frac{16}{3} - \frac{1 \div 9}{20 \div 3} = \frac{16}{3} - \frac{1}{60} = \frac{319}{60}$$

$$\approx 5.31667$$

$$X3 = \frac{319}{60} - \frac{(319 \div 60)^2 - 4(319 \div 60) - 7}{2(319 \div 60) - 4}$$

$$= \frac{319}{60} - \frac{1 \div 3600}{398 \div 60}$$

$$\approx 5.31662$$

We can stop now, because the thousandth and ten-thousandth digits of x2 and x3 are the same. If we were to continue, they would remain the same because we have gotten sufficiently close to the root:

$$x4 = 5{:}31662 - \frac{(5{:}3362)^2 - 4(5{:}3362) - 7}{2(5{:}3362) - 4}$$

$$= 5{:}31662$$

Our final answer is therefore 5.317

VI. DRAWBACK OF THE NEWTON-RAPHSON METHOD

A. Divergence at Inflection Points

If the selection of the initial guess or an iterated value of the root turns out to be close to the inflection point of the function f(x) in the equation f(x) = 0, Newton-Raphson method may start diverging away from the root.

B. Root Jumping

In some case where the function) (xf is oscillating and has a number of roots, one may choose an initial guess close to a root. However, the guesses may jump and converge to some other root.

Notice that in figure 1 our initial guess was 2:3π (say) and we wish to find the root 2π, the closest one or 3π second close root of this initial guess for this function. But we got x = 0 as our root which one we don't desire as our solution. This phenomena is known as root jumping.

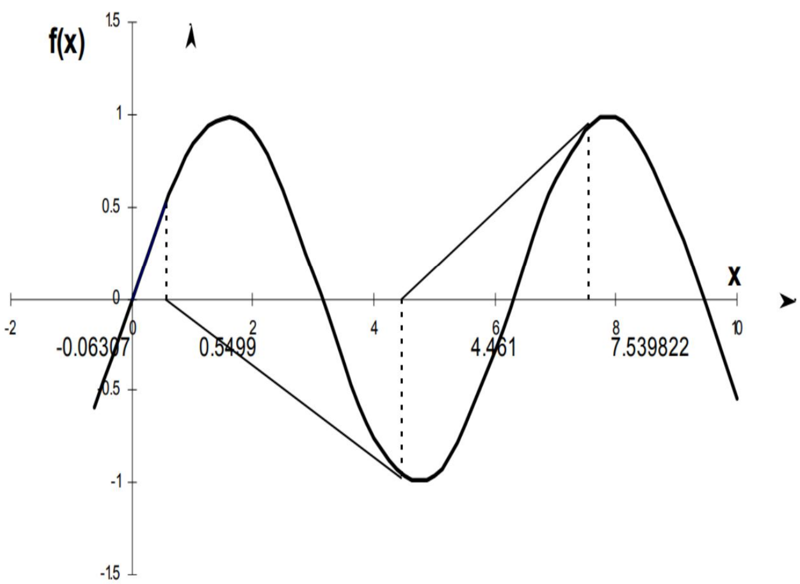


Figure 5.2: f(x) = Sin(x)

C. Oscillations near Local Maximum and Minimum

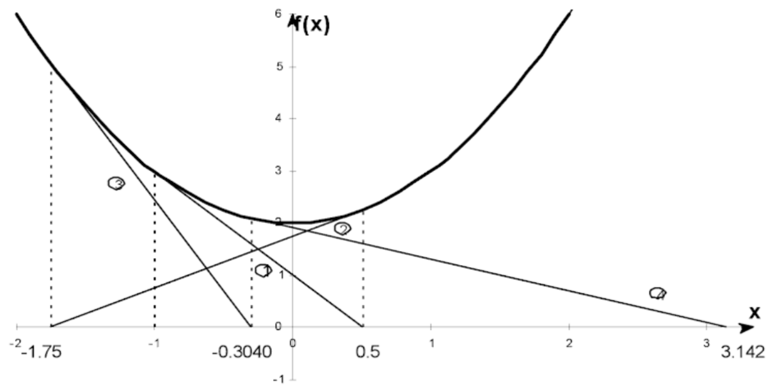


Figure 5.3: $f(x) = x^2 + 2$

f(x) = x2 + 2

Here in this case (figure 5.3) though our function have no real solution nevertheless if we wish to find imaginary root of this equation it never gives us the root. It oscillate around the local extreme but can't approach to the root.

D. Division By Zero

It is clear that if the value of $f'(x_n) = 0$ for the initial guess then the Newton-Raphson equation 4 becomes undefined. That's why finding root become impossible by this method.

VII. CONCLUSION

In this review paper we have discussed about Newton Raphson method. This method is an application of differentiating calculus. It is efficient with some limitation and an easy algorithm to find root of polynomial equation. We have taken examples and have also derived its formula.

VIII. ACKNOWLEDGEMENT

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