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Exploring Ratio and Unit Circle Methods in Understanding Trigonometric Functions: Towards Enriched Student Learning in Solving Right-Angled Triangles

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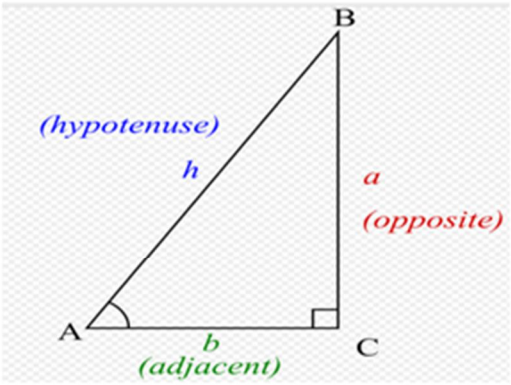
Abstract: This is an experimental study, made used of the non-randomized experimental and control groups, pretest-posttest designs. The main purpose of this research was to compare the two methods of teaching trigonometric functions to see which promotes better understanding of the underlying concepts and mastery of skills. The experimental and control groups were four separate intact classes in Plane Trigonometry of two semesters. For a period of nine sessions, the experimental group was subjected to the unit circle method, but the control group instead was given direct instruction using unit ratio in understanding trigonometric functions of solving right-angled triangles. Using a non-parametric statistical test, particularly the Wilcoxon Rank-Sum test, results showed that students in the control group performed significantly better than the experimental group in the post-test. On the other hand, the Fisher's Exact Probability Test yielded very statistically significant different proportions in the post-test scores between the experimental and control group.
Keywords: Trigonometric functions, unit ratio, unit circle, right-angled triangle

I. INTRODUCTION

The word *trigonometry* comes from the Greek words *trigonon* ("triangle") and *metron* ("to measure"). Until about the 16th century, trigonometry was chiefly concerned with computing the numerical values of the missing parts of a triangle (or any shape that can be dissected into triangles) when the values of other parts were given. In broader sense, it is that branch of mathematics which deals with the measurement of the triangle and which deals with the relations between the sides and the angles of a triangle [1]. Historically, triangle trigonometry and circle trigonometry emerged from separate traditions and were developed for different purposes [2][3][4]. Triangle trigonometry, which focuses on ratios between side lengths in a right triangle, has its origins in the surveying and measurement techniques used by the Babylonians and Egyptians. Circle trigonometry, which focuses on chords and their associated arcs, was developed by the Greeks in their study of the heavens [4].

Trigonometry plays a very important role in any curriculum framework. It is one of the foundation courses of science, technology, engineering and mathematics curriculum. The concepts of trigonometry links about shape and space with other mathematical ideas such as ratio, deduction and mathematical proof. Likewise, it provides an opportunity to link what is observed in real life with the world of the mathematics classroom. Unfortunately, many students do not experience the richness, connections or creativity that trigonometry allows. Instead they often perceive it as another memory exercise where rules and formulae must be learnt 'by rote', along with methods for working out problems [5].

It's no secret that right triangle trigonometry is a notoriously difficult subject. Both students and teachers struggle through the unit [6]. The study outlines two distinct methods of solving right-angled triangles - the ratio and circle methods. The ratio method, where trigonometric functions are defined as the ratios of pairs of sides in a right angled triangle. Given an acute angle A of a right-angled triangle, the hypotenuse *h* is the side that connects the two acute angles. he side *b adjacent* to *A* is the side of the triangle that connects *A* to the right angle. The third side *a* is said *opposite* to *A*. If the angle *A* is given, then all sides of the right-angled triangle are well defined up to a scaling factor. This means that the ratio of any two side lengths depends only on *A* [7][12]. These six ratios define thus six functions of *A*, which are the trigonometric functions. Students are often taught to remember the definitions of the ratios using a mnemonic such as SOHCAHTOA (**S**ine =**O**pposite/**H**ypotenuse; **C**osine = **A**djacent/**H**ypotenuse and **T**angent = **O**pposite/**A**djacent).



$$\sin A = \frac{a}{h} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{h} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$$

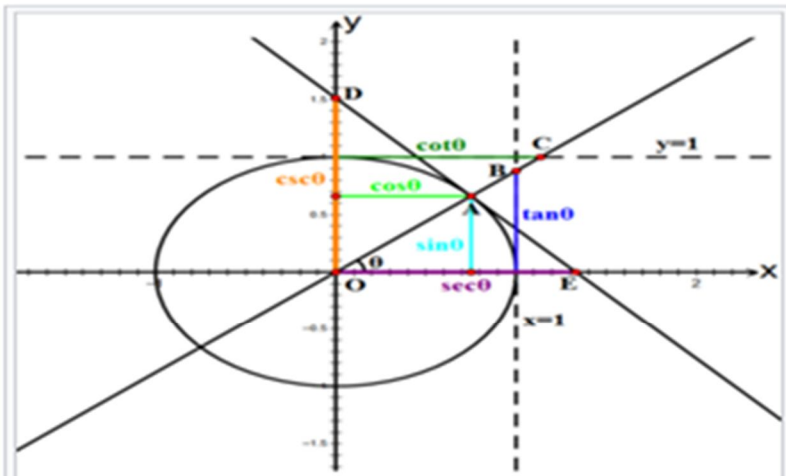
$$\csc A = \frac{h}{a} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\sec A = \frac{h}{b} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot A = \frac{b}{a} = \frac{\text{adjacent}}{\text{opposite}}$$

A right triangle always includes a 90 degree angle ($\pi/2$ radians), here labeled C, Angles A and B may vary. Trigonometric functions specify the relationships among side lengths and interior angles of a right triangle [3].

The Unit circle method is defined cosine and sine as the x and y co-ordinates of a point on a unit circle. The six trigonometric functions can be defined as coordinate values of points on the Euclidean plane that are related to the unit circle, which is the circle of radius one centered at the origin O of this coordinate system. While right-angled triangle definitions permit the definition of the trigonometric functions for angles between 0 and $\pi/2$ radian (90°), the unit circle definitions allow to extend the domain of the trigonometric functions to all positive and negative real numbers [7][11][13].

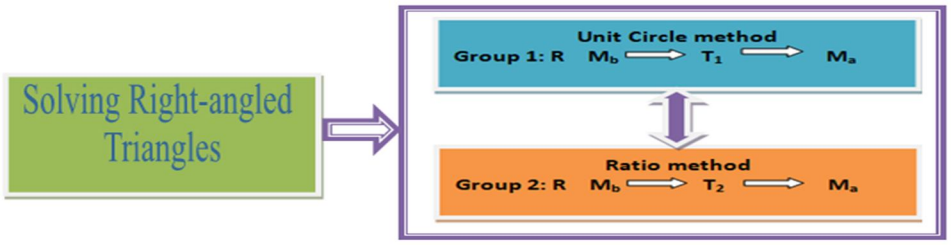


In this illustration, the six trigonometric functions of an arbitrary angle θ are represented as Cartesian coordinates of points related to the unit circle. The ordinates of A, B and D are sin θ, tan θ and csc θ, respectively, while the abscissas of A, C and E are cos θ, cot θ and sec θ, respectively.

The main purpose of this research is to compare the two methods of teaching trigonometric functions to see which promotes better understanding of the underlying concepts and mastery of skills. Specifically, it attempts to investigate the unit circle method to the teaching of solving right-angled triangles as compared to the ratio method, and to assess as a result, the effect on their achievement in Trigonometry.

A. *Conceptual Framework*

The Pretest – Posttest Control Group Design was used in this study. It was depicted schematically by the following figures:



The R in the model indicates that the two groups were assigned randomly, Mb corresponds to measurement before the experiment (pretest), T1 and T2 refer to the two different teaching methods and Ma signifies measurement after the teaching session (posttest). This study rests on the concept that the performance of the students taking Plane trigonometry can be affected by the manner of application activity afforded to the learners after the lesson in solving right-angled triangles had been conducted. Unit circle method was used as a treatment in the experimental group and ratio method was used in the control group. In mathematics, the time for students to try out what they have learned in the day's lesson is very important. Thus, the improvements of the students' performance in solving right-angled triangles were determined through the result of the evaluation.

II. METHODOLOGY

Research Design A quasi – experimental design was utilized with an experimental group of students which was taught by the unit circle method, while the control group was taught using ratio method. Both groups were taught with solving right-angled triangles. The Pretest – Posttest Control Group Design was used in this study. It depicts schematically by the following figures:

1) *Group 1:* R Mb ⟹ T1 ⟹ Ma
2) *Group 2:* R Mb ⟹ T2 ⟹ Ma

The R in the model indicates that the two groups were assigned randomly, Mb corresponds to measurement before the experiment (pretest), T1 and T2 refer to the two different teaching methods and Ma signifies measurement after the teaching session (posttest).

The respondents of this study were the identified four intact Plane Trigonometry classes. The groups assignment were selected through a toss coin. The pertinent data needed for the study was collected with the use of the following data gathering instruments. After the identification of respondents, the researcher conducted the experiment proper and personally administers the sets of questionnaires. In the experiment proper, the groups were taught separately, meeting three hours weekly for a three-week period. One class taught using unit circle method and the other class was taught using ratio method. Appropriate achievement test was administered to both classes before and after the session per semester. The data gathered from the two classes of experimental group and two classes of the control group of the two semesters were tabulated, computed and interpreted.

Descriptive statistics such as the mean and standard deviation was used to describe the level of performance of the respondents in Plane Trigonometry. Due to discrepancy in sample size, a non-parametric statistical test particularly the Wilcoxon Rank-Sum test was used to test the difference in the level of performance in Plane Trigonometry between the control and experimental groups. On the other hand, the Fisher's Exact Probability Test was used to find the difference in the proportion of passers in Trigonometry between the control and experimental groups. This test is appropriate for two independent samples where the data are nominally casted in a 2x2 table. The null hypotheses were tested at the 0.05 level of significance.

III. RESULTS AND DISCUSSION

A series of comparisons were made to establish which method was more successful and students' performance on the trigonometry test were also examined.

Table 1. Mean Scores in Solving Right-angled Triangles between the Experimental and Control Groups

Dependent Measure	Experimental Group (Unit Circle Method)		Control Group (Ratio Method)	
	Mean	SD	Mean	SD
Pre-test	9.794	6.619	9.706	6.053
Post test	15.529	9.873	23.118	8.676

The table presents the data on the level of performance of the two groups of respondents in trigonometry classes. Despite the difference in sample size, the distribution of the two sets of data was generally comparable as indicated by the small discrepancies between the standard deviations. It can be observed from Table 1 that the mean scores in trigonometry exhibited by both group of students on the pre-test indicator of the dependent variable were generally below average, the results indicated that the students generally below average in terms of skills in solving right-angled triangles and mathematics competencies in the said subject before the conduct of the experiment.

The same Table shows that the experimental group exhibited better mean scores in trigonometry pre-test than the control group. However, after the experiment proper the control group exhibited better mean scores in solving right-angled triangles post-test than the experimental group. Hence, The results of the data analysis revealed that the performance of the students in trigonometry specifically in the control group could be attributed largely in the ratio method of teaching- solving right-angled triangles. The results of the research coincides with the claimed of [8], students of all ability levels performed better with the ratio method, but the low ability students benefited the most. Further, using the ratio method were more successful at identifying which of the trigonometric functions were required in solving the question. There are also fewer steps using this method and therefore less opportunity for error. Furthermore, with the ratio method students remembered their knowledge for longer and more successful in terms of the amount of enjoyment and motivation students felt. An additional benefit of it is the extra practice it gives at rearranging equations, particularly those with the unknown as a divisor. On the other hand the unit circle method is preferable for conceptual development, but it holds no advantages for solving triangles. A good balance may be struck by introducing the concepts using the unit circle method (thus laying the foundations for further study or functional trigonometry) but then relating them to the ratio definitions. Students should then be encouraged to solve triangles using ratio method techniques [9].

Table 2. Comparison between the Experimental and Control Groups

Dependent Measure	Experimental Group	Control Group	Computed Z-value[a]	p-value
	Mean Rank	Mean Rank		
Pre-test	235	281	-0.0983	0.92034(ns)
Post test	223	298	-2.8038	0.00512*

[a]Wilcoxon Rank Sum Test

A non-parametric statistical test particularly the Wilcoxon Rank-Sum test was used to test the score comparison between the control and experimental groups. The table shows the test of significance conducted and compared the mean ranks of the scores of the students in trigonometry. The test of significance conducted and which compared the mean ranks of the scores of the students in trigonometry, as shown in Table 2, yielded p-values which was greater than $\alpha = .05$ (two tailed) in the pre-test. It clearly shows that there was no significant difference between mean rank scores of both groups before the experiment proper. However, in the same table, the test of significance conducted and which compared the mean ranks of the scores of the students in trigonometry yielded p-values which was smaller than $\alpha = .05$ (two tailed) in the post-test. This means that control group performed significantly better than the experimental group of students. Hence, using the ratio method, students were able to set up ratios of sides of right triangles to determine angles and lengths of other sides and using information they previously discovered to solve the ratios without explicit steps This pattern is observable on the post-test indicator of the dependent variable. Hence the null hypothesis which states that there is no significant difference in the performance in trigonometry between students in the experimental and control group is rejected. The test of significance conducted revealed that students exposed in the ratio method in teaching "solving right-angled triangles" performed significantly better than those who were exposed in the unit circle method of teaching the topic. However, from the study conducted in Indonesia, suggested that, [10] the design can link between trigonometry as ratios of right-angle triangle context and unit circle context.

Table 3. Proportion of Students who passed the assessment in the Experimental and Control Groups

Dependent Measure (academic performance in solving right-angled trigonometry)	Experimental Group	Control Group	Fishers Exact Test (2-tailed)
Passed	45.71%	78.57%	0.006* Very statistically significant
Failed	54.29%	21.43%	

The Fisher's Exact Probability test was used to test the proportion of passers in trigonometry between the control and experimental groups. A validation of the data analysis was conducted by classifying the post-test scores into pass or fail criterion. Operationally, a transmuted grade of 50% or better is considered a passing grade while less than 50% is considered a failure. The student respondents in each group were then classified based on this criterion. The objective of the analysis is to compare the proportion of students in each group who obtained passing scores in the post-test. Because the data generated resulted into a 2x2 table and the data were nominal, the Fisher's Exact Probability test was used. The results of the data analysis are presented in Table 3. It can be gleaned from the above table, that the proportion of passers in trigonometry is higher for students in the control group than the experimental group. The Fisher's Exact Test yielded significantly different proportions post-test scores ($p < 0.05$, two-tailed). The two-tailed P value equals 0.006.The association between rows (groups) and columns (outcomes) is considered very statistically significant. The validation of the data analysis conducted and which compared the number of passers in trigonometry between the two groups of students also yielded consistent results that a greater proportion of students in the control group than the experimental group.

IV. CONCLUSIONS AND RECOMMENDATIONS

The analysis of the data indicated that both the ratio method (control group) and the unit circle method class (experimental group) scored significantly better on the post-test than they did on the pre-test relative to mathematical achievement. The control group however, out-performed the experimental group on the post-test. The results of this study indicate clear evidence that ratio method is superior to the unit circle method in solving right-angles trigonometry. A future study could yield more meaningful results would be one that would measure more long-range effects that different methods of teaching would have on students' success in mathematics.

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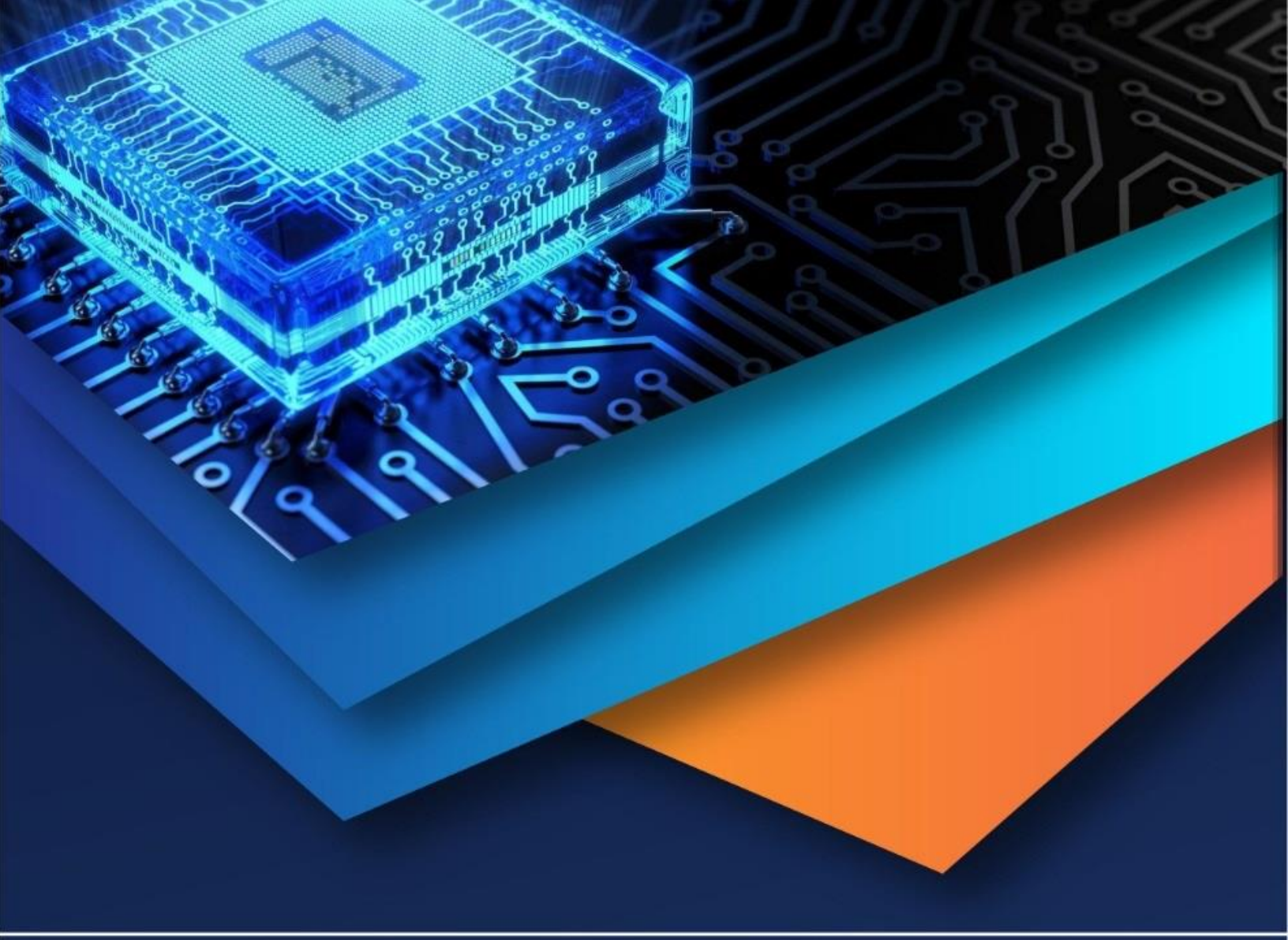

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