# Design and Kinematic Analysis of 4 DOF Parallel Manipulator 

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#### Abstract

A 4 degrees-of-freedom (DOFs) parallel manipulator with four fixed-length PUU limbs is proposed. The mobility of this parallel manipulator is dissected by means of screw theory. The forward and inverse kinematics problems are fathomed in closed-forms. The velocity condition of this new parallel manipulator is given. The singularity is contemplated in detail. The workspace for this manipulator is analyzed systematically. This parallel manipulator has the benefits of huge tilting angle and huge workspace along the rail. Along these lines, it is a reasonable contender for some mechanical applications.


Keywords: Parallel manipulator, Screw theory, Kinematic analysis, singularity.

## I. INTRODUCTION

Parallel manipulators have the advantages of high stiffness, large load capacity, low inertia, high accuracy, high velocity, high acceleration and no accumulation of positional error. Therefore, they are widely employed in many fields: medical robots, machine tools, pick-and-place, packaging, assembly etc. [1]. Specially, compared with 6-DOFs parallel manipulator, the low-DOF parallel manipulators have received much attention for the advantages of simpler mechanical design, lower manufacturing cost, larger workspace, and simple controller, playing more and more important role in many fields [2]. For example, 3T1R (3-DOFs for translation and I-DOF for rotation) manipulators are very suitable for pick-and-place tasks. Up to now, however, relatively few attempts have been made at 4-DOFs parallel manipulator. This is because a general 4-DOFs parallel manipulator cannot be constructed with identical limb structure, as pointed out by Hunt and Tsai [3] [4].
Several 4-DoF parallel manipulators with unsymmetrical limbs have been reported recently [5]. Chen Wen-jia et al proposed a 4-DOFs parallel manipulator with two PRS and two PSS limbs for a 5 -axis parallel machine tool [6].
In this paper, a novel 4-DOFs parallel manipulator with four identical fixed-length PUU kinematic chains is proposed. The mobility of this parallel manipulator is analysed by means of screw theory. The forward and inverse kinematics problems are solved analytically in terms of geometrical characteristic of the manipulator. The velocity equation of this new parallel manipulator is given. The singularity is also studied In detail. The workspace for the parallel manipulator is analyzed systematically.

## II. MECHANISM DESCRIPTION

Fig. 1 illustrates the CAD model of this novel parallel manipulator. The sketch of the 4-DOFs parallel manipulator is shown in Fig. 2, where $A, B, C, D, a, b, \mathrm{c}$ and $d$ represent the centres of the Hooke joints. The manipulator consists of a movable platform (rectangle $A B C D$ ), a base and four fixed- length limbs which connect the movable platform at point $A, B, \mathrm{C}$ and $D$ with a Hooke joint and connect the base at point $a, b, \mathrm{c}$
and $d$ with a Hooke joint and a prismatic joint, respectively. The lengths of the limbs are $/$, ( $\mathrm{i}=1,2,3,4$ ). The four prismatic joints are the active joints and are located on three parallel rails where the distances are $k$ and n , respectively. So, the manipulator can have large workspace along the rails. A reference frame (O-XYZ) is established with the start point O of the first rail being taken as the origin. The X -axis is coincident with the rail, the Z -axis is perpendicular to the base, and the Y -axis satisfies the right-hand rule. Whilst, a body-fixed coordinate system $P$ - $u v w$ is created with the geometry centre $P$ of the movable platform being taken as the origin.
The $u$-axis is parallel to the side $A B$, the $v$-axis is parallel to the side $A D$ and the $w$-axis is perpendicular to the movable platform. The lengths of the side M and $A D$ are $2 L$ and $2 H$. Let $\alpha$ denote the turn angle around the X -axis of the movable platform, and let x ,, y , and $\mathrm{z},(i=A, B, C, D)$ denote their coordinates in $O-X Y Z$, respectively. The coordinates of point $a, b, c$ and $d$ are ( $\mathrm{x}_{\mathrm{a}} 00$ ), ( $\mathrm{x}_{\mathrm{b}} 00$ ), ( $\mathrm{x}_{\mathrm{c}}, \mathrm{n} 0$ ) and ( $\mathrm{x}_{\mathrm{d}} \mathrm{k} 0$ )


Fig. 1 CAD model of the 4-DOFs parallel manipulator


Fig 2 sketch of four DOF parallel manipulator

## III. KINEMATIC ANAYLSIS OF MANIPULATOR

## A. Mobility Analysis

To study the mobility of this parallel manipulator, we have to first decompose it four kinematic chains and then study the inverse screws of each kinematic chain because all the kinematic chains connecting the movable platform with the base are identical, the process of analysis can be simplified. For example, one can analyse chain $a B$ ( $\operatorname{limb} 1_{1}$ ) and create the local frame $o_{1}-x_{1} y_{1} z_{1}$, as shown in Fig.3.


Fig. 3 The local coordinates of limb aB
Frame $o_{1}-x_{1} y_{1} Z_{1}$ is obtained by translating reference frame $O-X Y Z$ from point 0 to the centre of Hooke joint a. $B_{P}$ is the projection of $B$ in $y_{1} o_{1} z_{1}$ plane and $a B_{p}$ is the projective line $a B$ in $y_{1} o_{1} z_{1}$ plane $\beta_{1}$ is the angle from line $a B_{p}$ to line $a B$ and $\alpha_{1}$ is the angle from $z$-axis to the line $a B_{p}$. Note that $\alpha_{1}$ is also the angle from $z$-axis to the projective line of $a B$ in yoz plane By using Plucker coordinates, we determine the position of all the end points
B. Forward and Inverse Kinematics Analysis

Inverse position analysis of the parallel manipulator is concerned with the determination of the displacements of the four actuve joints when the position and the orientation is given In order to simply the analysis, we suppose that $l_{1}$ is equal to $l_{2}$ because point $P$ is the geometrical centre of the movable platform and side AB is parallel to the X -axis,one can obtain respectively.

$$
\left.\begin{array}{l}
x_{A}=x_{D}=x-L \\
x_{B}=x_{C}=x+L
\end{array}\right\}
$$

As $l_{1}=l_{2}$,it is proven that quadrangle abBA is an isosceles trapezoid,form this we obtain the following equations

$$
\left.\begin{array}{l}
\left(x_{B}-x_{a}\right)^{2}+y_{B}^{2}+z_{B}^{2}=l_{1}^{2} \\
\left(x_{A}-x_{b}\right)^{2}+y_{A}^{2}+z_{A}^{2}=l_{1}^{2} \\
\left(x_{C}-x_{d}\right)^{2}+\left(y_{C}-k\right)^{2}+z_{C}^{2}=l_{3}^{2} \\
\left(x_{D}-x_{c}\right)^{2}+\left(y_{D}-n\right)^{2}+z_{D}^{2}=l_{4}^{2} \tag{2}
\end{array}\right\}
$$

From the above equations (1),(2) we get the required actuators inputs can be directly derived as

$$
\begin{aligned}
& x_{a}=x+L \pm \sqrt{l_{1}^{2}-(y-H C \alpha)^{2}-(z-H S \alpha)^{2}} \\
& x_{b}=x-L \pm \sqrt{l_{1}^{2}-(y-H C \alpha)^{2}-(z-H S \alpha)^{2}} \\
& x_{c}=x-L \pm \sqrt{l_{4}^{2}-(y-n+H C \alpha)^{2}-(z+H S \alpha)^{2}} \\
& x_{d}=x+L \pm \sqrt{l_{3}^{2}-(y-k+H C \alpha)^{2}-(z+H S \alpha)^{2}}
\end{aligned}
$$

Velocity Equations
By differentiating the above equations w.r.t time, velocity equations are obtained

## C. Workspace Analysis

It is very important to analysis the volume and shape of the workspace for the manipulator with the give parameters in the context of industrial application
The reachable workspace is known as the region that can be reached by the reference point with the least one orientation from the below given equations

$$
\left.\begin{array}{l}
x_{1}=x_{a}-L, y_{1}^{2}+z_{1}^{2}=H^{2} \\
x_{2}=x_{b}+L, y_{2}{ }^{2}+z_{2}^{2}=H^{2} \\
x_{3}=x_{d}-L,\left(y_{3}-k\right)^{2}+z_{3}{ }^{2}=H^{2} \\
x_{4}=x_{c}+L,\left(y_{4}-n\right)^{2}+z_{4}^{2}=H^{2} \tag{3}
\end{array}\right\}
$$

From equations (1),(2),(3) we get
The workspace of the four limbs can also be written as

$$
\begin{aligned}
\Omega_{11}= & \left\{(x, y, z) \mid\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}+\left(z-z_{1}\right)^{2}=l_{1}^{2},\right. \\
& \varphi_{\min } \leq \varphi_{1} \leq \varphi_{\max }, \\
& x_{1}=x_{a}-L, \quad x_{a} \geq 0, \\
& \left.y_{1}{ }^{2}+z_{1}^{2}=H^{2}\right\} \\
\Omega_{21}= & \left\{(x, y, z) \mid\left(x-x_{2}\right)^{2}+\left(y-y_{2}\right)^{2}+\left(z-z_{2}\right)^{2}=l_{1}^{2},\right. \\
& \varphi_{\min } \leq \varphi_{2} \leq \varphi_{\max }, \\
& x_{2}=x_{b}+L, \quad x_{b} \leq L_{r}, \\
& \left.y_{2}{ }^{2}+z_{2}^{2}=H^{2}\right\} \\
\Omega_{31}= & \left\{(x, y, z) \mid\left(x-x_{3}\right)^{2}+\left(y-y_{3}\right)^{2}+\left(z-z_{3}\right)^{2}=l_{3}^{2},\right. \\
& \varphi_{\min } \leq \varphi_{3} \leq \varphi_{\max }, \\
& x_{3}=x_{d}-L, \quad x_{d} \geq 0, \\
& \left.\left(y_{3}-k\right)^{2}+z_{3}^{2}=H^{2}\right\} \\
\Omega_{41}= & \left\{(x, y, z) \mid\left(x-x_{4}\right)^{2}+\left(y-y_{4}\right)^{2}+\left(z-z_{4}\right)^{2}=l_{4}^{2},\right. \\
& \varphi_{\min } \leq \varphi_{4} \leq \varphi_{\max }, \\
& x_{4}=x_{c}+L, \quad x_{c} \leq L_{r}, \\
& \left.\left(y_{4}-n\right)^{2}+z_{4}{ }^{2}=H^{2}\right\}
\end{aligned}
$$

The reachable space can be given as $\Omega_{0}=\Omega_{1} \cap \Omega_{2} \cap \Omega_{3} \cap \Omega_{4}$

## IV. CONCLUSION

A tale 4 DOFs parallel manipulator is proposed.Its mobility is dissected by means of screw theory.The forward and inverse kinematics issues are comprehended by its geometry.Workspace for the given geometry of the limbs is additionally determined analytically.This parallel manipulator is an attractive contender for some modern applications since it has the bit of leeway enormous tilt point and huge workspace

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