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Model Order Reduction: An Approach towards a Simpler System Design

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***Abstract*– This paper proposes methods to convert a complex higher order system into a simple lower order system. In the design of control systems, you often need to work with mathematical models of high order. The analysis and synthesis of higher order systems are difficult and generally not desirable on economic and computational considerations. Therefore for, computation, control, and other purposes, it is often in demand adequately to comprise a high-order system by a low-order model. The reduced models are constructed such that all the parameters can be preserved with acceptable accuracy. Through model order reduction, a small system with reduced number of equations (reduced model) is derived. The reduced model is simulated instead, and the solution of the original differential equation can then be recovered from the solution of the reduced model. As a result, the simulation time of the original large-scale system can be shortened by several orders of magnitude.**
***Keywords*— Impulse Response Energy, Integral Square Error, Model Order Reduction, Order of System, Routh Approximation Method, stability.**

I. INTRODUCTION

Modelling and numerical simulation are unavoidable in many applications and research areas, e.g. reaction processes, micro-electro-mechanical systems (MEMS) design, and control design. The processes or devices can be modelled by partial differential equations (PDEs). To simulate such models, spatial discretization via e.g. finite element discretization is necessary, which results in a system of ordinary differential equations (ODEs), or differential algebraic equations (DAEs). After spatial discretization, the number of degrees of freedom is usually very high. It is therefore very time consuming to simulate such large-scale systems of ODEs or DAEs. Through model order reduction, a small system with reduced number of equations is derived. Electrical and electronics engineering problems most often involve large scale systems, or very fast processes that have to be controlled using low-order controllers. The analysis and synthesis of higher order systems are difficult and generally not desirable on economic and computational considerations. Thus, it is necessary to obtain a lower order system so that, it maintains the characteristics of the original system.

Main objectives of model order reduction are:
To reduce the computational complexity of the model for analysis of system.
To simplify the understanding of system for design purpose.
To economics in terms of hardware when realizing the system.
To reduce the simulation time with the model of system.
To reduce the controller size.

A. Approaches of Model Order Reduction

Generally, model order reduction is done in two ways:

1) *Time Domain Approach:* In this approach, first convert the system dynamics by first order linear differential equation (means state space form), let us take an nth order linear time invariant system which is represented by Eqns. 1.1 & 1.2 as:

$$x(t)= Ax(t) + Bu(t) \quad (1.1)$$

$$y(t)= Cx(t) + Du(t) \quad (1.2)$$

Where, x(t) - n dimensional state vector
u(t) - m dimensional input vector
y(t) - p dimensional output vector
A, B, C and D are matrices, which have size of (n X n), (n X m), (p X n) and (p X m) respectively.

The model order reduction problem consists of finding an approximate system of order r <n, given by:

$$x_r(t)= A_r x(t) + B_r u(t) \quad (1.3)$$

$$y_r(t)= C_r x(t) + D_r u(t) \quad (1.4)$$

Eqns. (1.3 & 1.4) show the reduced model of system which is approximate of the original system.

B. Frequency Domain Approach

The second and important approach is frequency domain. In this approach we take Laplace transform of differential equation, and find the transfer function of the system with all the initial conditions set to zero. Let the transfer function of higher order system is given by:

$$G(s) = \frac{b_1 S^{n-1}+\dots\dots\dots\dots+b_n}{a_0 S^n+\dots\dots\dots\dots+a_n} \quad (2.1)$$

Let us assume that a reduced order model R(s) of order 'r'<n which approximates the system G(s) is given by

$$R(s) = \frac{b_1 S^{r-1}+\dots\dots\dots\dots+b_r}{a_0 S^r+\dots\dots\dots\dots+a_r} \quad (2.2)$$

The coefficients of reduced order model which good approximates the original system are calculated.

The rest of this paper is summarized as follows. In section II, proposed method for reduced order system design is discussed; an overview of reduction methods is shown in section III, section IV describes performance measures for order conversion of the system, followed by conclusion in Section V.

II. PROPOSED WORK

Proposed work includes finding reduced order system. For reduction of system Routh approximation method is used. Routh approximation method (RAM) is preferred because it is one of the most elegant methods of model reduction. It involves no Eigen value evaluation and provides the capability to derive all stable lower order models simultaneously via a single set of computation. Due to the fact that the RAM retains some of the initial time moments only, the reduced model obtained by the RAM tends to approximate the steady state response of the original system in the time domain.

The requirement that an approximant to a stable system must be stable suggests the Routh Approximation Method. The idea underlying this method is to develop the well-known Routh Table for the original system and then to construct the approximant in such a manner that the coefficients of its Routh Table agree, up to a given order, with those of the original system. By this construction, it is obvious that any Routh approximant of a stable system is stable. The Routh approximation not only preserves stability but has other interesting and useful properties.

III. AN OVERVIEW OF REDUCTION METHODS

An extensive range of model order reduction methods have been anticipated by several authors for the time of last few decades. A diversity of model order reduction techniques are used for reduction propose and they give diverse reduced model for same system model but the value of a reduced model is finally judged by the manner it is utilized. Some of the well-known methods are introduced viz.

A. Padé Approximation Method

Padé approximation [1], [4] is the method of model order reduction of the higher order system. This gives the simplification of a model after converting it into a reduced order model. Padé approximant is the "best" approximation of a function by a rational function of given order. In this technique, the approximant's power series agrees with the power series of the function it is approximating. The Padé approximant often gives better approximation of the function than truncating its Taylor series, and it may still work where the Taylor series does not converge. For these reasons Padé approximants are used extensively in computer calculations. To understand Padé Approximation Consider a function

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$$f(s)= C_0 + C_1S + C_2S^2 + \ldots\ldots \qquad (1)$$

And rational function Um(s)/Vn(s) where Um(s) and Vn(s) are mth and nth order polynomials in s respectively, and m <= n. The rational function Um(s)/Vn(s) is said to be a Pade approximant of f(s) if and only if the first (m+n) terms of the power series expansions of f(s) and Um(s)/Vn(s) are identical. For the function *f*(*s*) in Eqn. (1) to be approximated, let the following Pade approximant be defined.

$$\frac{Um(s)}{Vn(s)} = \frac{a_0 + a_1S + \ldots\ldots + a_{n-1}S^{n-1}}{b_0 + b_1S \ldots\ldots + S^n} \qquad (2)$$

Drawback- Does not guarantee stability.

B. Pole Placement Method

The pole placement is one of the most important methods for designing controller for linear systems. A pole-placement method [7], which is agreeable to either a complete or incomplete state (output) feedback, is employed to drastically improve the dynamic stability characteristics of a practical power system (original open-loop system) by designing a suitable controller (i.e. a closed-loop system) with output feedback. Furthermore, an adequate reduced-order model of the original system is obtained by using three distinct pole selection criteria. The pole-placement method is also used to design an appropriate closed-loop system of the attained reduced-order model based on complete state feedback.

C. Particle Swarm Optimization

Particle swarm optimization is a population based stochastic optimization technique. In PSO [15], an aggregation of particles (or agents) swarms through an N-dimensional space. The principles for how the particles pass through the space are based on simple flocking rules that cause the particles to orbit around the amplest found solution in the promise of finding improved one. This method seems to be simple but it is efficient and can be used in many types of optimization troubles.

D. Routh Approximation Method

Let us consider a linear, time-invariant SISO system having the transfer function

$$H(s) = \frac{b_1S^{n-1} + \ldots\ldots + b_n}{a_0S^n + \ldots\ldots + a_n} \qquad (3)$$

A linear time invariant system by m inputs and I outputs can be symbolized by a matrix of transfer functions of the form (3) through the numerator coefficients b_i being I x m matrices. As the denominator of a Routh approximant depends merely on the denominator of H(s), a Routh approximant [2], [3] to an I-output, m-input system is computed by computing the Routh approximant for every one term in the matrix of transfer functions. The denominator of the Routh approximant merely has to be computed once since it is similar for every term.

1) *Advantages:* Routh approximation method has following advantages-
   a) It guarantees that stability of system is preserved, when the original system is stable.
   b) The poles and zeros of the approximants approaches the poles and zeros of the original system function as the order of the approximation is improved.
2) *Disadvantages:*
   a) Only guarantees that the steady state of system is preserved and gives no guarantee that transient state of system is preserved.

IV. PERFORMANCE PARAMETERS

A. Impulse Response Energy

Impulse response describes the reaction of the system as a function of time(or possibly as a function of some other independent variable that parameterizes the dynamic behaviour of the system). Let h(t) be the impulse response of transfer function *H(s)*, and from it the impulse response energy be defined by the integral of impulse response and given by

$$\|h^2\| = \int h^2(t)\, dt \qquad (4)$$

Assuming that *H(s)* is the transfer function of a stable system.

B. Integral Square Error (ISE)

Integral squared error is a measure of system performance formed by integrating the square of the system error over a fixed interval of time; this performance measure and its generalizations are frequently used in linear optimal control and estimation theory.

$$ISE = \int \epsilon^2\, dt \qquad (5)$$

V. CONCLUSIONS

This paper focuses on the study of Model Order Reduction to reduce the order of a system using suitable reduction method, preserving stability with acceptable accuracy of the reduced system. A survey of some pre-existing model order reduction methods is done and after that with a proper study, it is concluded that Routh approximation method retains some of the initial time moments only and the reduced model obtained by routh approximation method tends to approximate the steady state response of the original system in some classical problems. For such classical cases, the Routh approximation method always used to produce good approximants.

Reduced system obtained by Routh Approximation Method will always be stable provided the original system is stable. This method of model order reduction can be used to extend the research further to make improvements in the existing method.

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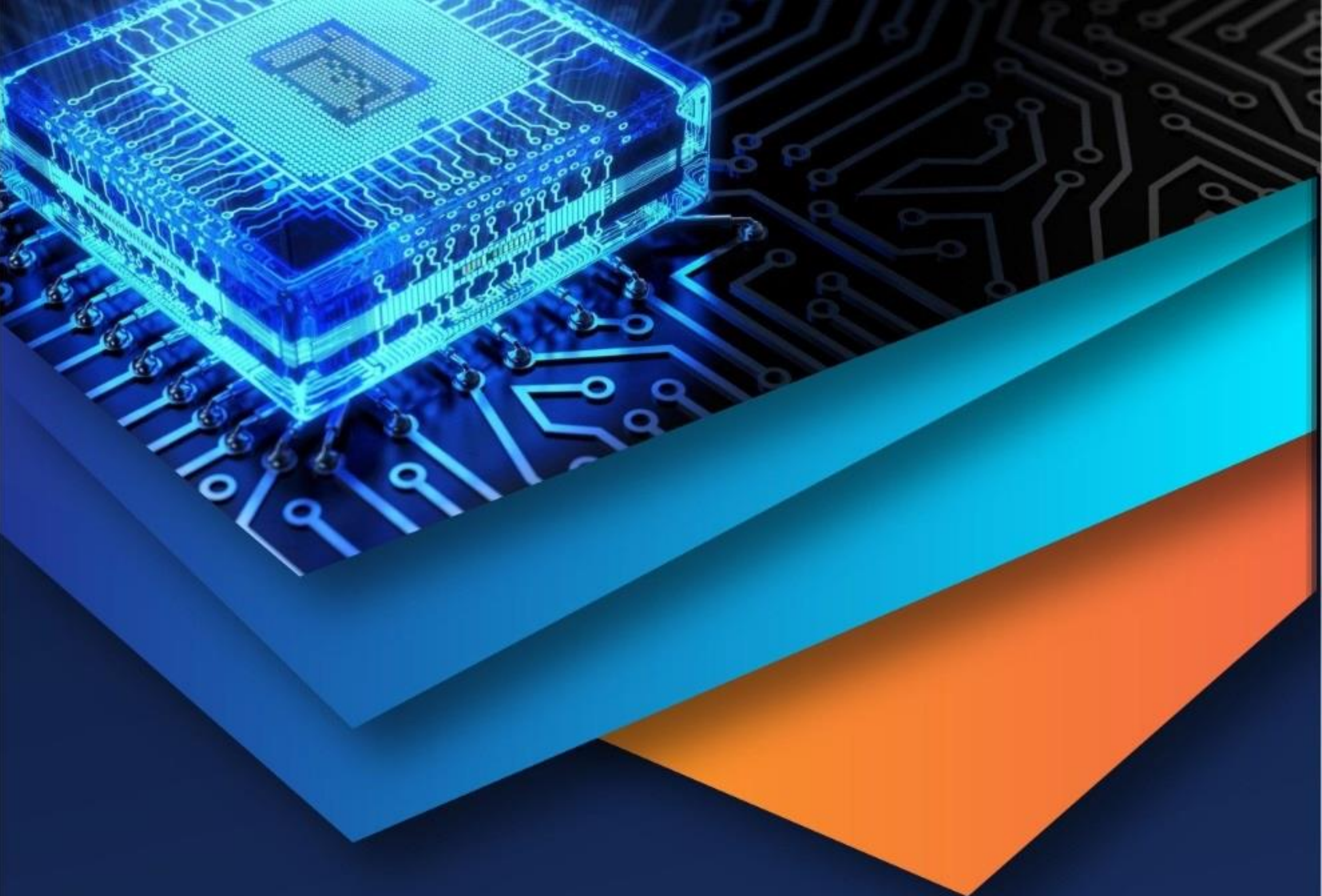

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