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Stress Analysis of a Variable Thickness Rotating FGM Disc

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Abstract: A mathematical model to describe stress analysis in a functionally graded rotating disc having linearly varying thickness has been investigated. The properties of the disc material like density and young's modulus are assumed as power functions of disc radius. The stress analysis of the FGM disc with variable thickness has been compared with a similar disc but having constant thickness. It is observed that compared to uniform thickness disc the radial and tangential stresses in the variable thickness disc decreases throughout. The strain rates in the FGM disc with variable thickness are significantly reduced over the entire radius than the constant thickness disc.
Key words: Rotating disc, functionally graded material, Stress analysis, variable thickness

I. INTRODUCTION

Rotating disc is a widely used component in engineering and structural applications, involving severe mechanical loadings, thereby causing significant deformations. In most of these applications, the disc is subjected to several mechanical and/or thermal loadings, thereby causing significant deformations. Therefore, it attracts the attention of many researchers [1-4]. Thus, a new material concept of Functionally Graded Materials (FGMs) has been developed. In FGMs the constituents or their contents vary with respect to position coordinates, which enable them to offer unique performance [5-7].

Yeh and Han [8] proposed a symmetric formulation to predict elastic stresses in an inhomogeneous, rotating disc with arbitrary thickness under thermal loading. Durodola and Adlington [9] investigated the effects of imposing various forms of gradation in material properties on the elastic stresses and strains in rotating axisymmetric components such as disc and rotor. Zenkour [10] presented a solution procedure to estimate the distribution of stresses in rotating FG disc of constant thickness. The study indicates that the stresses and deformations in the disc are lower when the material of the disc is tailored in such a way that the elastic modulus and density are more near the inner radius than those at the outer radius.

Callioglu [11] obtained closed-form solutions for rotating annular FGM disc subjected to internal and external pressures. The elastic modulus and density of the disc were assumed to vary radially according to three different forms of power law functions. It is observed that with the increase in gradient parameter (n), the stresses and deformations in the disc change significantly. Bayat *et al* [12] obtained the exact solutions for axi-symmetric rotating disc made of FGM and having variable thickness. The study reveals that the FGM disc having parabolic and hyperbolic convergent thickness profiles exhibits smaller stresses and displacements as compared to a constant thickness disc. Nejad *et al* [13] performed elastic analysis of FGM disc subjected to internal and external pressures with exponential varying properties. The radial displacement, radial stress, and circumferential stress were estimated for different values of material inhomogeneity constant. The analytical results are observed to be in good agreement with those estimated by FEM.

The literature consulted reveals that a number of studies have been undertaken to investigate the stress analysis of rotating FGM disc having constant thickness. The variable thickness disc performs better than a constant thickness disc [14]. Therefore, it has been decided to investigate the stress response of variable thickness FGM disc.

II. DISC PROFILE AND MATERIAL PROPERTIES

Let us consider a variable thickness functionally graded rotating disc with the inner radius a (=0.2 m) and the outer radius b (=0.5 m). The thickness $h(r)$ of the disc is assumed to vary radially, according to the following equation,

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$$h(r) = h_b \left(\frac{r}{b} \right)^k \tag{1}$$

where h_b is thickness of disc at the outer radius and k(=-0.5) is the disc thickness gradation index. The thickness of the disc is assumed to be significantly small compared to its diameter, therefore condition of plane stress (σ_z =0) can be assumed [6].

In this study, we have compared the elastic stresses in variable thickness FGM disc with a similar FGM disc of uniform thickness (t). For the purpose of comparison, the volume of both the FGM discs are kept equal *i.e.*,

$$\int_a^b 2\pi\, r h(r) dr = \pi (b^2 - a^2) t \tag{2}$$

Substituting $h(r)$ from Equation (1) into above equation and simplifying, we get,

$$h_b = \frac{(b^2 - a^2)\, t\, (2+k)\, b^k}{2\, (b^{2+k} - a^{2+k})} \tag{3}$$

Assuming thickness of uniform disc (t) = 5 *mm*, one may estimate the value of h_b from equation (3), after substituting the numerical values of a, b and k, as mentioned earlier.

The properties of disc material, like density, Young's modulus, coefficient of thermal expansion and thermal conductivity are assumed to vary radially, according to the following relation,

$$\rho(r) = \rho_0 \left(\frac{r}{b} \right)^{n_1},\ E(r) = E_0 \left(\frac{r}{b} \right)^{n_2} \tag{8}$$

where ρ_o (=5600 Kg/m^3) and E_o (= 150 *GPa*) are respectively the values of density, young's modulus and coefficient of thermal expansion at the outer radius of FGM disc. The exponents n_1 (= -0.4873) and n_2 (= -0.5194) denotes gradation indices respectively for density and young's modulus.

III. MATHEMATICAL FORMULATION

The radial strain (ε_r) and tangential strain (ε_θ) are related to the radial displacement (u_r) by,

$$\varepsilon_r = \frac{du_r}{dr},\ \varepsilon_\theta = \frac{u_r}{r} \tag{10}$$

The above relations yield the following compatibility equation,

$$\varepsilon_r = \varepsilon_\theta + r \frac{d\varepsilon_\theta}{dr} \tag{11}$$

For elastic deformations, the constitutive equations between elastic stresses and strains for an isotropic FGM disc under plane stress condition are given by [15],

$$\varepsilon_r = \frac{1}{E(r)} (\sigma_r - \nu \sigma_\theta) \tag{12}$$

$$\varepsilon_\theta = \frac{1}{E(r)} (\sigma_\theta - \nu \sigma_r) \tag{13}$$

where σ_r and σ_θ are respectively the radial and tangential stresses in the disc and v (=0.3) is the Poisson's ratio.

The force equilibrium equation for a variable thickness rotating FGM disc is given as [16],

$$\frac{d}{dr} \left[r h(r) \sigma_r \right] - h(r) \sigma_\theta + \rho(r) \omega^2 r^2 h(r) = 0 \tag{14}$$

where $\rho(r)$ is the density and ω is angular velocity of the disc. The angular velocity (ω) is taken 650 *rad/s* in this study.

The equilibrium equation (14) is satisfied by the stress function F, defined as,

$$\sigma_r = \frac{F}{r\,h(r)}, \quad \sigma_\theta = \frac{1}{h(r)}\frac{dF}{dr} + \rho(r)\omega^2 r^2 \tag{15}$$

Substituting the values of ε_r and ε_θ from equations (12) and (13) respectively and using equation (15) into the compatibility equation (11), we can get on simplification,

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr}\left[1 - \frac{r}{h(r)}\frac{dh}{dr} - \frac{r}{E(r)}\frac{dE}{dr}\right] + F\left[\frac{\nu r}{h(r)}\frac{dh}{dr} + \frac{\nu r}{E(r)}\frac{dE}{dr} - 1\right] = -\rho(r)\omega^2 r^3 h(r)\left[3 + \upsilon - \frac{r}{E(r)}\frac{dE}{dr}\right] - \omega^2 r^4 h(r)\frac{d\rho}{dr} \tag{16}$$

Substituting the values of properties from equation (8) into equation (16), we get,

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr}\left[1 - k - n_2\right] + F\left[vk + vn_2 - 1\right] = -\frac{\rho_o\omega^2 h_b r}{b^{n_1+k}}(3 + \upsilon + n_1 - n_2)r^{3+n_1+k} \tag{17}$$

Solving the above this differential equation, the stress function (F) can be obtained as,

$$F = C_1 r^{\frac{n_2+k+m}{2}} + C_2 r^{\frac{n_2+k-m}{2}} + Ar^{3+n_1+k} \tag{18}$$

where C_1 and C_2 are the constants of integration and A, B, C and m are given by,

$$A = \frac{-\rho_o\omega^2 h_b(3 + v + n_1 - n_2)}{b^{n_1+k}(8 + n_1{}^2 + n_1k + 3k + 6n_1 - n_1n_2 - n_2k - 3n_2 + vn_2 + vk)} \quad ; \; m = \sqrt{(n_2 + k)^2 - 4(vn_2 + vk - 1)}$$

The radial and tangential stresses can be obtained by substituting the values of stress function (F) in equation (15) as,

$$\sigma_r = \frac{1}{h(r)}\left[C_1 r^{\frac{n_2+k+m-2}{2}} + C_2 r^{\frac{n_2+k-m-2}{2}} + Ar^{2+n_1+k}\right] \tag{19}$$

$$\sigma_\theta = \frac{1}{h(r)}\left[\left(\frac{n_2 + k + m}{2}\right)C_1 r^{\frac{n_2+k+m-2}{2}} + \left(\frac{n_2 + k - m}{2}\right)C_2 r^{\frac{n_2+k-m-2}{2}} + (3 + n_1 + k)Ar^{2+n_1+k}\right] + \rho(r)r^2\omega^2 \tag{20}$$

The disc is assumed to operate under fixed-free conditions, as expressed below [8],

$$\sigma_r = 0 \text{ at } r = a \text{ and } \sigma_r = 0 \text{ at } r = b \tag{21}$$

Using the above boundary condition in equation (19), we get,

$$C_1 = \frac{D_2 b^{\frac{2+m-n_2-k}{2}} - D_1 a^{\frac{2+m-n_2-k}{2}}}{(b^m - a^m)} \text{ and } C_2 = \frac{D_1 b^m a^{\frac{2+m-n_2-k}{2}} - D_2 b^{\frac{2+m-n_2-k}{2}} a^m}{(b^m - a^m)} \tag{22}$$

where,

$$D_1 = -Aa^{2+n_1+k} \text{ and } D_2 = -Ab^{2+n_1+k} \tag{23}$$

IV. RESULTS AND DISCUSSION

A computer code based on the mathematical formulation presented in previous section has been developed to obtain the distribution of stresses and strains in the FGM disc having variable thickness. For comparison, the results have also been obtained for a similar FGM disc but having constant thickness. Before presenting the results obtained in the present study, it is necessary to validate the analysis carried out. Therefore, the radial stress has been computed for a rotating with constant thickness disc. The results obtained

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are compared with those reported in literature [17] for the same disc. A close agreement is observed between the present study and results reported earlier (refer Figure 1).

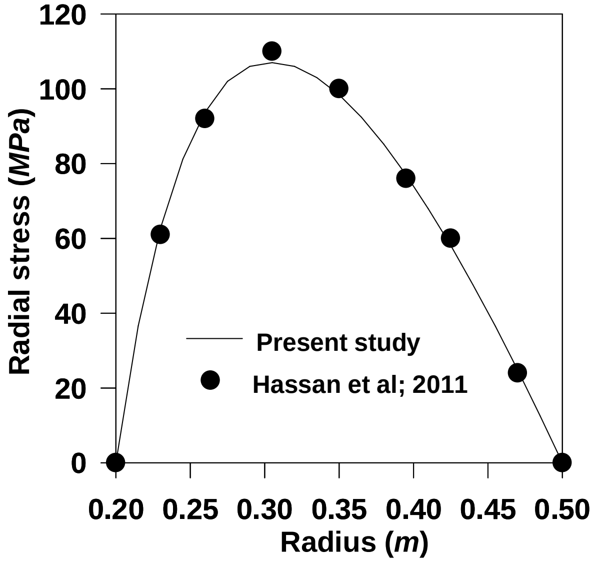

Fig. 1: Comparison of radial stress in disc: Present study vs results reported earlier

A. Effect of varying disc thickness profile on elastic stresses and strains

Figure 2 shows the variation of radial and tangential stresses in the FGM disc. The effects of thickness gradation index (*k*) on radial stress distributions in FGM disc are presented in Fig. 2 (a) and 2(b). It is observed that the radial stresses at the inner and outer radius vanish, under the imposed boundary conditions Eq. (21). The radial and tangential stresses in the FGM disc decreases throughout for the FGM disc with decreasing thickness profile (*k* = -0.5) as compared to FGM disc having constant thickness (*k* = 0). The variation in the radial stress is observed only in the inner portion of the disc. In Fig. 2(b), the tangential stress is highest at the inner radius and it decreases along the radius.

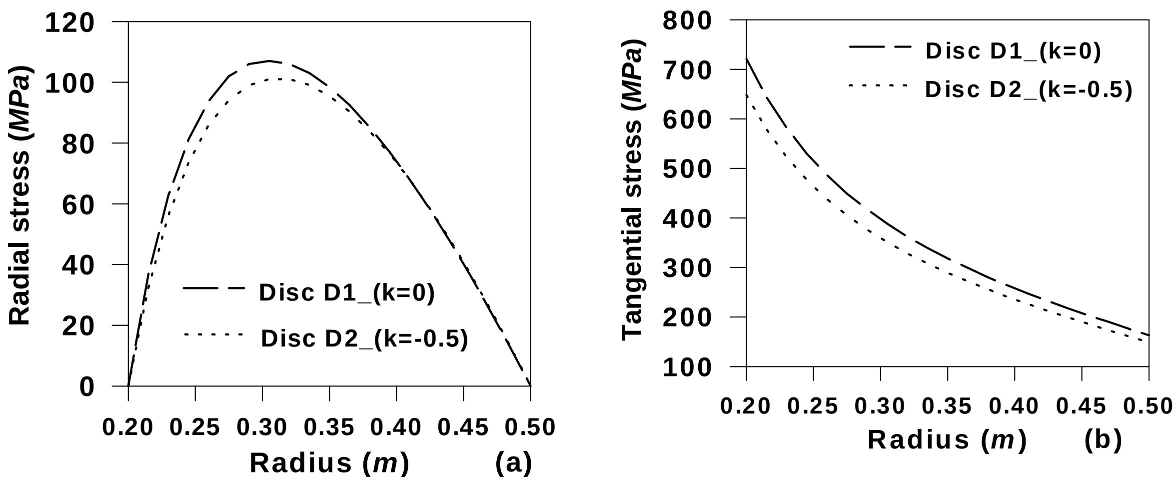

Fig. 2: Variation of: (a) Radial and (b) Tangential stresses in FGM discs.

The variation of strain in the FGM disc is shown in Figs. 3(a) and 3(b). The radial strain is compressive and tangential strain is tensile. It is revealed from the Fig. 3 (a) that the radial strain in the FGM disc (*k* = -0.5) decreases throughout when compared to constant thickness FGM disc (*k* = 0). The variation in the tangential strain (Fig. 3 b) is similar to the radial strain. From the above discussion, it is concluded that the mechanical response of the FGM disc is superior when the disc have decreasing thickness profile as compare to the disc with constant thickness profile.

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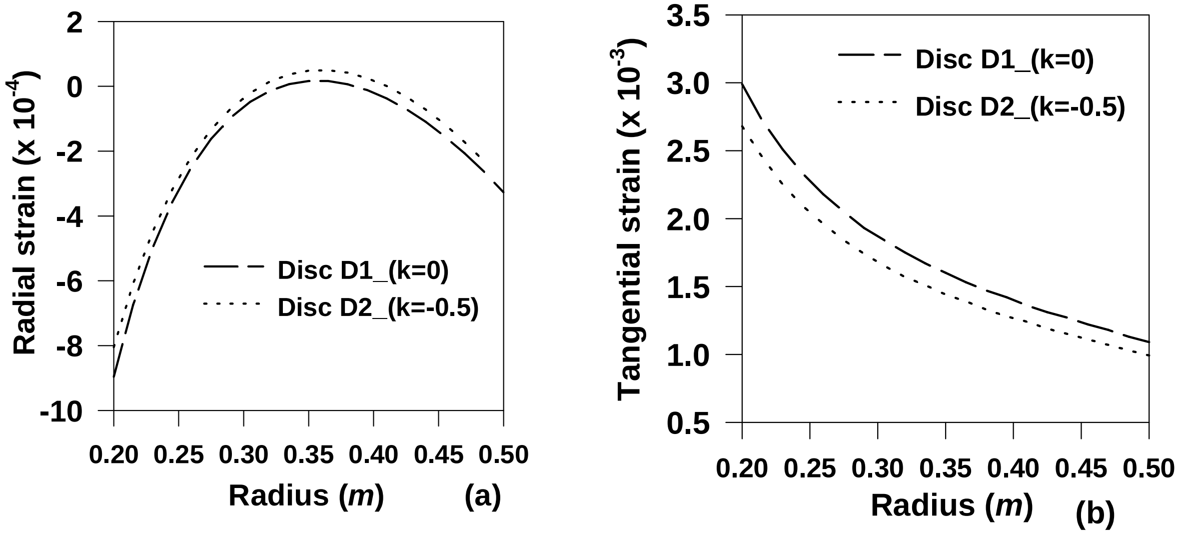

Fig. 3: Variation of: (a) Radial and (b) Tangential strains in FGM discs.

V. CONCLUSIONS

In this paper, effect of variation of thickness profile in FGM disc has been analyzed on the stress strain analysis of rotating FGM disc. For the varying material properties, we have obtained the exact solutions by solving the governing equation. It is concluded that, radial and tangential stresses in the FGM disc is significantly reduced for FGM disc having variable thickness as compared to similar FGM disc having constant thickness. It is also observed that radial and tangential strain also decreases for FGM disc with variable thickness disc as compare constant thickness disc. Therefore, FGM disc with decreasing thickness profile have relatively lesser chances of distortion.

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