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Roman Coloring of Graphs and Application to Military Strategy

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Abstract: In this paper, we introduce a new type of graph coloring, Roman coloring, motivated from the traditional Roman military defense strategy. A related parameter called Roman Chromatic Number is also introduced and its properties and values for special types of graphs are investigated.
Keyword: Graph, Coloring, Roman coloring, Roman Chromatic Number

I. INTRODUCTION

The majority of early graph theory research on graph coloring pays attention only to finding some possible solution to the Four Color Conjecture. After Appel and Haken gave a computer verification proof of the Four Color Conjecture, research focus on graph coloring was shifted to vertex coloring that satisfies some specified property for the induced edge coloring [5]. The coloring is also played an important role in combinatorial optimization problems and critical graphs were crucial in the Chromatic number Theory [7, 8, 9, 10, 11].

Jason Robert Lewis [1] suggested several new graph parameters in his Doctoral Thesis. Several studies were made in applying such parameters to Roman defense strategy [2, 3, 4, 5, 6]. The basic idea was that in a specified city, if the streets are considered as the edges of a graph and the meeting points of the streets, called the junctions, as the edges of the graph, then we can color each vertex by the number of soldiers deployed at that junction and require that every street (edge) should be guarded by at least one soldier using a strategy that if any street have no soldier, then there myst be an adjacent junction with two soldiers so that one among them may be deployed to the former junction in case of emergency. This motivated us to define a new type of graph coloring, namely, Roman Coloring and a related parameter, Roman Chromatic number. We study the properties of Roman Coloring and the values of Roman number for some special graphs. For the terms and definitions not explicitly here, refer Harary [12].

II. MAIN RESULTS

Let G be a connected graph. Roman coloring of G is an assignment of three colors {0, 1, 2} to the vertices of G such that any vertex with color, 0 must be adjacent to a vertex with color, 2. The color classes will be denoted as V_0, V_1, V_2 which are the subsets of V(G) with colors 0, 1, 2 respectively.

Weight of a Roman coloring is defined as the sum of all vertex colors. Roman Chromatic number of a graph G is defined as the minimum weight of a Roman coloring on G and is denoted by $R(G)$. a Roman coloring of G with the minimal weight is called a minimal Roman coloring of G.

It can be easily seen that if G has a Roman coloring, then for any edge e = (u, v}, either the edge e is incident with a vertex with color, 2 or both u and v are adjacent to vertices with colors at least one.

1) Theorem 2.1. In a minimal Roman coloring of a graph G, there is no edge connecting $V_1 and\ V_2$

Proof. Consider a minimal Roman coloring of a graph G. If possible, suppose that there is an edge e connecting a vertex $u \in V_1$ and a vertex $v \in V_2$. Since v is adjacent to u, this edge is incident to all edges incident at u. So we can change the color of u from one to zero, which is a contradiction to the minimality of the Roman coloring of a graph G. Hence the theorem follows.

2) Theorem 2.2. Let $E(v) = \{e \in E(G): v\ is\ incident\ to\ e\}$. Then $R(G) = 1$ for a graph G if and only if $E(v) = E(G)$ for some vertex v of G,

Proof. If R(G) = 1, then all edges of G are adjacent to v. Hence, $E(V) = E(G)$. Conversely, if $E(v) = E(G)$, then v is incident with all the edges of G, then the coloring of v with the color, 1 and coloring all other vertices of G with 0 is a Roman coloring with R(G) = 1.

3) Theorem 2.3. For a graph G, R(G)= 1 if and only if G is a star graph.

Proof. First suppose R(G) = 1. Then there is no vertex with color 2 and exactly one vertex, v, with color 1. Then, v is incident with all the edges of G. Then, G is a star graph. Conversely, if G is a star graph, then, the coloring that assigns the color 1 to v and the color 1 to all other vertices of G is a Roman coloring of G with R(G)=1.

4) Theorem 2.4. If R(G)=2, then $E(v) = E(G)$, for some vertex v of G.

Proof. If R(G)= 2 and f is a Roman coloring of G, then we have to consider two cases.

Case-1: Exactly one vertex v has the color, 2. Then all vertices with color 0 must be adjacent to v and since R(G)= 2, they constitutes the entire set of vertices, so that $E(v) = E(G)$,

Case: 2: There are exactly two vertices, say u, v with color, 1. Then, all other vertices with color 0 and no vertex must have color 2. Hence G must precisely be the edge {u, v}, so that $E(G) = E(v)$.

5) Theorem 2.5. If G is not a star graph and the minimum eccentricity and the maximum eccentricity of vertices in G are 1 and 2 respectively, then R(G)= 2.

Proof. Since G is not a star graph and minimum eccentricity is 1, there exists a vertex v, which is adjacent to all other vertices. So the coloring that assigns the color 2 to v and the color, 0 to u is a Roman coloring with R(G)=2.

6) Theorem 2.6. If R(G) = 3, then there exists a Roman coloring and exactly two vertices u, v with the colors 1, 2 respectively.

Proof. In a minimal Roman coloring of the given graph G, R(G)= 3 is possible in two ways:

Case-1: There exists exactly two vertices u, v with the colors 1, 2 respectively. This is the required condition in the theorem.

Case-2: There exist three vertices u, v, w all with the same color, 1. Then we can find another vertex x and a vertex among u, v, w, say u, such that the new coloring that assigns the color 2 to x, the color 1 to u and the color 0 to all the remaining vertices is a Roman coloring of G. Since G is connected, there exist a pair of vertices among u, v, w, say u, v, such that the induced subgraphs induced by the neighborhoods, N[u] and N[v] have one vertex in common. Then we can assign the color 2 to this common vertex, the color 1 to the third vertex and the color 0 to all the remaining vertices which will give a new Roman coloring of G in which there are exactly two vertices u, v with the colors 1, 2 respectively. Then, this case reduces to case-1 and hence the theorem follows.

7) Theorem.2.7. If $G = K_n$, n >2, then R(G)= 2

Proof. In $G = K_n$, $E(v) \neq E(G)$ for all vertices of G, the result follows.

8) Theorem.2.8. For $G = P_n$, R(G)= 2r or 2r-1 according as if n = 5r, 5r – 1 or n = 5r − 2, 5r – 3, 5r − 4

Proof. Along the path we take a block of four edges. Let $v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5$ be the vertices and edges on the path. Arranging two guards at the vertex v_3 we can guard all the edges in the block. We can do the same for each block of five vertices and four edges. We cannot guard a block by less number of guards. Thus for a path which contains n = 5r vertices, only 2r guards are needed.

For a path containing 5r − 2, 5r − 3 or 5r − 4 vertices, there are r – 1 complete blocks of five vertices. We require 2(r − 1) guards to take care of the edges in the blocks. After taking the blocks from one end if there remains either one or two vertices at the other end, one guard arranged at the first vertex on the last section can protect the last edges. If there are three vertices at the last portion, say u1, u2, u3, one guard is needed at u3. In all these cases R(P_n) = 2r − 1. If there are four vertices in the remaining block, then two guards are to be placed at the vertex u4. Hence R(P_n) = 2r.

Using a similar argument used in the previous proof we get

9) Theorem 2.9. For $G = C_n$, R(G)=2r or 2r–1 according as n=5r, 5r–1 or n=5r−2, 5r–3, 5r−4.

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