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Unique Minimal Proper Roman Coloring of a Graph

Dr. J. Suresh Kumar
Assistant Professor, PG and Research Department of Mathematics, N.S.S. Hindu College, Changanacherry, Kerala, India 686102

Abstract: Suresh Kumar [12] introduced the notion of Roman Coloring of a graph, G which is an assignment of three colors {0, 1, 2} to the vertices of G such that a vertex with color, 0 must be adjacent to a vertex with color, 2 motivated from the Roman military defense strategy. The Proper coloring version of the Roman Coloring of a graph was introduced and studied by Suresh Kumar and Preethi K Pillai [13]. In this paper, we investigate the graphs having a unique minimal Proper Roman Coloring.
Keyword: Graph, Coloring, Proper Roman coloring, Minimal Proper Roman Coloring

I. INTRODUCTION

Early graph theory research on graph coloring pays attention to find some possible solution to the Four Color Conjecture. After Appel and Haken gave a computer verification proof of the Four Color Conjecture, research focus was shifted to coloring that satisfies some specified property [5]. The coloring plays an important role in the combinatorial optimization problems and critical graphs (optimal graphs) are crucial in the Chromatic number Theory [7, 8, 9, 10, 11].

Several studies were made in applying graph theory parameters to Roman defense strategy [1, 2, 3, 4, 5, 6]. The basic idea was that in a specified city, if the streets are considered as the edges of a graph and the meeting points of the streets, called the junctions, as the edges of the graph, then we can color each vertex by the number of soldiers deployed at that junction and require that every street (edge) should be guarded by at least one soldier using a strategy that if any street have no soldier, then there must be an adjacent junction with two soldiers so that one among them may be deployed to the former junction in case of emergency. This motivated us to define a new type of graph coloring, namely, Roman Coloring and a related parameter, Roman Chromatic number. We study the properties of Roman Coloring and the values of Roman number for some special graphs. For the terms and definitions not explicitly here, refer Harary [14].

II. MAIN RESULTS

Let G be a connected graph. A Proper Roman Coloring of a graph, G is an assignment of three colors {0, 1, 2} to the vertices of G such that (1) adjacent vertices should have distinct colors and (2) any vertex with color, 0 must be adjacent to a vertex with color, 2. The color classes will be denoted as V_0, V_1, V_2 which are the subsets of V(G) with colors 0, 1, 2 respectively.

Weight of a Roman coloring is defined as the sum of all vertex colors. Roman Chromatic number of a graph G is defined as the minimum weight of a Roman coloring on G and is denoted by $X_R(G)$. a Roman coloring of G with the minimal weight is called a minimal Roman coloring of G.

1) *Definition 2.1.* A graph, G is said to have a unique minimal Proper Roman Coloring if any minimal Proper Roman Coloring of G is unique. For example, the cycle graph, C_3 has a unique minimal Proper Roman Coloring of assigning all the three vertices by the three colors 0, 1 and 2. We prove it now.
2) *Theorem 2.2.* The cycle graph C_3 has a unique minimal Proper Roman Coloring.
a) *Proof.* Since $X_R(C_3) = 3$ and partitions of 3 into {1 2} are 2+1 or 1+1+1, a minimal Proper Roman coloring of a graph C_3 can be in only two ways:
b) *Case-1:* There exist a vertex with color, 2 and another with color, 1. Then, the third vertex must have the color, 0. Thus, all the three vertices by the three colors 0, 1 and 2.
c) *Case-2:* There exist 3 vertices with color, 1 each. This case is not possible, since adjacent vertices must have distinct colors. Hence the graph C_3 has a unique minimal Proper Roman Coloring.
3) *Theorem 2.3.* The cycle graph C_4 has a unique minimal Proper Roman Coloring.
a) *Proof.* Since $X_R(C_4) = 3$ and partitions of 3 into {1 2} are 2+1 or 1+1+1, a minimal Proper Roman coloring of a graph C_4 can be in only two ways:
b) *Case-1:* There exist a vertex with color, 2 and another with color, 1
c) *Case-2:* There exist 3 vertices with color, 1 each

But case-2 is not possible, since if 3 vertices have the color, 1, in the graph C_4 then two of these vertices will be adjacent, which is

a contradiction to the Proper Roman coloring of C_4. Hence the only way of Proper Roman coloring of C_4 is by coloring a vertex with color, 2 and the non-adjacent vertex with color, 1 are remaining 2 vertices by the color, 0. Hence the graph, C_4 has a unique minimal Proper Roman Coloring.

4) *Theorem 2.4.* The cycle graph C_5 has a unique minimal Proper Roman Coloring.

a) *Proof.* Since $X_R(C_4) = 5$ and partitions of 5 into {1 2} are 1+1+1+1+1, 1+1+1+2, 2+2+1, a minimal Proper Roman coloring of a graph C_5 can be in only three ways.

b) *Case-1:* All the five vertices have the color, 1.

c) *Case-2:* Three vertices have the color, 1 and one vertex each has color 2 which is adjacent the 5th vertex which has color 0.

d) *Case-3:* Two vertices have color, 2 and one vertex have color, 1.

But since C_5 has only 2 independent vertices, no color can be assigned to more than 2 vertices. Thus, the first two cases are not possible. Hence the only one possible way for namely, color the five vertices in the order: (2,0,2,1,0). Hence the graph, C_5 has a unique minimal Proper Roman Coloring.

Suresh Kumar and Preethi K Pillai [12] proved that $X_R(C_{2k+1}) = 2k + 1$, $X_R(C_{2k}) = 2k - 1$.

5) *Theorem.2.5.* The cycle graph C_n , $n \geq 3$ is unique minimal Proper Roman Colorable.

a) *Proof.* We have $X_R(C_{2k+1}) = 2k + 1$ and $X_R(C_{2k}) = 2k - 1$. Since C_n has at most $\left\lfloor \frac{n}{2} \right\rfloor$ independent vertices so that no color can be assigned more than $\left\lfloor \frac{n}{2} \right\rfloor$ times. So, we need to consider the Partitions of n into 1s and 2s such that neither 1 nor 2 appears more than $\left\lfloor \frac{n}{2} \right\rfloor$ times. So both 1 and 2 appears at most $\left\lfloor \frac{n}{2} \right\rfloor$ times. If 2 is the number in a partition that is appearing less than $\left\lfloor \frac{n}{2} \right\rfloor$ times, then, it will imply that 1 is appearing more than $\left\lfloor \frac{n}{2} \right\rfloor$ times. Hence 1 must be the number in a partition that is appearing less than $\left\lfloor \frac{n}{2} \right\rfloor$ times. Thus the Partitions is $(2,2,\ldots,2,1)$, which consists of $(n-1)$ 2s and exactly one 1s. The corresponding coloring is

$$v_1 = 0, v_2 = 2, v_3 = 0, v_4 = 2, \ldots, v_{n-1} = 0 \text{ or } 2, v_n = 1. \text{ Hence the theorem.}$$

Suresh Kumar and Preethi K Pillai [12] proved that, $X_R(K_{1,n}) = 2$.

6) *Theorem. 2.6.* The star graph $K_{1,n}$, $n \geq 1$ is unique minimal Proper Roman Colorable

a) *Proof.* Since $X_R(K_{1,n}) = 2$, the only two ways of coloring the star graph are:

b) *Case-1:* Assign the color 1 to exactly 2 vertices and 0 to all others

c) *Case-2:* Assigning the color 2 to exactly 1 vertex and 0 to all others

In the case-1, the assignment of the color 1 to exactly 2 vertices is possible in the star graph, only if they are end-vertices. But then we cannot color any of the remaining vertrx with color, 0 since any such vertex must be adjacent to a vertex of color, 2. So, the only way of coloring the star graph is to assign the color 2 to the unique vertex of degree, n and color all the others by 0. This is the unique minimal Proper Roman Coloring of the star graph.

Suresh Kumar and Preethi K Pillai [13] proved that $(P_n) = n - 1$, $n \geq 2$.

7) *Theorem.2.7.* Path graphs P_n , $n \geq 2$ are unique minimal Proper Roman Colorable.

a) *Proof.* Since, $X_R(P_n) = n - 1, n \geq 2$, the only way of coloring P_n by $n - 1$ colors is $v_1 = 0, v_2 = 2, v_3 = 0, v_4 = 2, \ldots, v_{n-1} = 2 \text{ or } 0$, according as n is even or odd. Then color the last vertex as $v_n = 0 \text{ or } 1$, according as n is even or odd. Hence the theorem.

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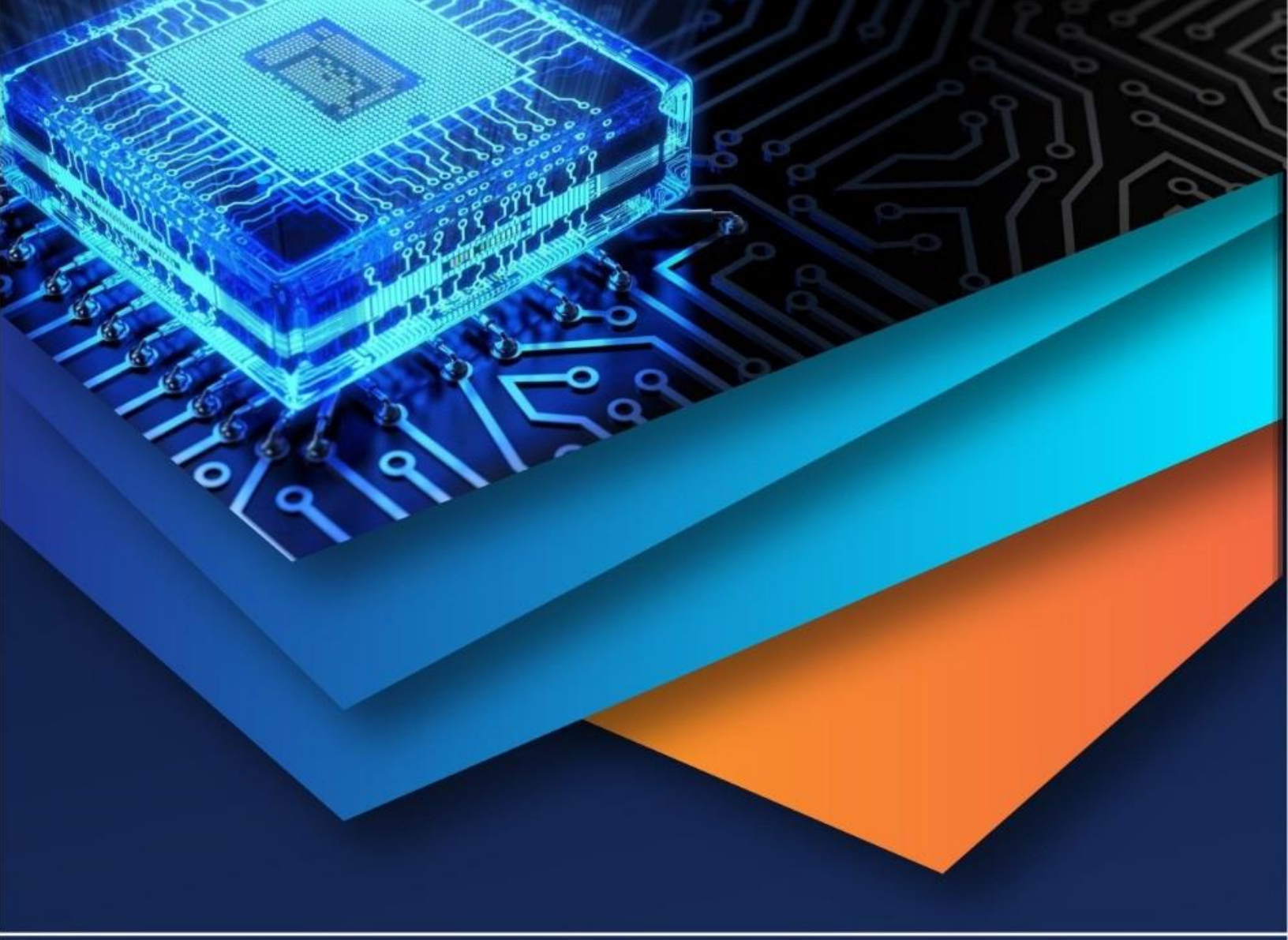

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