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Roman Coloring of Bull Graphs and related Graphs

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Abstract: The Bull graph is a graph with 5 vertices and 5 edges consisting of a triangle with two disjoint pendant edges. In this paper, we obtain the Roman coloring of the Bull graph and related graphs such as Middle graph, Total graph, Splitting graph, Degree splitting graph, Shadow graph and Litact graph of the Bull graph.
Keywords: Graph, Roman coloring, Bull graph

I. INTRODUCTION

Graph coloring take a major stage in Graph Theory since the advent of the famous four color conjecture. Several variations of graph coloring were investigated [5] and still new types of coloring are available such as σ −(Sigma) coloring [4] and Roman coloring [6, 7]. In this Paper, we consider the proper coloring for the Bull graph and related graphs. By a graph, we mean a finite undirected graph without loops and parallel edges.

The Bull graph is a planar undirected graph with five vertices and five edges in the form of a triangle with two disjoint pendant edges. There are three variants of Bull graph, the name being derived from the fact that the structure of the graph is meant to represent the face of a "Bull".

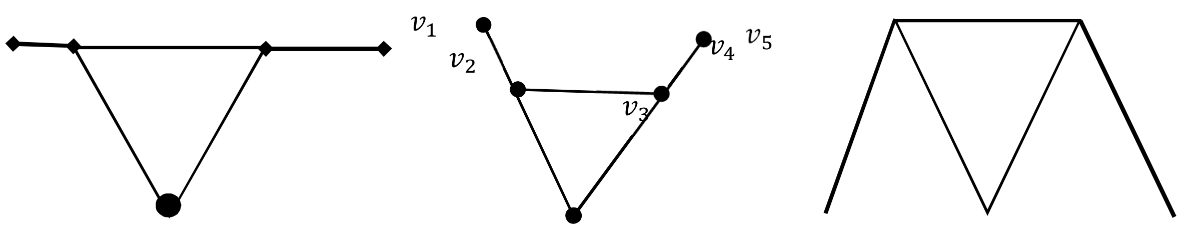

Figure.1. Bull Graph

The Bull graph was introduced by Weisstein [2]. It is a simple graph with chromatic number 3, chromatic index 3, radius 2, diameter 3 and girth 3. The concept of Bull-free graphs was studied, but there are no works related to the coloring of Bull graph and its related graphs. In this Paper, we investigate the coloring of Bull graph and its related graphs. We recall some basic definitions.

1) *Definition 1.1.* Roman coloring of a graph G is an assignment of three colors, namely {0, 1, 2}, to the vertices of G such that every vertex with the color, 0 must be adjacent to a vertex of color 2. The weight of a Roman coloring is defined as the sum of all vertex colors. The Roman Chromatic number of a graph G is defined as the minimum weight of a Roman coloring on G and denoted by $R(G)$.
2) *Definition. 1.2.* The Middle graph M(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of M(G) are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge of G incident to it.

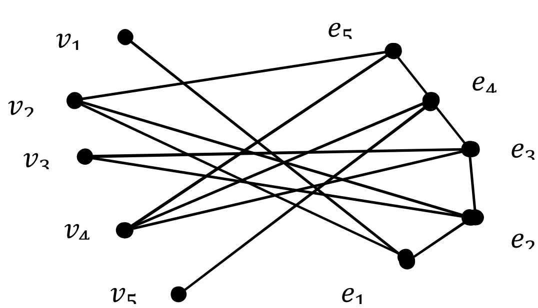

Figure.2. Middle graph of the Bull Graph

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3) *Definition. 1.3.* The Total graph T(G) of a graph G is the graph whose vertex set is V(G)∪E(G) and in which two vertices of T(G) are adjacent whenever they are adjacent or incident in G..

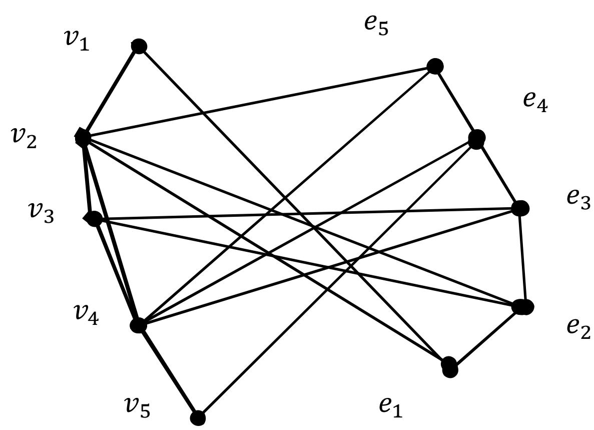

Figure. 3. Total graph of the Bull graph

4) *Definition.1.4.* The Splitting graph S(G) of a graph G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v)=N(v′) where N(v) and N(v′) are the neighbourhood set of v and v′ respectively.

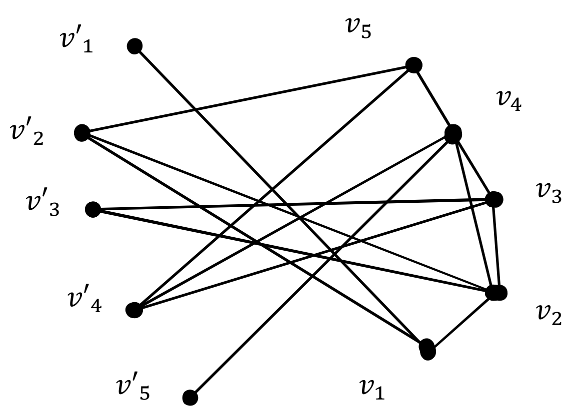

Figure.4. Splitting graph of the Bull graph

5) *Definition 1.5.* Let G = (V, E) be a graph with V= $S_1 \cup S_2 \cup \ldots\ldots \cup S_t \cup$ T, where each S_i is a set of vertices having at least two vertices and having the same degree and T=V-∪ S_i. The Degree Splitting graph DS(G) of a graph G is obtained from G by adding vertices $w_1, w_2, \ldots\ldots w_t$ and joining w_i, to each vertex of S_i (1≤ i≤ t)

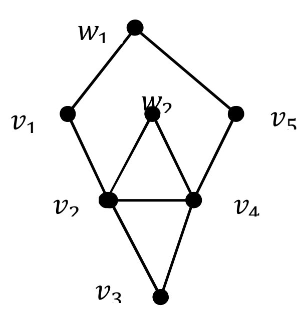

Figure.5. Degree Splitting graph of Bull graph

6) *Definition.1.6.* The Shadow graph, D_2(G) of a connected graph G is constructed by taking two copies of G, say G' and G" and joining each vertex u′ in G' to all the adjacent vertices of the corresponding vertex, u" in G".

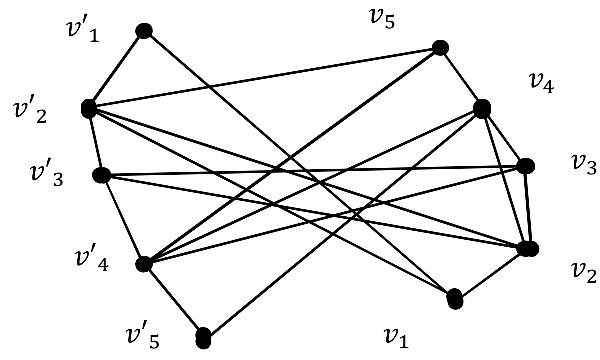

Figure.6. Shadow graph of Bull graph

7) *Definition.1.7.* Let c(G) denotes the set of all cut vertices of Bull graph G. The Litact graph m(G) of a graph G is the graph whose vertex set is V(G)∪c(G) in which two vertices are adjacent if they are adjacent edges of G or adjacent cut-vertices of G or one is an edge e_i of G and the other is a cut-vertex c_j of G and e_i is incident with c_j.

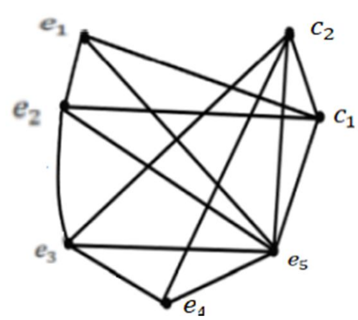

Figure.7. Litact graph of Bull graph

II. MAIN REESULTS

In this section, we discuss the Roman Coloring of Bull Graphs and related Graphs.

1) *Theorem. 2.1.* Roman Chromatic number of the Bull graph is 4.

a) *Proof.* Assign the color 0 to the two vertices of degree 3 each and the color 2 to the unique vertex of degree, 2 and assign the color 1 to the remaining 2 pendant vertices. Then it is a roman coloring with weight, 4. Clearly, it is minimal so that the Roman chromatic number of the Bull graph is 4.

2) *Theorem. 2.2.* Roman chromatic number of Middle graph of the Bull graph is 5.

a) *Proof.* In the Middle graph of Bull graph, let the vertices be v_i for i = 1 to 5 and e_j for j= 1 to 5. Assign the color 2 to the two vertices $e_2$and e_4 and assign the color 0 to all the remaining tvertices except v_1. We can assign the color 1 to v_1. Then it is a roman coloring with weight, 5. Clearly, it is minimal so that the Roman chromatic number of the Middle graph of the Bull graph is 5.

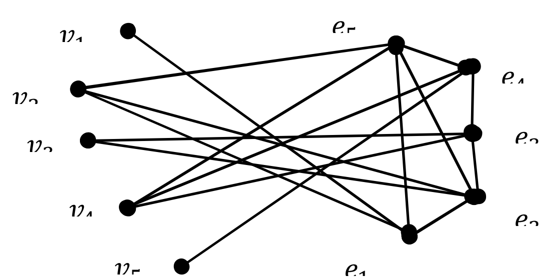

Figure.8

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3) *Theorem. 2.3.* The Roman chromatic number of the Total graph of the Bull graph is 4.
a) *Proof.* In the Total graph of Bull graph, let the vertices be v_i for i = 1 to 5 and e_j for j= 1 to 5. Assign the color 2 to the two vertices $e_1$and v_4 and assign the color 0 to all the remaining tvertices. Then it is a roman coloring with weight, 4. Clearly, it is minimal so that the Roman chromatic number of the Total graph of the Bull graph is 4.

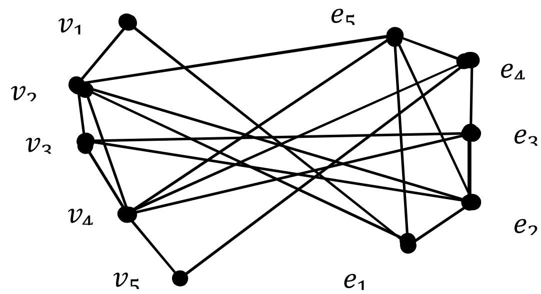

Figure.9

4) *Theorem. 2.4.* The Roman chromatic number of Splitting graph of the Bull graph is 5.
a) *Proof.* Let the vertices of the Splitting graph of Bull graph be v_i for i = 1 to 5 and $v'_j$for j= 1 to 5. Assign the color 2 to the vertex v_4, color 1 to the vertices v_1, v'_1 v'_4 and assign the color 0 to all the remaining vertices. Then it is a roman coloring with weight, 5. Clearly, it is minimal so that the Roman chromatic number of Splitting graph of the Bull graph is 5.

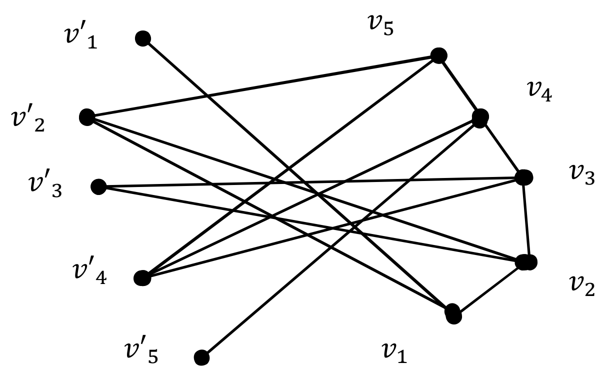

Figure.10

5) *Theorem. 2.5.* The Roman chromatic number of Degree Splitting graph of the Bull graph is 4.
a) *Proof.* Let the vertices of the Degree Splitting graph of Bull graph be v_i for I = 1 to 5 and $w_j$for j= 1 to 2. Assign the color 2 to the vertex v_4, color 1 to the vertices v_1, w_1 and assign the color 0 to all the remaining vertices. Then it is a roman coloring with weight, 4. Clearly, it is minimal so that the Roman chromatic number of Degree Splitting graph of the Bull graph is 4.

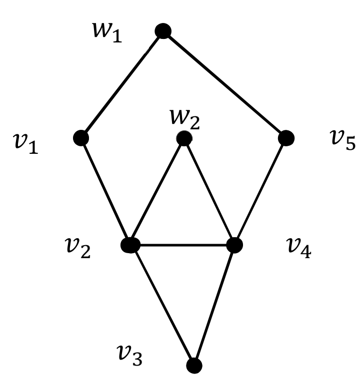

Figure.11

6) *Theorem. 2.6.* The Roman chromatic number of shadow graph of the Bull graph is 5.

a) *Proof.* Let the vertices of the Shadow graph of Bull graph be v_i for i = 1 to 5 and v'_j for j= 1 to 5. Assign the color 2 to the vertex, v'_2, color 1 to the vertices v_2, v_5, v'_5 and assign the color 0 to all the remaining tvertices. Then it is a roman coloring with weight 5. Clearly, it is minimal so that the Roman chromatic number of shadow graph of the Bull graph is 5.

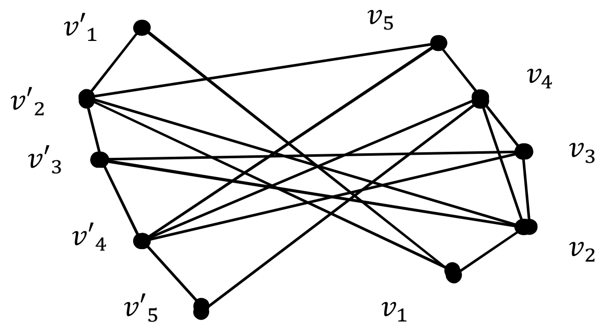

Figure.12

7) *Theorem. 2.7.* The Roman chromatic number of Litact graph of the Bull graph is 2.

a) *Proof.* Let the vertices of the Litact graph of Bull graph be e_i for i = 1 to 5 and c_j for j= 1 to 2. Assign the color 2 to the vertex, e_5 and assign the color 0 to all the remaining vertices. Then it is a roman coloring with weight 2. Clearly, it is minimal so that the Roman chromatic number of Litact graph of the Bull graph is 2.

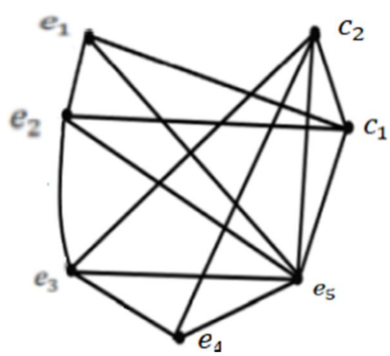

Figure.13

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