ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

Effect of Angle of Incidence on Stiffness And Damping Derivatives for Oscillating Hypersonic Non-Planar Wedge

Asha Crasta[1] and S. A. Khan [2]

1. Research Scholar, Department of Mathematics, Jain University, Bangalore, Karnataka, India,

2. Principal, Department of Mechanical Engineering, Bearys Institute of technology, Mangalore, Karnataka,India,

Abstract: ***A similitude has been obtained for a pitching oscillating Non planar wedge with attached bow shock at high angle of attack in hypersonic flow. A strip theory in which flow at a span wise location is two dimensional and independent of each other is being used. This combines with the similitude to lead to a one-dimensional piston theory. Closed form of simple relations is obtained for stiffness and damping derivatives in pitch. The present theory is valid only when the shock wave is attached with the nose of the wedge. With the increase in semi vertex angle of the wedge the stiffness as well as the damping derivatives assumes high value, and the center of pressure of the wedge also shift towards the trailing edge. For high Mach numbers the stiffness and damping derivatives become independent of the Mach numbers. From the theory developed some of the results are obtained for wide range of Angle of incidence with remarkable computational ease.***

Keywords: Hypersonic Flow, Non Planar wedge, Piston Theory, pitch, angle of incidence

1. INTRODUCTION

High incidence hypersonic similitude of Sychev's [1] is applicable to a wing provided it has an extremely small span in addition to small thickness. The unsteady infinite span case has been analyzed, but mostly for small flow deflections. The piston theory of Light hill [2] neglects the effects of secondary wave reflection. Appleton [3] and McIntosh [4] have included these effects. Hui's [5] theory is valid for wedges of arbitrary thickness oscillating with small amplitude provided the bow shock remains attached. Erricsson's [6] theory covers viscous and elastic effects for airfoils with large flow deflection. Orlik-Ruckemann [7] has included viscous effect and Mandl [8] has addressed small surface curvature effect for oscillating thin wedges. Ghosh's [9] similitude and piston theory for the infinite span case with large flow deflection is valid for airfoils with planar or non-planar surfaces whereas Hui's theory [10] is for plane wedges. Ghosh's piston theory has been applied to non-planar cases, both steady and unsteady. The effect of viscosity and secondary wave reflection has not been included. Crasta and Khan have studied the hypersonic and supersonic similitude for planar wedge ([17], [12]), for Delta wing ([11], [13]) and for Delta wing with curved leading edges ([14], [16]). Crasta and Khan have further extended the similitude to study the stability derivatives for Newtonian limit for planar wedge[19], delta wing [18] and delta wing with curved leading edges [15]. In the present work the similitude of Hypersonic planar wedge has been extended for Hypersonic flows past a non-planar wedge and the effect of Mach number and angle of incidence on stability derivatives has been studied.

II PISTON THEORY

A thin strip of the wing, parallel to the centerline, can be considered independent of the z dimension when the velocity component along the z direction is small. This has been discussed by Ghosh's [9]. The strip theory combined with Ghosh's large incidence similitude leads to the "piston analogy" and pressure P on the surface can be directly related to equivalent piston mach no. M_p. In this case both Mp and flow deflections are permitted to be large. Hence light hill piston theory [2] or miles strong shock piston theory cannot be used but Ghosh's piston theory will be applicable.

$$\frac{P}{P_\infty} = 1 + AM_P{}^2 + AM_P(B + M_P{}^2)^{\frac{1}{2}} \qquad (1)$$

, Where P_∞ is free stream pressure

ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The angle of incidence is same as that of wing. Angle ϕ is the angle between the shock and the strip.

$A=\frac{(\gamma+1)}{4}, B=(4/(\gamma+1)^2, \gamma$ is the specific heat ratio and M_p = the local piston Mach number normal to the wedge surface.

Pitching moment derivatives

Let the mean incidence be α_0 for the wing oscillating in pitch with small frequency and amplitude about an axis X_0. The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length 2L at x, the pressure on the lee surface is assumed zero. Therefore, the nose up moment is

$$m=-\int_0^L p.(x-x_0)dx \tag{2}$$

A non planar wedge is obtained by superimposing parabolic arcs on the two sides of the plane wedge. Therefore The Stiffness and damping derivatives are respectively

$$-C_{m_\alpha}=\frac{2}{\frac{1}{2}\rho_\infty U_\infty^2 L^2}\left(-\frac{\partial m}{\partial \alpha}\right)_{\alpha=\alpha_0, q=0} \tag{3}$$

$$-C_{m_q}=\frac{2}{\frac{1}{2}\rho_\infty U_\infty^2 L^3}\left(-\frac{\partial m}{\partial q}\right)_{\alpha=\alpha_0, q=0} \tag{4}$$

Where ρ_∞, a_∞ are density and velocity of sound in the free stream Combining (1) through (4), differentiation under the integral sign is performed. Therefore

$$-C_{m_\alpha}=\frac{p_\infty . A}{\frac{1}{2}\rho_\infty U_\infty^2 L^2 Cos^2\alpha_0}\int_0^L (x-x_0).\frac{\partial p}{\partial \alpha}.dx$$

And

$$-C_{m_q}=\frac{p_\infty . A}{\frac{1}{2}\rho_\infty U_\infty L^3 Cos^3\alpha_0}\int_0^L (x-x_0).\frac{\partial p}{\partial q}.dx$$

On Simplifying we get,

The Stiffness and damping derivative are given by

$$-C_{m_\alpha}=\frac{\gamma+1}{2M_\infty^2\cos^2\alpha_0}(I_1+I_2+I_3) \tag{5}$$

$$-C_{m_q}=\frac{\gamma+1}{2M_\infty\cos^3\alpha_0}(J_1+J_2+J_3) \tag{6}$$

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

Where

$$I_1 = M_\infty{}^2\left[H\sin 2\alpha_0 - \frac{2\lambda\cos 2\alpha_0}{3}\right]$$

$$I_2 = \left[\left\{-M_\infty\frac{(z)^{\frac{3}{2}}}{3b}\right\}\left\{H\cos\alpha_0 - (H\lambda\sin\alpha_0 + \cos\alpha_0)\left(\frac{5a-3z}{5b}\right) + \lambda\sin\alpha_0\left(\frac{15z^2-42az+35a^2}{3b^2}\right)\right\}\right]_{z=a+b}^{z=a-b}$$

$$I_3 = \left[\left\{\frac{M_\infty z^{\frac{1}{2}}}{2\lambda\cos\alpha_0}\right\}\left\{-H\sin\alpha_0\cos\alpha_0 + \left(\begin{array}{l}(\sin\alpha_0\cos\alpha_0 - \lambda H\left(\cos^2\alpha_0 + \cos 2\alpha_0\right))\left(\frac{z-3a}{3b}\right)\\ +\lambda(\cos^2\alpha_0 + \cos 2\alpha_0)\left(\frac{3z^2-10az+15a^2}{15b^2}\right)\end{array}\right)\right\}\right]_{z=a+b}^{z=a-b}$$

$$J_1 = M_\infty\left[H^2\sin\alpha_0 + \left(\frac{\sin\alpha_0 - 2H\lambda\cos\alpha_0}{3}\right)\right]$$

$$J_2 = \frac{-2z^{\frac{3}{2}}}{3b}\left[\frac{H^2}{4} + H\left(\frac{5a-3z}{10b}\right) + \frac{15z^2-42az+35a^2}{140b^2}\right]_{z=a+b}^{z=a-b}$$

ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

$$J_3 = \left[\left\{ \frac{-z^{\frac{1}{2}}}{\lambda \cos\alpha_0} \right\} \left\{ \begin{array}{l} \left(\frac{H^2 \sin\alpha_0}{4} \right) - H\left(\sin\alpha_0 - \lambda H \cos\alpha_0\right)\left(\frac{z-3a}{6b}\right) \\ +\left(\sin\alpha_0 - 4\lambda H \cos\alpha_0\right)\left(\frac{3z^2 - 10az + 15a^2}{60b^2}\right) \\ +\lambda \cos\alpha_0 \left(\frac{5z^3 - 21z^2 a + 35za^2 - 35a^3}{70b^3}\right) \end{array} \right\} \right]_{z=a+b}^{z=a-b}$$

Where

$$H = 1 - 2h\cos^2\alpha_0$$

$$a = \left(\frac{4}{\gamma + 1}\right)^2 + M^2_\infty \sin^2\alpha_0 ,$$

$$b = \lambda M^2_\infty \sin 2\alpha_0$$

Some of the results have been obtained for various Mach Numbers and studied

RESULTS AND DISCUSSIONS:

In Fig.1 the variation of stiffness derivatives with respect to the pivot position is presented for α = 10, 15, 20, and 25 degrees. The location of center of pressure is at 35 %, 45 %, 50 %, and 55 % respectively from the nose and values are on the higher side as compared to the values for planar wedge for the same Mach number. The reasons for this increase may be that in case of non-planar wedges there is a shift of additional area towards the nose and at the same time the area at the trailing edge is decreased.

Fig. 2 shows the variation of damping derivatives with respect to the pivot position is presented for α= 10, 25, 20 and 25 degrees. The location of center of pressure is remains in the range from 20% to 30% from the nose. From the figure it is seen that the increase in the damping derivatives is very high for the pivot position for h = 0 to 0.4, this means up to the location of center of pressure trends in the pressure distribution are totally different when we compare them with that at the trailing edge. The convex shape of the non-planar wedge as well as the pressure distribution there on may be responsible for this trend in the damping derivatives.

Similar results are for stiffness and damping derivatives are seen in Figs. 3 and 4.

Results for Mach number 9 are shown in Figs. 5 and 6. The variation of the damping derivative is on the similar lines as that of at lower Mach numbers namely 7 and 8. However, there is drastic change in the behavior of the damping derivative for all the Mach numbers at lower values of semi vertex angle. The trend tends to become linear for the distance from the nose till the location of center of pressure. But for the rest of the distance of pivot position, the trend remains as expected and the increase in the magnitude of the damping derivative is uniform. For the higher values of the semi vertex angle namely 15, 20, and 25 the behavior is on the expected lines.

Results for Mach number 10 are presented in Figs. 7 and 8. As far as stiffness derivative is concerned, the behavior is similar to that of at lower Mach numbers. In case of damping derivative, the increase in the magnitude is more for the forward pivot position and for the rear pivot

ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

position it remains the same as that for lower Mach numbers as discussed earlier.
Results for Mach number 13, 15, 18 and 20 are presented in Figs. 9, 10, 11, 12, 13, 14, 15 and 16. Since this Mach number is very high and the effect of the temperature will be predominant compared to that of at the lower Mach numbers, the shock wave will be very close to the body even though it will never collapse with the surface of the non-planar wedge as the Mach number independence principle will come in to play. At a very high Mach number there will be interaction between the boundary layer, entropy layer and the shock wave. The above discussed explanation may be the reasons for the trends and behavior of the stability derivatives.

CONCLUSION:

Present theory demonstrates its application for a wide range of the Mach number, angle of attack and the semi vertex angle of the wedge. For semi vertex angle five to ten degrees the variation in the stiffness and damping derivatives is substantial, however; for large values of semi vertex angle the variation stiffness and damping derivative is only marginal. The variation in the center of pressure for stiffness and damping derivatives is from 20 % to 55 % for all the Mach numbers and semi vertex Angle of the present study. The theory is valid only when the shock wave is attached with the nose of the wedge. In the present study the effect of Lee surface has been neglected as it is well known that the pressure on the lee surface will be negligible for hypersonic Mach numbers. The present theory could be handy at the initial design stage of the Aerospace Vehicles. Effects of viscosity & wave reflection are also neglected in the present study. The present theory is simple and yet gives good results with remarkable computational ease with the error around ten percent.

Results and Discussions:

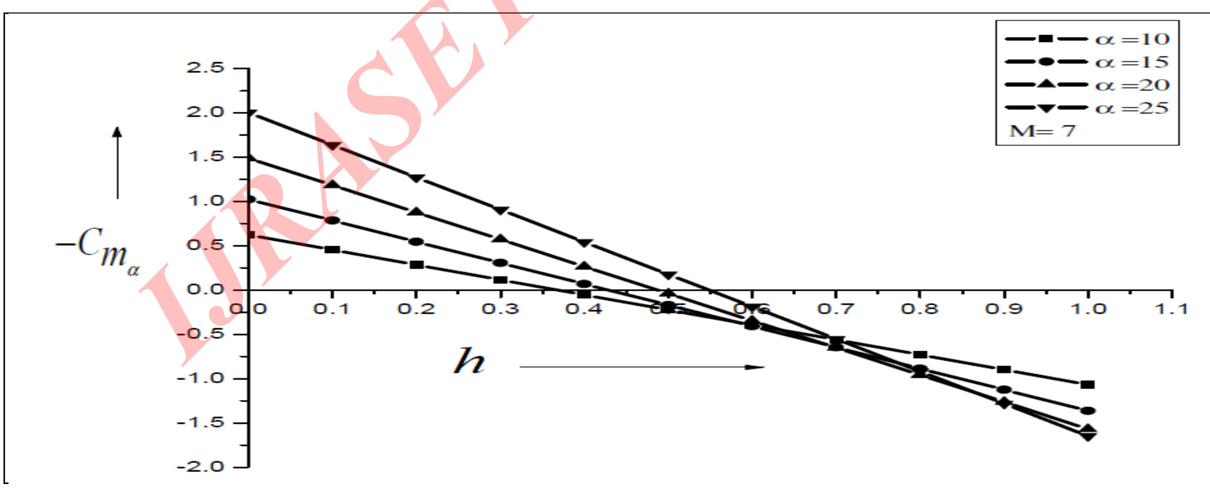

Fig.1:

Variation of Stiffness derivative with pivot position for M = 7

ISSN: 2321-9653

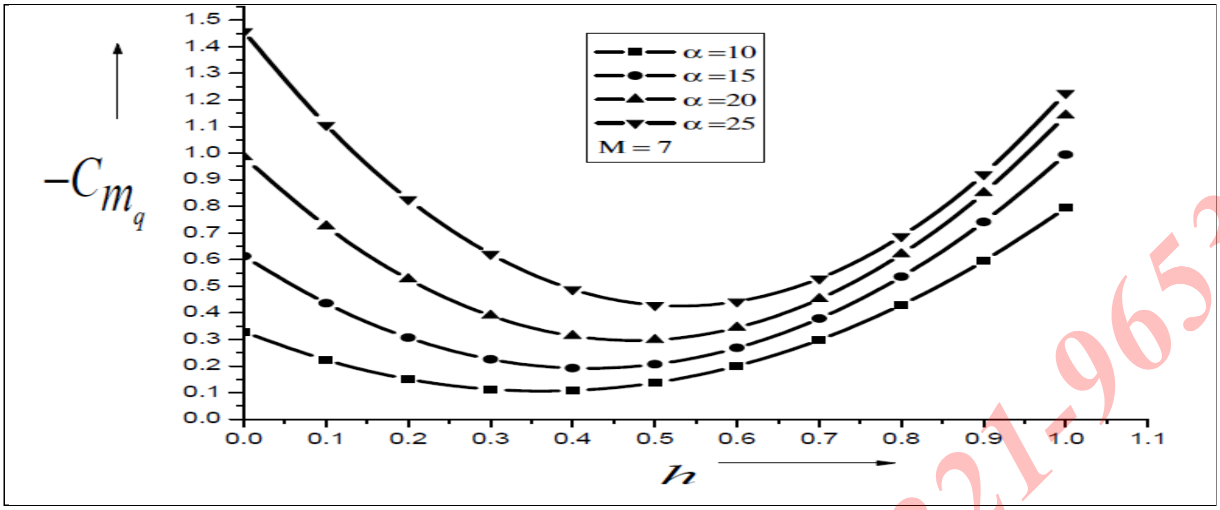

Fig. 2: Variation of damping derivative with pivot position M = 7

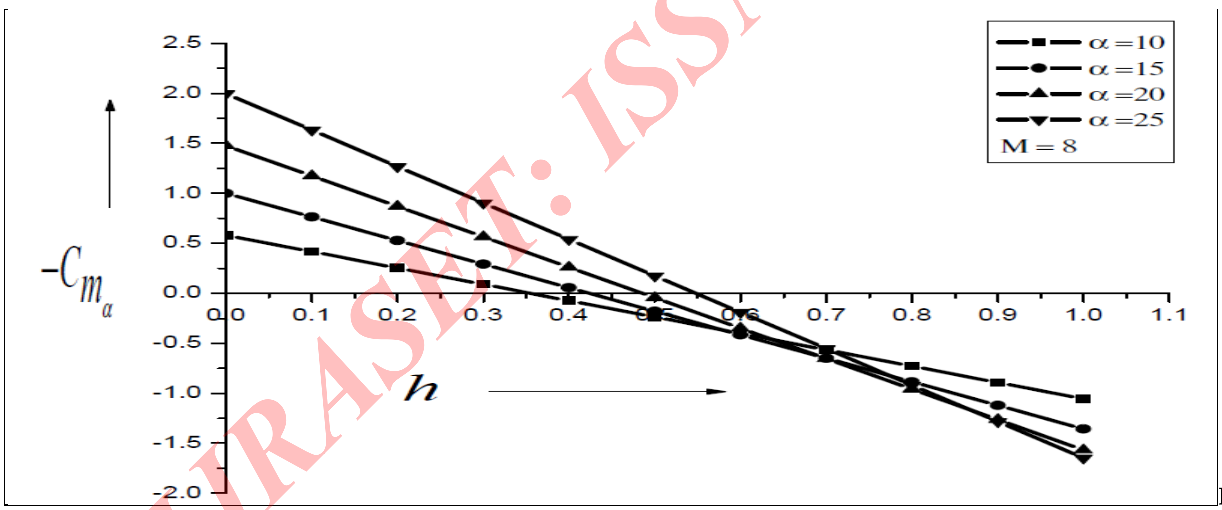

Fig. 3: Variation of Stiffness derivative with pivot position M=8

ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

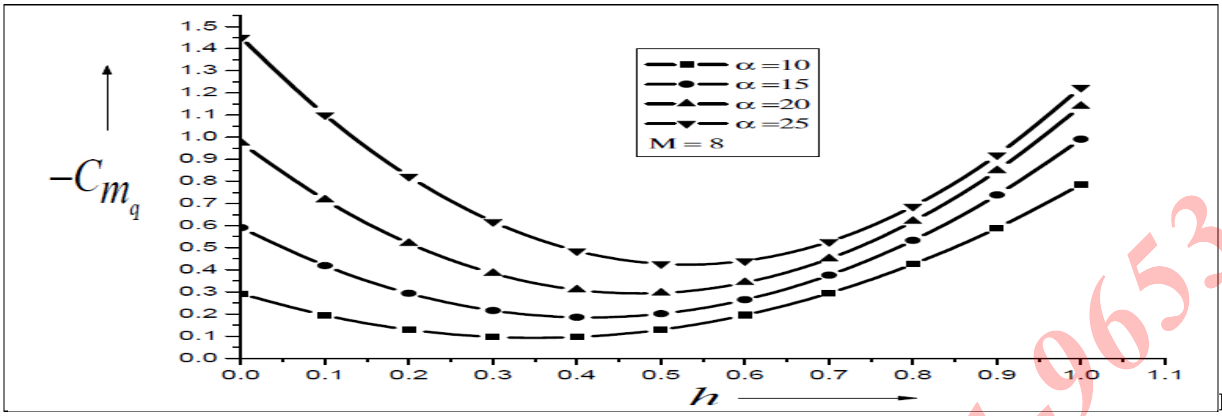

Fig. 4:

Variation of Damping derivative with pivot position M = 8

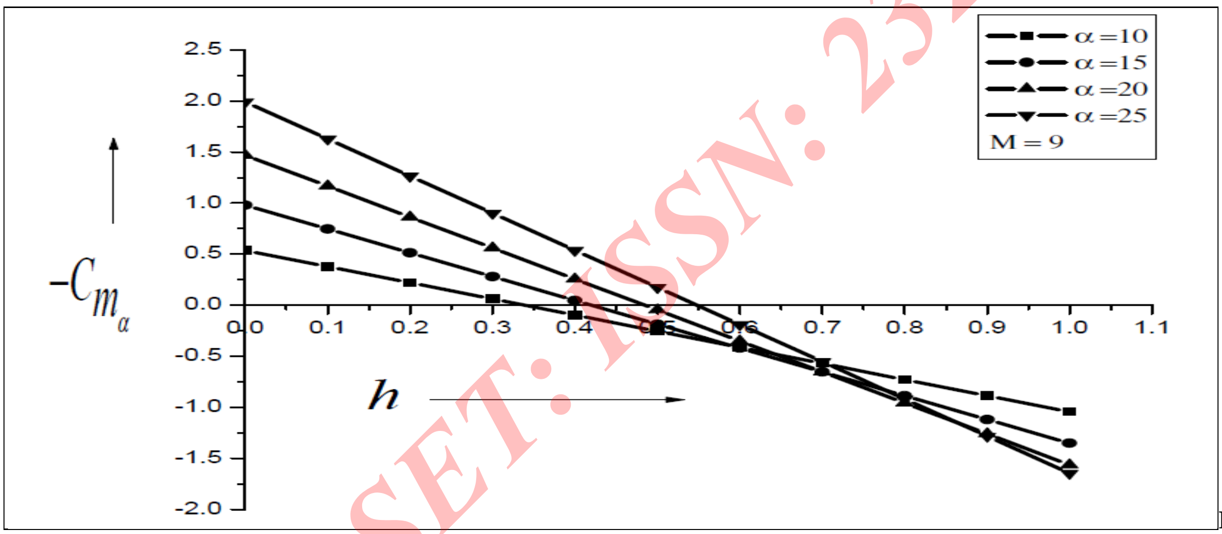

Fig. 5:

variation of stiffness derivative with pivot position for M = 9

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

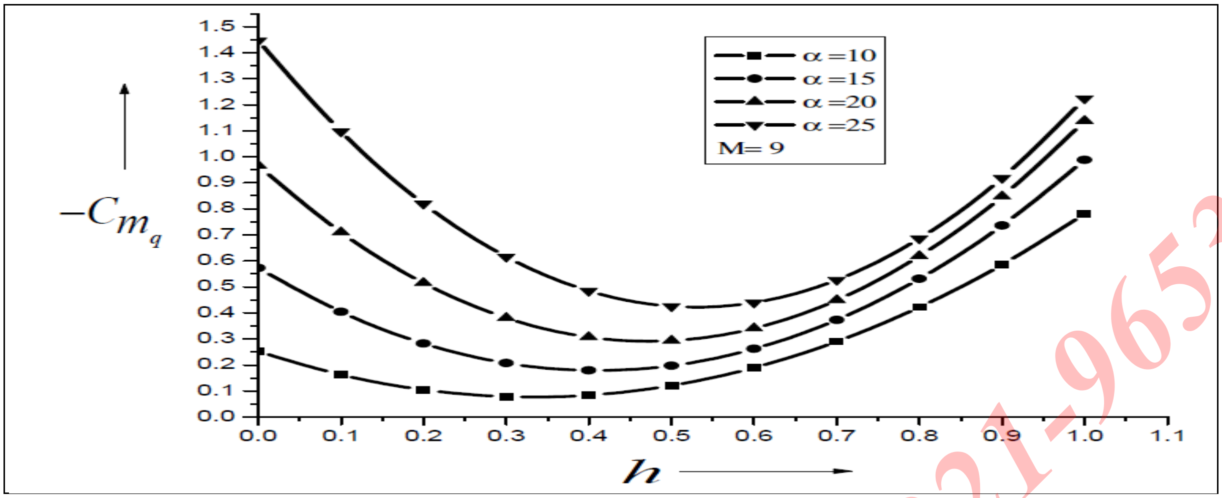

Fig. 6: variation of Damping derivative with pivot position for M = 9

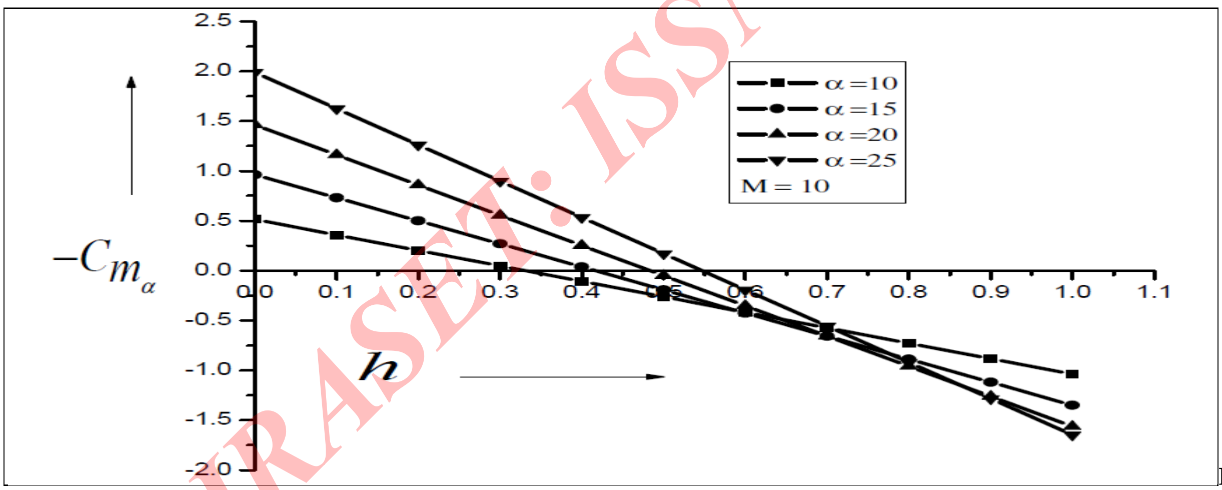

Fig. 7: variation of stiffness derivative with pivot position for M = 10

Vol. 2 Issue IV, April 2014
ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

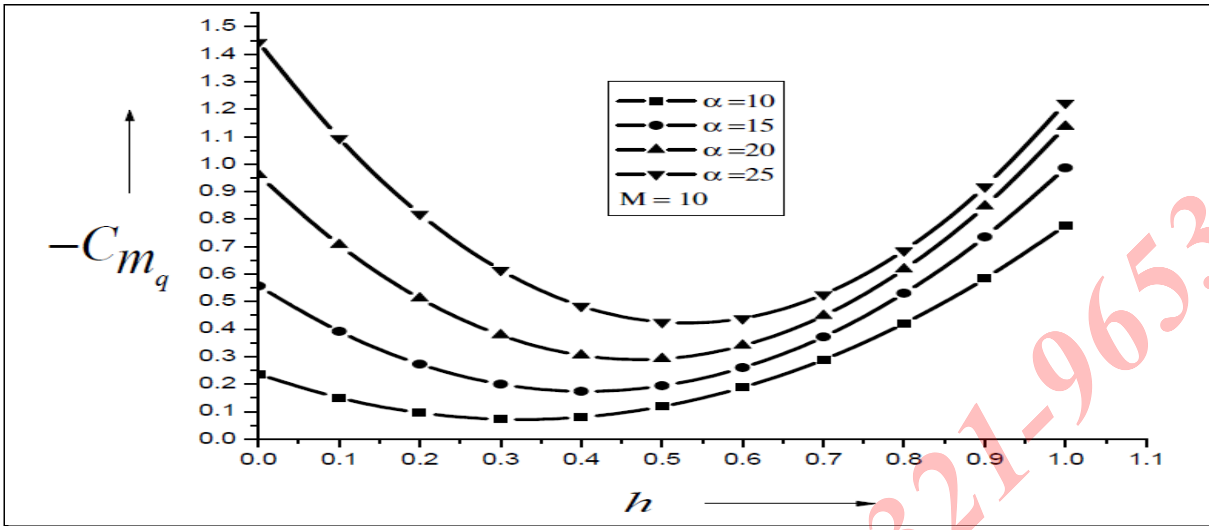

Fig. 8:

Variation of damping derivative with pivot position for M = 10

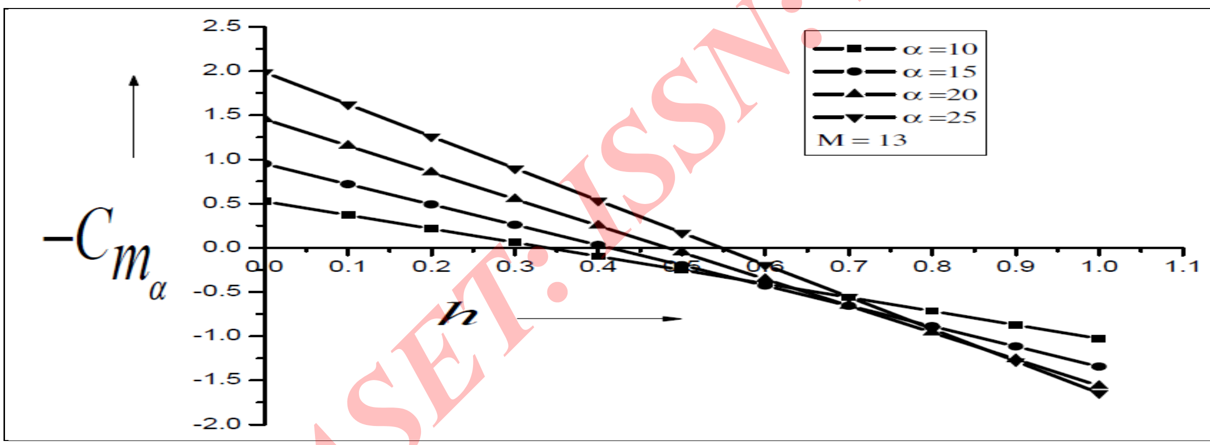

Fig 9:

variation of Stiffness derivative with pivot position for M = 13

ISSN: 2321-9653

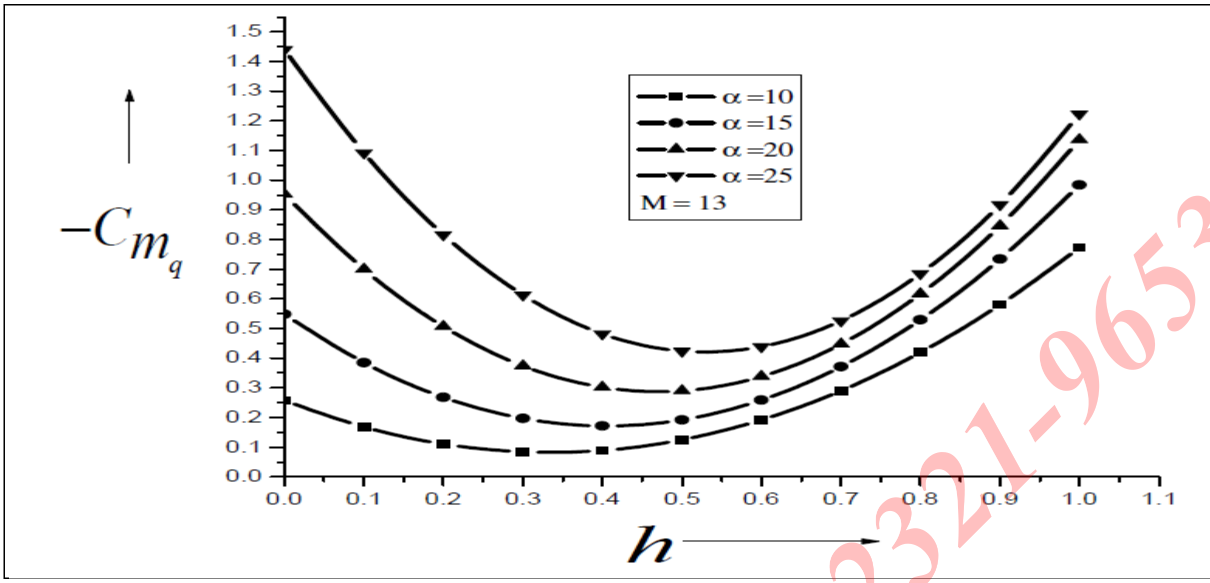

Fig. 10: variation of damping derivative with pivot position for M = 13

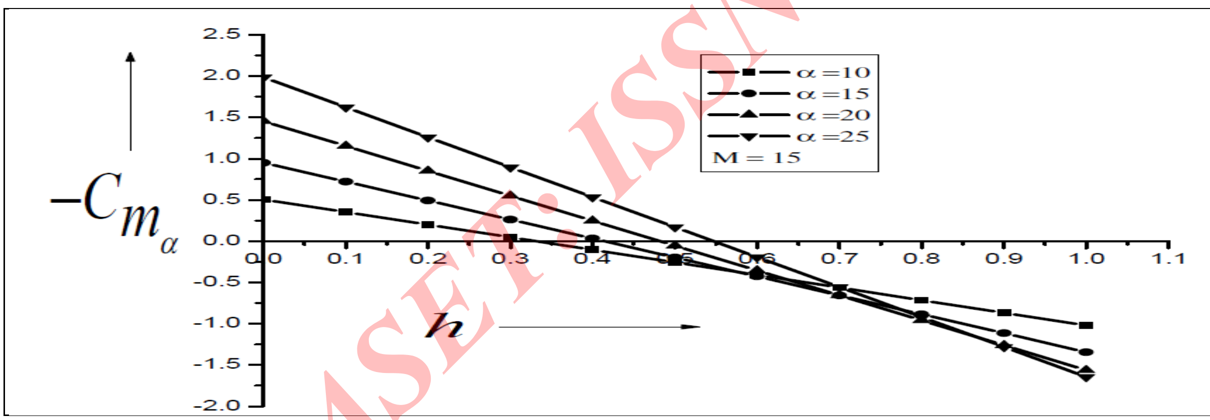

Fig. 11: variation of stiffness derivative with pivot position for M = 15

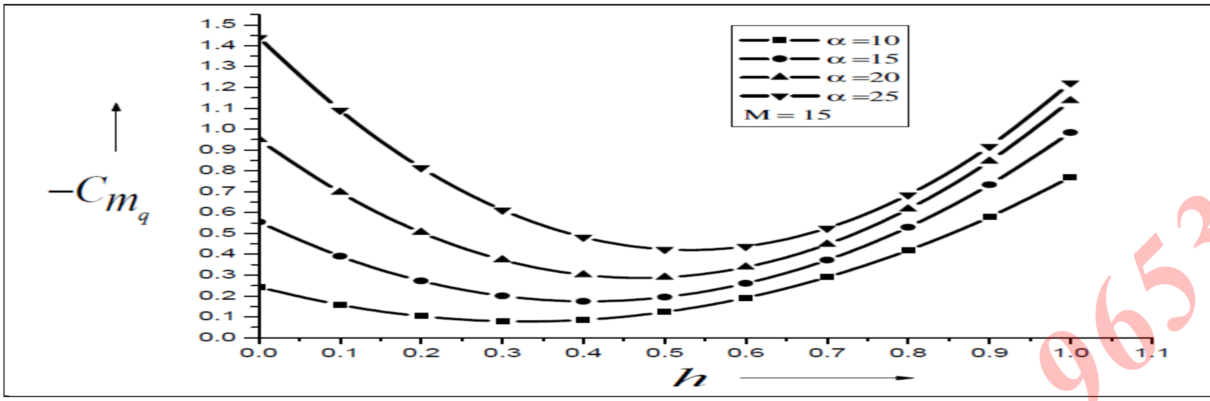

Fig. 12: variation of damping derivative with pivot position for M = 15

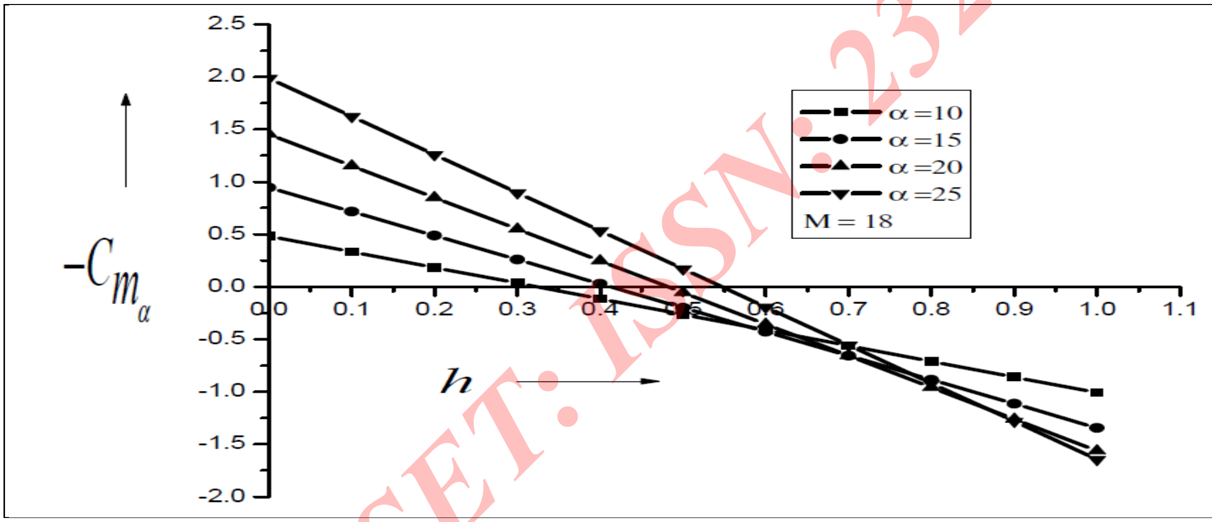

Fig. 13: variation of Stiffness derivative with pivot position for M=18

ISSN: 2321-9653

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

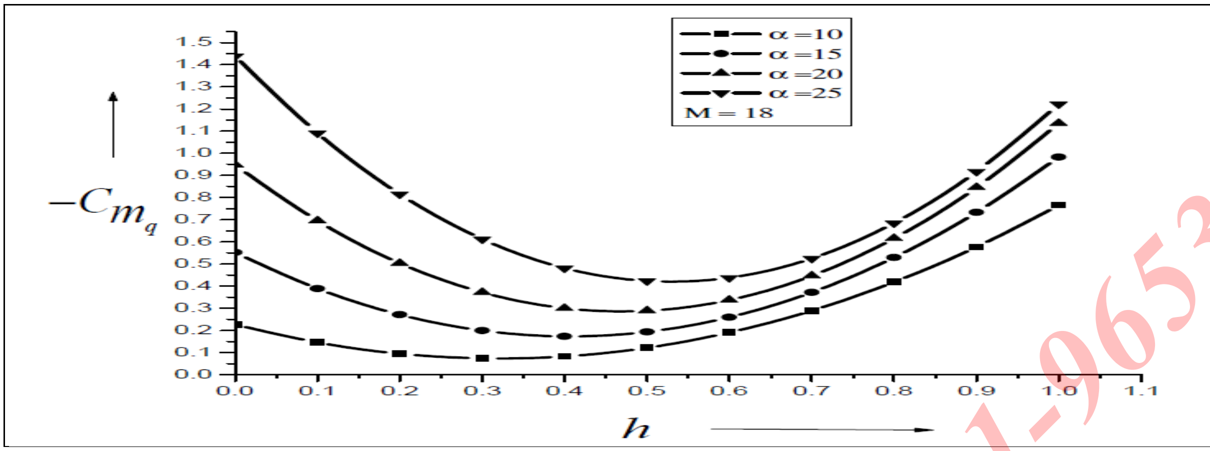

Fig. 14: variation of Damping derivative with pivot position for M =18

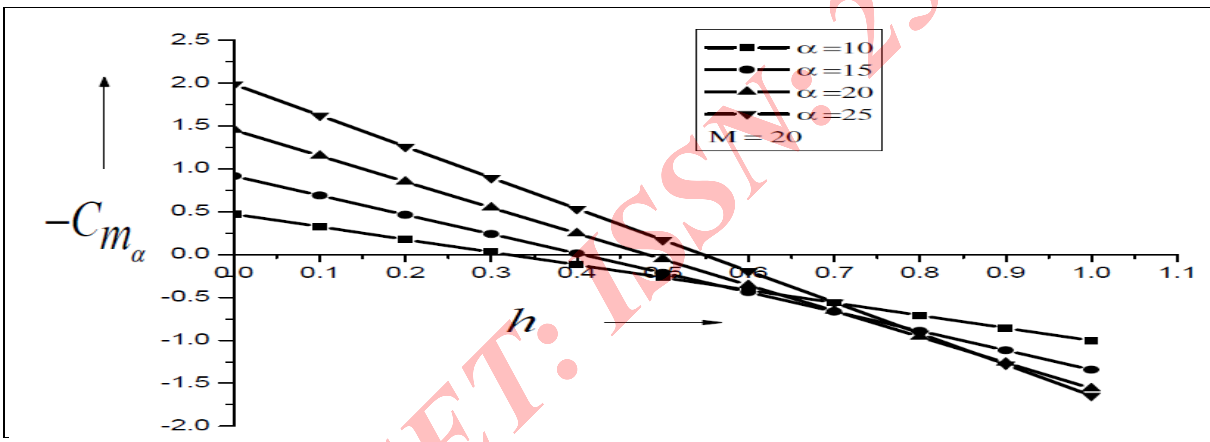

Fig. 15: variation of Stiffness derivative with pivot position for M = 20

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

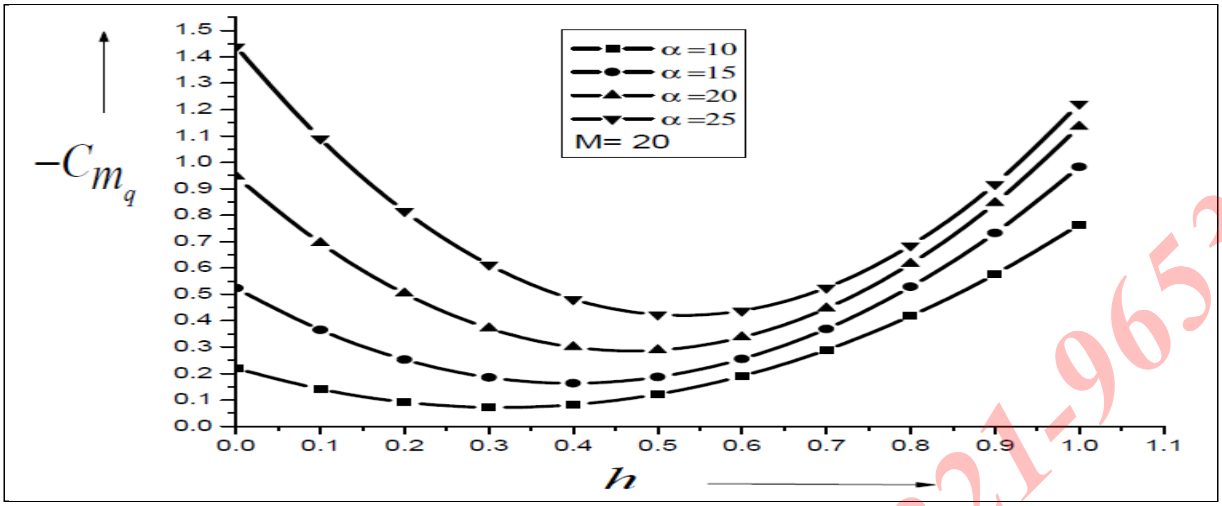

Fig. 16: Variation of Damping Derivative with pivot position for M = 20

REFERENCES:

[1] Sychev, V. V, Three Dimensional Hypersonic Gas Flow Past Slender Bodies at High Angles of Attack, Journal of Applied Mathematics and Mechanics, Vol. 24, August 1960, pp. 296- 306.

[2] Light Hill, M. J., Oscillating Aerofoil at High Mach Numbers, Journal of Aeronautical Sciences, Vol. 20, June 1953, pp. 402-406.

[3] Appleton, J. P. , Aerodynamic Pitching Derivatives of a wedge in Hypersonic Flow, AIAA Journal, Vol. 2, November 1964, pp. 2034-2036.

[4] Mc Inthosh, S. C., Jr., Studies in Unsteady Hypersonic Flow Theory, Ph.D. Dissertation Stanford Univ., California, Aug. 1965.

[5] Hui, W.H., Stability of Oscillating Wedges and Caret Wings in Hypersonic and Supersonic Flows, AIAA Journal, Vol. 7, Aug. 1969, pp. 1524-1530.

[6] Ericsson, L. E., Viscous and Elastic Perturbation Effects on Hypersonic Unsteady Airfoil Aerodynamics, AIAA Journal , Vol. 15 Oct. 1977, pp. 1481-1490.

[7] Orlik-Ruckemann, K. J., Stability Derivatives of Sharp Wedges in Viscous Hypersonic Flow, AIAA Journal, Vol. 4June 1966, pp. 1001-1007.

[8] Mandl, P., Effect of Small Surface Curvature on Unsteady Hypersonic Flow over an Oscillating Thin Wedge, C. A. S. I. Transactions, Vol. 4, No. 1, March 1971, pp. 47-57.

[9] Ghosh, Kunal, Hypersonic large deflection similitude for oscillating delta wings, Aeronautical Journal, Oct. 1984, pp. 357-361.

[10]Lui, D. D. and Hui W. H., Oscillating delta wings with attached shock waves, AIAA Journal, June 1977, 15, 6, pp. 804-812.

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

[11] Asha Crasta and Khan S. A., Oscillating Supersonic delta wing with Straight Leading Edges, International Journal of Computational Engineering Research, Vol. 2, Issue 5, September 2012, pp. 1226-1233, ISSN:2250-3005.

[12] Asha Crasta and Khan S. A., High Incidence Supersonic similitude for Planar wedge, International Journal of Engineering research and Applications, Vol. 2, Issue 5, September-October 2012, pp. 468-471, ISSN: 2248-9622.

[13] Asha Crasta, M. Baig, S. A. Khan, Estimation of Stability derivatives of a Delta wing in Hypersonic flow, International Journal of Emerging trends in Engineering and Developments, Vol.6, Issue 2, September 2012, pp. 505-516, ISSN:2249-6149.

[14] Asha Crasta, S. A. Khan, Estimation of stability derivatives of Oscillating Hypersonic delta wings with curved leading edges, International Journal of Mechanical Engineering & Technology, Volume 3, Issue 3, Dec 2012, pp. 483-492, ISSN 0976 – 6340 (Print).

[15] Asha Crasta and S. A. Khan, Stability Derivatives in the Newtonian Limit, The International Journal of Advanced Research in Engineering and Technology, Volume 4, Issue 7, Nov-Dec 2013, pp. 276-289, ISSN 0976 - 6480 (Print),

[16] S. A. Khan, Asha crasta, Oscillating supersonic delta wings with curved leading edges, International journal of Advanced studies in Contemporary Mathematics. Volume 20, No. 3, July 2010, pp. 359-372.

[17] Asha Crasta and S. A. Khan, Hypersonic Similitude for Planar Wedges, The International Journal of Advanced Research in Engineering and Technology, Volume 5, Issue 2, February 2014, pp. 16-31. ISSN: 0976 - 6480 (Print).

[18] Asha Crasta and Khan S. A., Stability derivatives of a delta wing with Straight leading edge in the Newtonian limit, International Journal of Engineering research and Applications, Volume 4, Issue 3, March 2014, pp. 32-38, ISSN: 2248-9622.

[19] Asha crasta and Khan S. A., Estimation of Stability Derivatives for a Planar Wedge in the Newtonian Limit, IOSR Journal of Mathematics (IOSR-JM), Volume 10, Issue 2, (March-April 2014), pp. 01- 06.