

Comprehensive Review on Band Structure, Density of States and Wave Propagation inside One-Dimensional Photonic Crystal

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Abstract— In this paper, a comprehensive review work is carried out on the computation of band structure and density of states of one-dimension photonic crystal, and propagation of normal and polarized electromagnetic wave inside the structure. Different numerical approaches are made by eminent researchers to compute the first Brillouin zone for understanding the band diagram, and corresponding density of states. Plane wave expansion method is explained in detail, as it yields the most accurate result comparing all the numerical techniques. A brief mathematical model is also presented to study the wave propagation, and a few applications are mentioned where the essential characteristics can be obtained from the calculation of these fundamental properties. This paper aims to capture the important and significant developments which took place in the field of photonics for the last two decades and present a n overall technical review on that matter.

Keywords— Photonic crystal, Band structure, Density of states, Wave propagation, Photonic bandgap, Optical nanostructure

I. INTRODUCTION

Photonic crystals can generally be regarded as an optical media with spatially periodic properties [1]. However, this definition is too general and too vague to be used in almost all possible contexts and there have been a considerable amount of debate about the conditions under which it is suitable to use the term. However a more formal definition can be given as, photonic crystals are periodic optical nanostructures that affect the motion of photons in much the same way that ionic lattices affect electrons in solids [2]. They are synthetic crystals that can manipulate or be sensitized to respond to specific wavelengths of light. Its development suggests the possibility of increased miniaturization and efficiency of computing components and other technologies.

The year 1987 was significant, as in that very year two seminal papers published in the same volume of Physical Review Letters laid the foundation of the field of photonics. Still today the interest of the researchers in the field of “photonic crystal” has been incessantly growing since then. The number of papers coming out each year is so high that it becomes very difficult to keep track of even the most significant ones. However long before the usage of the term

“photonic crystals” was common; a significant amount of work was already done in the field. In 1887, Lord Rayleigh [3] laid the foundation of what we know today as a 1D photonic crystal. However for nearly 100 years after that this field was lying dormant until the year 1987. In that very year, both Yablonovitch and John independently published two very similar papers. Yablonovitch’s main motivation in his paper had been to engineer a crystal’s density of states, to control the spontaneous emission of materials embedded within the crystal [4]; in comparison to that John’s idea was to use crystals to affect the localization and control of light. In spite of such differences, both the papers addressed the engineering of a structured material exhibiting ranges of frequencies at which the propagation of electromagnetic waves should not be allowed i.e. the so called band gaps and their employment in the emission control of optically active materials [4].

Rudziński [5] was one of first person who calculated the density of states of a defected photonic crystal by analytical method. On similar lines, Kano [6] calculated DOS for anisotropic 3D photonic crystal for thermally pumped terahertz emission. Boundary effects on DOS are computed in [7] using Green’s function. Dispersion relation of an N-period

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crystal was theoretically investigated by Dios-Leyva [8] and compared with the result of an infinite one for finite and large values of unit cells. Mode spectrum is also calculated in [5], [8] for 1D structure by several workers. Scotognella [9] suggested that 1D PC with suitable material composition can be used as DFB laser. In addition to such notable work the authors also studied the density of states profile in one-dimensional photonic structures for different both p- and s-polarized wave conditions for both conventional material compositions as well as for semiconductor heterostructure based material compositions [10], [11].

On similar lines as in the case of DOS, W. M. Robertson et al. [12] studied the photonic band structure in a 2D dielectric array using the coherent microwave transient spectroscopy technique. Tip et al. [13] analyzed the band structure of absorptive dielectric photonic crystals and investigated them. In their result, the frequency-dependent electric permeability $\epsilon(x, \omega)$ satisfied certain analyticity requirements as a function of frequency, they showed that no band gaps exist in frequency regions where absorption takes place, i.e. where $\epsilon(x, \omega)$ has a non-zero imaginary part. Bandgap of two dimensional photonic crystals had also previously been studied by varying column roundness by Hillebrand [14] using plane-wave expansion method. A. Huttunen, and P. Törmä, [15] both presented a method for calculating band structures for one-dimensional Kerr nonlinear photonic crystals, which exhibit an optical switching function. The band structure showed the allowed modes for the nonlinear photonic crystal as a function of the magnitude of the nonlinearity. Recently, finite-difference-time-domain method was used to analyze the forbidden region of photonic crystal with different geometries [16]. Zhao calculated the width of bandgap [17] using Bragg's principle of reflection. Men optimized the computational problem using semi-definite programming and subspace methods [18]. Evolutionary algorithm [20] and level-set method [19] have also been used for design of large bandgap crystal.

Pochi Yeh et al. [21] were among the first group of researchers who studied propagation of electromagnetic radiation in periodically stratified media. Media of finite, semi-infinite, and infinite extent were the subject of treatment in their seminal paper in 1977. Min Qui et al. [22] studied the wave propagation through a photonic crystal with a triangular lattice of air holes realized in the InP-InGaAsP heterostructure, theoretically for the transverse magnetic modes. S. Foteinopoulou and C. M. Soukoulis [23]

systematically studied a collection of refractive phenomena that can possibly occur at the interface of a two-dimensional photonic crystal with the use of the wave vector diagram formalism. Benoît Lombardet et al. [24] obtained the standard representation of an optical field propagating in a photonic crystal (PhC) is an electromagnetic Bloch wave. They presented a description of these waves based on their Fourier transform into a series of electromagnetic plane wave.

This organization of this paper is as follows, in section 2 we discuss the first and the most fundamental concept in photonic crystal i.e. the Density of States profile which lays down the foundation for the next section. We follow it up with a discussion on the importance of band structure studies and methods used in its calculation in section 3. Section 4 discusses the phenomenon of wave propagation in photonic crystals for different incident wave conditions. In section 5 we review some of the applications of photonic crystals. Lastly we conclude our paper.

II. DENSITY OF STATES

The photon density of states can be manipulated by using periodic structures i.e. photonic crystals with length scales on the order of the wavelength of light. Some of these structures can completely inhibit the propagation of light of certain colors (energies), creating a photonic bandgap: the DOS is zero for those photon energies. Other structures inhibit the propagation of light only in certain directions to create mirrors, waveguides, and cavities. In the nanostructure media the concept of Local density of states (LDOS) is often more relevant than that of DOS, as the DOS varies considerably from point to point.

We know that the electromagnetic density of states (DOS) characterizes the mode density of the fluctuating vacuum fields and a thorough knowledge of how to control the DOS enables us to engineer various quantum-mechanical processes which depend on it. Tuning of the local density of states function are very important for fabrication of micro-laser or optical memory. Also threshold voltage of a laser may be reduced by modifying the DOS function. Hence the accurate evaluation of density of states and its dependence on structural parameters are very important when the structure is subjected to polarized wave incidence.

A. Calculating DOS

The density of states DOS in a medium equals the density per unit volume and energy of the number of solutions to

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Maxwell's equations. A direct consequence of the existence of PBG in the dispersion relation of light through photonic crystals is that, for those energy intervals, the density of available electromagnetic states in the system is zero. This effect can be engineered in photonic crystals and has two crucial implications: the first one over the spontaneous emission of an emitter placed in a photonic crystal, the second is related to light localization in three dimensional systems.

A complete photonic bandgap entails a complete suppression of the density of states, a modification of the electromagnetic vacuum density of states. A small impurity inside a photonic band gap material will give rise to a confined mode around this impurity, as in atomic crystals. On the other hand, the spontaneous emission of an emitter (atom, molecule, quantum dot) can be controlled and tailored by modification of the properties of the radiation field. This is due to the fact that the total radiative rate Γ of the spontaneous emission is given by the well-known Fermi golden rule:

$$\Gamma(\omega) = \frac{2\pi}{\hbar} \rho(\omega) \quad (1)$$

where $\rho(\omega)$ is the photon density of states. For photons in ordinary vacuum ($\omega = ck$),

$$\rho(\omega) = \frac{\omega^2}{\pi^2 c^3} \quad (2)$$

In 1950's, E. Purcell proposed the enhancement of spontaneous emission rates of atoms when they are matched in a resonant cavity (the Purcell Effect). A two level system will decay spontaneously by interaction with a vacuum continuum at a rate proportional to the spectral density of modes per volume evaluated at the transition frequency. If a local source is placed in a photonic crystal with an electromagnetic band gap, which has zero local density of states, then the spontaneous emission can be rigorously forbidden. Rather, a bound photon-atom state is formed. This effect will occur for an emitter placed in the photonic crystal if the transition energy lies within a complete PBG. However, if the transition energy is near a band edge, where $\rho(\omega)$ is enhanced, an enhancement of the radiative decay is expected. Many different kinds of systems in which the rate of spontaneous emission is modified by the environment are reported, including microcavities, two and three-dimensional photonic crystals to give just a few examples.

Other crucial effect which can be strongly affected by the engineered $\rho(\omega)$ in a photonic crystal is the, so-called, strong or Anderson light localization. The inhibition of light propagation was predicted to occur also in the opposite case, as an effect of disorder in some random systems. In analogy to the phenomenon of Anderson localization originally predicted for electrons, if the transport mean free path becomes as short as the wavelength of light itself, interference dominates in the scattering process. One may assist to the formation of localized states, in which light remains trapped, inhibiting light transport. Such an effect has been subject of great interest in disordered materials and can be reached if:

$$\pi^2 c \rho_{loc}(\omega) l_t^2 \cong 1 \quad (3)$$

where c is the speed of light in vacuum, $\rho_{loc}(\omega)$ is the photon local density of states at frequency ω , and l_t is the transport mean free path for photons, determined by the degree of disorder in the medium. For photons in a disorder effective medium with refractive index n , this condition reduces to the Ioffel-Regel criterion, $kl_t(\omega) \approx 1$.

However, in a photonic crystal, a decrease of $\rho(\omega)$ in the band-gap and an enhancement at the band-edge reflects the modified phase space available Δk for light scattering when the photonic modes are concentrated around few k -directions or the available scattered states are reduced. This is consistent with John's seminal prediction of a need for a modified Ioffe-Regel criterion for scattering in photonic crystals, to include Δk . The very low density of states near the complete band gap provides a very favorable scenario for the photon localization according to criterion, even when $kl_t(\omega) \gg 1$. Localization in these structures arises from a delicate interplay between order and disorder.

III. BAND STRUCTURE

Band structure is one of the most important concepts in solid state physics as well as in photonics. It gives the photonic levels or modes in ideal crystal structures, which are characterized by a Bloch vector k as well as a band index n . Here the Bloch vector is an element of the reciprocal space and is typically restricted to the first Brillouin zone. We can say that photonic band structure computations determine the dispersion relation of infinitely extended defect free photonic crystals. They also allow one to design photonic crystals that exhibit photonic band gaps; accurately interprets measurements on photonic crystal samples. As a consequence

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photonic band structure calculations represent important predictive as well as interpretative basis for photonics crystals research and therefore lie at the very core of theoretical investigations of photonic crystals. More specifically the goal of the photonic band structure computations is to find the eigenfrequencies as well as eigen-modes of the wave equation for the perfect photonic crystal i.e. for an infinitely extended periodic array of dielectric material. In essence photonic band structure analysis, allows us to engineer the photonic band gaps with great accuracy.

Till date numerous methods have been used to calculate the photonic band structure with different geometries, such as the Plane Wave expansion Method (PWM), Transfer Matrix Method (TMM), Finite Difference Time Domain (FDTD) method, Order-n spectral method, KKR method and also the Bloch wave – MoM method. However not all of these are widely used in industry and academia alike for band structure studies. With respect to all of these methods, the plane wave expansion method (PWM) is the most popular of all the methods because it is relatively easy for the people to understand the derivation due to its similarity with the electron energy band gap which is studied in solid state physics.

Besides PWM method, another method namely the TMM method is also a popular method which is extensively used in one dimensional photonic crystals band structure calculations due to its maturity with the results being directly applied in the optical industry as well as in academic research. The FDTD method is one of the dominant computational electrodynamics techniques primarily used in two or three dimensional crystal structures. Being a numerical analysis technique it is used for finding approximate solutions to the associated system of differential equations. Since it is a time-domain method, FDTD solutions can cover a wide frequency range with a single simulation run, and treat nonlinear material properties in a natural way. Other methods like the Order-n spectral method, KKR method and also the Bloch wave – MoM method are nothing but specialized versions of the PWM, TMM and FDTD techniques and require much deeper understanding of the electromagnetic theory.

A. Plane Wave Expansion Method

Lying on the fundamental principles of the Bloch function and the Fourier transform, the plane wave method (PWM) applies to a periodic structure of a photonic crystal in the wave vector-space. On solving the Maxwell equations, eigen-

values can be obtained to extract the dispersive feature of the photonic energy band. In terms of an approach the way to solve, Maxwell equations can be converted into mathematical constructs to calculate the eigen-values of the system and to obtain the dispersive relation of the energy band of the photonic crystal which has a periodic $\epsilon(r)$ structure that change in real space. On the assumption that both $E(r, t)$ and $H(r, t)$ have sine surge modes with respect to real space r and time t , then Maxwell equations can be written as:

$$\nabla \cdot \epsilon(r) \cdot E(r) = 0 \quad (4)$$

$$\nabla \cdot B(r, t) = 0 \quad (5)$$

$$\nabla \times H(r) = i\omega \epsilon_0 \epsilon(r) E(r) \quad (6)$$

$$\nabla \times E(r) = -i\omega \mu_0 H(r) \quad (7)$$

where ω is the oscillation frequency of the electromagnetic field, $\epsilon(r)$ is the corresponding dielectric constant of the crystal and is a function of space r , also ϵ_0 and

μ_0 denote the dielectric constant and permeability in vacuum, respectively. On mathematically solving these equations in addition to the consideration of the harmonic mode, a simple important equation obtained can be written as:

$$\nabla \times \frac{1}{\epsilon(r)} \nabla \times \vec{H}(r) = \frac{\omega^2}{C^2} \vec{H}(r) \quad (8)$$

Using Bloch's theorem, essentially in the case of an infinite periodic photonic crystal, a mode in a periodic structure can be expressed as a sum of infinite number of plane waves:

$$H(r) = \sum_{\vec{G}_i, \lambda} h_{\vec{G}_i, \lambda} e^{i(\vec{k} + \vec{G}_i) \cdot \vec{r}} \hat{e}_\lambda \quad (9)$$

where $\lambda=1, 2$, also k denotes the wave vector of the plane wave, \vec{G} is the reciprocal lattice vector, \hat{e}_λ is used to denote the two unit axis perpendicular to the propagation direction $\vec{k} + \vec{G}$. $h_{\vec{G}_i, \lambda}$ is used to represent the coefficient of the H

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component along the axes \hat{e}_λ . One thing to note here is that $(\hat{e}_1, \hat{e}_2, \vec{k} + \vec{G})$ are perpendicular to each other.

Now, using the Fourier transform, the dielectric function can also be written as,

$$\varepsilon(r) = \sum_{\vec{G}} \varepsilon(\vec{G}) \exp(i\vec{G}.r) \quad (10)$$

$$\varepsilon(\vec{G}) = \frac{1}{V} \iiint_{\Omega} \varepsilon(r) \exp(-i\vec{G}.r) \quad (11)$$

where Ω is the unit cell and V is the volume of the unit cell.

Eventually, Helmholtz's equation can be expressed in a form which is standard eigen-value problem,

$$\sum_{\vec{G}'} \begin{bmatrix} |k + G||k + G'| \varepsilon^{-1}(G - G') \times \\ \hat{e}_2 \hat{e}'_2 & -\hat{e}_2 \hat{e}'_1 \\ -\hat{e}_1 \hat{e}'_2 & \hat{e}_1 \hat{e}'_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \frac{\omega^2}{C^2} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \quad (12)$$

Here, $\begin{bmatrix} \hat{e}_2 \hat{e}'_2 & -\hat{e}_2 \hat{e}'_1 \\ -\hat{e}_1 \hat{e}'_2 & \hat{e}_1 \hat{e}'_1 \end{bmatrix}$ matrix gives us the direction of

the wave vector propagation when it strikes a lattice site; i.e. causes diffraction on hitting a Bragg plane in the first Brillouin zone. Certain simplifications exist for both 1D and 2D case as well as for both in-plane and off-plane light propagation.

B. Transfer Matrix Method

This method is based on the principles of the Maxwell equations and the boundary conditions; the transfer matrix method has been widely used to calculate the light path, amplitude and phase spectra of the light wave propagating in a one dimensional photonics material, which is also called a periodic multi-layered structure. For the one dimensional photonic crystal structure consisting of a single layer, the transfer matrix M to present the propagation property of the light wave can be written as

$$M = M_1.M_2.M_3.....M_j...M_n \quad (13)$$

In the transfer matrix M , transmission as well as the reflection intensities of the light, including the amplitude and

the phase change of the electric field under all incidence conditions, can be calculated as functions of the wavelength. The TMM has an obvious merit in one dimensional photonic crystal band gap calculations because it can be used to produce a result with high precision.

C. Finite Difference Time Domain Method

By volumetric sampling of unknown fields, the finite difference time domain method imitates the electromagnetic wave. In photonic crystals, where the dielectric constant is periodically modulated, the electric and magnetic fields of the electromagnetic waves, $E(r)$ and $H(r)$, the structure can be described by band index n and a wave vector k in irreducible Brillouin zone due to Bloch's theorem. Now, one can find eigen-values of the structure for a given periodic boundary condition. Random initial condition is used in order to compute the eigen-modes of the structure by means of FDTD method. Also sufficient amount of time is needed to catch enough accuracy. For attaining eigen-modes we choose various low-symmetry locations in the unit cell as probes, in order to record peaks of the Fourier transform of the complex field components in the time domain for a given propagation constant. We should notice that, probes at high-symmetry locations of the unit cell are not able to detect all eigen-modes.

IV. WAVE PROPAGATION

Among the different photonic bandgap structures, one-dimensional periodic photonic bandgap microstructures had been studied previously by various researchers in the last decade due to the advantage of theoretically analysing optical characteristics with precision in regard to the lack of confinement in two spatial dimensions. Some of the important contributions in this regard were made by Foteinopoulou who analysed the effect of surface defect on backward wave, and showed that surface mode manipulation is possible with dispersion. Propagating wave analysis is useful for designing four-wave mixing analysis in nonlinear photonic crystal. Suitable dielectric material is used to characterize modal dispersion in 1D crystal which is why the computation of wave profile is much easier in this structure, making it more interesting when filter characteristics is considered including the effect of polarization of incident light. Variation of wave profiles can provide the idea about Brillouin zone of the block, which is fundamental in studying the optical characteristics like transmittivity, reflectivity of the structure. Henceforth, the study is helpful for analysis of usability of photonic crystal at optical communication.

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A. Method for Estimating Propagating Modes

Coupled mode theory is being applied to counter-propagating waves in a single mode one dimensional periodic structure i.e. Bragg grating with a periodic corrugation as shown in fig. 1.

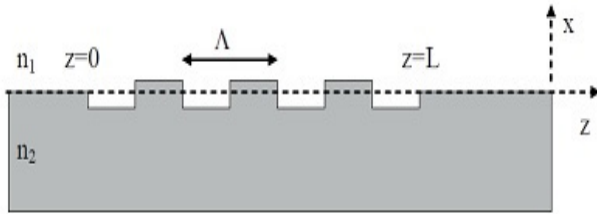


Figure 1: Periodic corrugation in one of the core-cladding interfaces. The grating is single mode, and we make the assumption that the only significant coupling is between counter propagating guided modes

The corrugation is scalar and we don't expect coupling between TE and TM modes, so in the following treatment we'll consider TE modes. We start by describing the field in the corrugated structure as a sum of the forward and backward propagating modes

$$E_y = A(z)u(x)\exp[j(\omega t - \beta \cdot z)] + B(z)u(x)\exp[j(\omega t + \beta \cdot z)] \quad (14)$$

where A and B are the amplitudes of the forward and backward propagating waves, and u(x) is the mode profile.

The perturbation in the corrugated region is

$$\vec{P}_{pert} = \Delta n(x, z)^2 \epsilon_0 \vec{E} \quad (15)$$

We now substitute the expression for the field into this expression to get

$$P_{pert} = \frac{1}{2} \Delta n(x)^2 \epsilon_0 \times \left\{ \begin{aligned} &A(z)u(x)\exp[j(\omega t - \beta \cdot z)] + \\ &B(z)u(x)\exp[j(\omega t + \beta \cdot z)] \end{aligned} \right\} \quad (16)$$

$$P_{pert} = \frac{1}{2} \Delta n(x)^2 \epsilon_0 e^{j\omega t} e^{-j\beta \cdot z} \times \{A + B e^{j2\beta \cdot z}\} u(x) \quad (17)$$

Recalling the fundamental coupled mode equation

$$-\frac{dA_i^+}{dz} \cdot \exp[j(\omega t - \beta \cdot z)] + \frac{dA_i^-}{dz} \cdot \exp[j(\omega t + \beta \cdot z)] + c.c = \frac{-j}{2\omega} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} P_{pert}(x, z, t) u_i(x) dx \quad (18)$$

which then simplifies to

$$-\frac{dA}{dz} + \frac{dB}{dz} e^{j2\beta \cdot z} = \frac{-j\omega \epsilon_0}{4} \times \{A + B e^{j2\beta \cdot z}\} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (19)$$

We will now assume that the corrugation has a square-wave shape as indicated in Fig. 1. The general conclusions are not dependent on the exact shape, so the following treatment, with appropriate adjustments, is valid also for non-square corrugations. The square-wave corrugations can be expressed as a series in the following form

$$\Delta n^2(x, z) = \Delta n^2 \sum_m C_m e^{j \frac{2m\pi \cdot z}{\Lambda}} \quad (20)$$

$$C_m = \begin{cases} -\frac{j}{m\pi} & m = \text{odd} \\ 0 & m = \text{even} \end{cases} \quad (21)$$

By comparing this expression to the above coupled mode equations, we realize that only modes that are close to phase matched will experience significant coupling. In other words, we need only keep terms of the same periodicity. In a range of wave vectors, the equations can be simplified to

$$\frac{dA}{dz} = \frac{j\omega \epsilon_0}{4} B e^{j2\beta \cdot z} C_m e^{-j \frac{2m\pi \cdot z}{\Lambda}} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (22)$$

$$\frac{dB}{dz} = \frac{j\omega \epsilon_0}{4} A e^{-j2\beta \cdot z} C_m e^{j \frac{2m\pi \cdot z}{\Lambda}} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (23)$$

$$\frac{dA}{dz} = K^* B e^{j2\Delta\beta \cdot z} \quad (24)$$

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$$\frac{dB}{dz} = KAe^{-j2\Delta\beta \cdot z} \quad (25)$$

$$K = C_m e^{j \frac{2m\pi \cdot z}{\Lambda}} \int_{-\infty}^{\infty} \Delta n^2 u^2(x) dx \quad (26)$$

where

$$\Delta\beta = \beta - \frac{m\pi}{\Lambda} \quad (27)$$

Let us check energy conservation in the systems of equations we have found for modes in a Bragg grating i.e. one-dimensional periodic dielectric array. We start by deriving expression for the energies in the forward and backward propagating waves. Based on Eq. (14) we can write

$$\frac{d}{dz} |A|^2 = \frac{d}{dx} (A.A^*) = \quad (28)$$

$$A.B^* .Ke^{-j2\Delta\beta \cdot z} + A^* .B.K^* e^{j2\Delta\beta \cdot z}$$

$$\frac{d}{dz} |B|^2 = \frac{d}{dx} (B.B) = \quad (29)$$

$$B.A^* .K^* e^{j2\Delta\beta \cdot z} + B^* .A.Ke^{-j2\Delta\beta \cdot z}$$

The difference between the rates of change in the forward-propagating and backward- propagating energy is then

$$\frac{d}{dz} |A|^2 - \frac{d}{dz} |B|^2 = 0 \quad (30)$$

We see that the rate of change in forward-propagating energy is exactly balanced by the rate of change in backward-propagating energy, which is the correct result for loss-less, counter-propagating waves.

The set of equations describing the modes of the Bragg grating (Eq. 9-13) can now be solved. Assuming that the forward propagating mode has an amplitude A_0 at $z=0$, and that the backward propagating wave is zero at $z=L$, we find

$$A = A_0 e^{j\Delta\beta \cdot z} \cdot \frac{-\Delta\beta \sinh[S(z-L)] + jS \cosh[S(z-L)]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \quad (31)$$

$$A = A_0 e^{j\Delta\beta \cdot z} \left[\begin{array}{l} \frac{-\Delta\beta \sinh[S(z-L)]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \\ + \frac{jS \cosh[S(z-L)]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \end{array} \right] \quad (32)$$

$$B = A_0 \cdot jK \cdot e^{-j\Delta\beta \cdot z} \frac{\sinh[S(z-L)]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \quad (33)$$

where

$$S = \sqrt{K^2 - \Delta\beta^2} \quad (34)$$

when $\Delta\beta > 0$, this simplifies to

$$A = A_0 \frac{\cosh[K(z-L)]}{\cosh[KL]} \quad (35)$$

$$B = A_0 \cdot \frac{\sinh[K(z-L)]}{\cosh[KL]} \quad (36)$$

The expressions we have found for the field amplitudes in the periodically corrugated waveguide allow us to calculate the reflection and transmission spectra of the Bragg grating. For example, the field reflection is simply the ratio of the forward propagating and backward propagating wave at the input to the Bragg grating:

$$r = \frac{A_0 \cdot jK \frac{\sinh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]}}{A_0 \cdot \frac{-\Delta\beta \sinh[SL] + jS \cosh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]}} \quad (37)$$

which may be written as

$$r = jK \frac{\sinh[SL]}{-\Delta\beta \sinh[SL] + jS \cosh[SL]} \quad (38)$$

V. APPLICATIONS

One dimensional photonic crystal have widespread use in thin-film optics with applications ranging from low and high reflection coatings on lenses and mirrors to color changing paints and inks. The products involving two-dimensional

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periodic photonic crystals are already available in the form of photonic-crystal fibers. Photonic crystal fibres are developed on the physics required for the primary purpose of optical communication [25], optical nonlinearity [26], which use a microscale structure to confine light with radically different characteristics. These novel microstructures have already replaced conventional optical fiber for efficient communication. One- as well as two- dimensional photonic crystals have been already been used to construct optical waveguides [27], photonic band-edge laser [28], high efficient LED [29], filter [30], switches [31], integrated photonics [32], sensing [33], high power technology [34], quantum information science [35]. The three-dimensional structures are still far from commercialization but have the potential to offer additional features like optical nonlinearity required for the operation of optical transistors used in optical computers, when some technological aspects such as manufacturability and principal difficulties such as disorder are under control.

VI. CONCLUSIONS

In the last two decades the field of photonics has had such rapid development that it is truly quite a cumbersome task to touch upon all the major significant developments associated with it. We hope that students and researchers alike who are new to the world of photonics would benefit from this short review and gain a clear insight of the subject. Still today, a lot of work needs to be done especially in the area of three dimensional photonic crystals and challenges in fabricating such structures with precision in the optical regime. Use of group theory in photonics is an interesting and active area of study from both the theoretical and experimental point of view. Similarly, another area of active research is semiconductor heterostructure based photonic crystals. Tailoring as well as tuning the photonic band gaps with precision as well as accuracy is the most important thing which anybody working in the field of photonics must keep in the back of their mind while designing any of the photonic devices. Structuring the material properties to control the flow of light has paved way to an alternate dimension altogether due to its difference from the classical theory of electronic band gap physics. Light has surprised mankind from the very beginning and now it's our turn to see what more surprises can light offer; just when we are beginning to modulate it.

REFERENCES

- [1] John D. Joannopoulos, Steven G. Johnson, Joshua N. Winn, and Robert D. Meade. Photonic crystals: molding the flow of light. Princeton University press, 2011.
- [2] John D. Joannopoulos, Pierre R. Villeneuve, and Shanhui Fan. "Photonic crystals: putting a new twist on light." *Nature* 386, no. 6621 (1997): 143-149.
- [3] Lord. Rayleigh, "XVII. On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure." *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 24, no. 147 (1887): 145-159.
- [4] Eli Yablonovitch. "Inhibited spontaneous emission in solid-state physics and electronics." *Physical review letters* 58, no. 20 (1987): 2059.
- [5] A. Rudzinski, A. Tyszcza-Zawadzka, and P. Szczepanski, Simple Model of the Density of States in 1 D Photonic Crystal, *Proceedings of SPIE*, vol. 5950, p. 59501A, 2005.
- [6] P Kano, D Barker, M Brio, "Analysis of the Analytic Dispersion Relation and Density of States of A Selected Photonic Crystal", *Journal of Physics D: Applied Physics*, vol. 41, p. 1 85106, 2008.
- [7] V. Prosentsov, and A. Lagendijk, Periodicity Enclosed in Boundaries: Local Density of States in Photonic Clusters, *Optics Express*, vol. 1 6, pp. 6974-6984, 2008.
- [8] M. de Dios-Leyva, and J. C. Drake-Perez, "Properties of the Dispersion Relation in Finite One-dimensional Photonic Crystals", *Journal of Applied Physics*, vol. 1 09, p. 1 03 526,201 L
- [9] F. Scotognella, A. Monguzzi, M. Cucini, F. Meinardi, D. Comoretto, and R. Tubino, One Dimensional Polymeric Organic Photonic Crystals for DFB Lasers, *International Journal of Photoenergy*, [hnp://dx.doi.org/10.1155/2008/389034](http://dx.doi.org/10.1155/2008/389034), 2008.
- [10] S. Banerji, A. Deyasi, A. Halder, and S. Bose, "Comparative Study of Density of States of 1D Photonic Crystal for Different Polarization Conditions of Incident Wave", *International Conference on Electronics, Communication and Instrumentation [ICECI: 14]*, pp. 1-4, Jan 2014 [IEEE Xplore, 978-1-4799-3982-4, doi: 10.1109/ICECI.2014.6767359]
- [11] A. Halder, S. Banerji, S. Bose, S. Mandal, and A. Deyasi "Computing Density of States of One-Dimensional Photonic Crystal under P-Polarized Incident Wave", *International Journal of Modern Communication Technologies & Research (IJMCTR)*, Volume-2, Issue-3, pp:38-41, March 2014. [ISSN: 2321-0850]
- [12] W. M. Robertson, G. Arjavalingam, R. D. Meade, K. D. Brommer, A. M. Rappe, and J. D. Joannopoulos, "Measurement of photonic band structure in a two-dimensional periodic dielectric array", *Phys. Rev. Lett.*, vol. 68, pp.2023-2026, 1992.

INTERNATIONAL JOURNAL FOR RESEARCH IN APPLIED SCIENCE AND ENGINEERING TECHNOLOGY (IJRASET)

- [13] A. Tip, A. Moroz, and J. M. Combes. "Band structure of absorptive photonic crystals." *Journal of Physics A: Mathematical and General* 33, no. 35 (2000): 6223.
- [14] R. Hillebrand, W. Hergert, W. Harm, "Theoretical Band Gap Studies of Two-Dimensional Photonic Crystals with Varying Column Roundness", *Physica Status Solidi (b)*, vol. 217, pp. 981-989, 2000.
- [15] A. Huttunen, and P. Törmä. "Band structures for nonlinear photonic crystals." *Journal of applied physics* 91, no. 7 (2002): 3988-3991
- [16] T.H. Zhang, D. S. Wang, M. L. Gong, and D. Z. Zhao. "Application of group theory to plane wave expansion method for photonic crystals." *Optics communications* 237, no. 1 (2004): 179-187.
- [17] C. Y. Kao, S. Osher, E. Yablonovitch, "Maximizing band gaps in two-dimensional photonic crystals by using level set methods", *Applied Physics B*, vol. 81, pp:235-244, 2005.
- [18] H. Men, N. C. Nguyenz, R. M. Freundx, P. A. Parrilo, J. Perairek, "Band Gap Optimization of Two-Dimensional Photonic Crystals Using Semidefinite Programming And Subspace Methods" arXiv: 0907.2267 V1 [Math.Oc], 13 Jul 2009.
- [19] J. Zhao, X. Li, L. Zhong, G. Chen, "Calculation of photonic band-gap of one dimensional photonic crystal" *Journal of Physics: Conference Series (Dielectrics 2009: Measurement Analysis and Applications, 40th Anniversary Meeting)*, vol. 183, p. 012018, 2009.
- [20] S. Preble, M. Lipson, "Two-dimensional photonic crystals designed by evolutionary algorithms", *Applied Physics Letters*, vol. 86, p. 061111, 2005.
- [21] P. Yeh, A. Yariv, and C. S. Hong, "Electromagnetic propagation in periodic stratified media - I. General theory," *J. Opt. Soc. Am.*, Vol. 67, No. 4, 423-438, 1977.
- [22] Benoît Lombardet, L. Andrea Dunbar, Rolando Ferrini, and Romuald Houdré. "Fourier analysis of Bloch wave propagation in photonic crystals." *JOSA B* 22, no. 6 (2005): 1179-1190.
- [23] Min Qiu, Lars Thylen, Marcin Swillo, and Bozena Jaskorzynska. "Wave propagation through a photonic crystal in a negative phase refractive-index region." *Selected Topics in Quantum Electronics, IEEE Journal of* 9, no. 1 (2003): 106-110.
- [24] S. Foteinopoulou, and C. M. Soukoulis. "Electromagnetic wave propagation in two-dimensional photonic crystals: A study of anomalous refractive effects." *Physical Review B* 72, no. 16 (2005): 165112.
- [25] J. Hansryd, P. A. Andrekson, M. Westlund, J. Li and P. O. Hedekvist, "Fiber-based Optical Parametric Amplifiers and their Applications", (2002), *IEEE Journal of Selected Topics on Quantum Electronics.*, vol. 8, pp. 506-520.
- [26] J. Limpert, A. Liem, M. Reich, T. Schreiber, S. Nolte, H. Zellmer, A. Tünnermann, J. Broeng, A. Petersson and C. Jakobsen, "Low-Nonlinearity Single-Transverse-Mode Ytterbium-Doped Photonic Crystal Fiber Amplifier", (2004), *Optic Express*, vol. 12, pp. 1313-1319.
- [27] A. Mekis, J. C. Chen, I. Kurland, S. Fan, P. R. Villeneuve and J. D. Joannopoulos, "High Transmission through Sharp Bends in Photonic Crystal Waveguides", (1996), *Physical Review Letters*, vol. 77, pp. 3787-3790.
- [28] P. Szczepanski, "Semiclassical Theory of Multimode Operation of a Distributed Feedback Laser", (1988), *IEEE Journal of Quantum Electronics*, vol. 24, pp. 1248-1257.
- [29] I. S. Fogel, J. M. Bendickson, M. D. Tocci, M. J. Bloemer, M. Scalora, C. M. Bowden and J. P. Dowling, *Pure and Applied Optics*, "Spontaneous Emission and Nonlinear Effects in Photonic Bandgap Materials", (1998), vol. 7, pp. 393-408.
- [30] M. Boroditsky, R. Vrijen, T. F. Krauss, R. Coccioli, R. Bhat and E. Yablonovitch, "Spontaneous Emission Extraction and Purcell Enhancement from Thin-Film 2-D Photonic Crystals", (1999), *Journal of Lightwave Technology*, vol.17, pp. 2096-2112.
- [31] K. Bayat, G. Z. Rafi, G. S. A. Shaker, N. Ranjkesh, S. K. Chaudhuri and S. Safavi-Naeini, "Photonic-Crystal based Polarization Converter for Terahertz Integrated Circuit", (2010), *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, pp. 1976-1984.
- [32] W. Belhadj, F. AbdelMalek and H. Bouchriha, "Characterization and Study of Photonic Crystal Fibres with Bends", (2006), *Material Science and Engineering: C*, vol. 26, pp. 578-579.
- [33] J. Limpert, T. Schreiber, S. Nolte, H. Zellmer, T. Tunnermann, R. Iliew, F. Lederer, J. Broeng, G. Vienne, A. Petersson and C. Jacobson, "High Power Air-Clad Large-Mode-Area Photonic Crystal Fiber Laser", (2003), *Optic Express*, vol. 11, pp. 818-823.
- [34] H. Azuma, "Quantum Computation with Kerr-Nonlinear Photonic Crystals", (2008), *Journal of Physics D: Applied Physics*, vol. 41, p. 025102.
- [35] J. C. Chen, H. A. Haus, S. Fan, P. R. Villeneuve and J. D. Joannopoulos, "Optical Filters from Photonic Band Gap Air Bridges", *Journal of Lightwave Technology*, (1996), vol. 14, pp. 2575-2580.