



IJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 4

Issue: II

Month of publication: February 2016

DOI:

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

Special pairs of rectangles and sphenic number

G.Janaki^{#1}, S.Vidhya^{*2}

[#]Department of Mathematics, Cauvery College for Women, Trichy – 620018.

^{*} Department of Mathematics, Cauvery College for Women, Trichy – 620018.

Abstract— We present pairs of Rectangles, such that in each pair, the sum of their areas is two times the Sphenic number minus one. Also we present the number of pairs of primitive and non-primitive rectangles.

Keywords— Pairs of Rectangles, Sphenic number, Primitive and non-primitive Rectangles.

I. INTRODUCTION

Number theory is a broad and diverse part of Mathematics that developed from the study of the integers. Diophantine equations are numerously rich because of its variety. Diophantine problems have fewer equations that unknown variables and involve finding solutions in integers. There is no universal method available to know whether a Diophantine equation has a solution or finding all solutions if it exists. In this context one may refer [1-4].

A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. These numbers have presented in [5-8]. In [9-14], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are presented.

In this communication, we search for pairs of Rectangles, such that in each pair, the sum of their areas is two times the sphenic numbers minus one, where the Sphenic number which are divisible by three satisfies this relation.

II. BASIC DEFINITIONS

A. Definition1: Sphenic Number

Sphenic number is a positive integer that is the product of three distinct prime numbers.

B. Definition 2 : Primitive Rectangle

A primitive rectangle is a rectangle having integer sides a and b such that $\gcd(a, b) = 1$.

III.METHOD OF ANALYSIS

Let R_1, R_2 be two distinct Rectangles with generators $u, w (u > w > 0)$ and $v, w (v > w > 0)$ respectively.

Let A_1, A_2 be the areas of R_1, R_2 such that

$$A_1 + A_2 = 2 \text{ times the Sphenic Number} - 1.$$

where the Sphenic number which are divisible by 3 satisfies the above relation.

The above relation which leads to the equation

$$u^2 + v^2 - 2w^2 = 2 \text{ Times the Sphenic Number} - 1.$$

After performing numerical computations we have presented below the values of u, v, w and the corresponding sphenic numbers.

Here we present 2-digit and 3-digit sphenic numbers which satisfies the above relation

TABLE I
 RECTANGLE PAIRS AND SPHENIC NUMBERS

| u | v | w | A_1 | A_2 | $\frac{A_1 + A_2 + 1}{2} = \text{Sphenic Number}$ |
|-----|-----|-----|-------|-------|---|
| 11 | 10 | 9 | 40 | 19 | 30 |
| 15 | 14 | 13 | 56 | 27 | 42 |

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

| | | | | | |
|-----|-----|-----|------|-----|-----|
| 23 | 22 | 21 | 88 | 43 | 66 |
| 27 | 26 | 25 | 104 | 51 | 78 |
| 35 | 34 | 33 | 136 | 67 | 102 |
| 36 | 35 | 34 | 140 | 69 | 105 |
| 39 | 38 | 37 | 152 | 75 | 114 |
| 47 | 46 | 45 | 184 | 91 | 138 |
| 56 | 55 | 54 | 220 | 109 | 165 |
| 59 | 58 | 57 | 232 | 115 | 174 |
| 63 | 62 | 61 | 248 | 123 | 186 |
| 66 | 65 | 64 | 260 | 129 | 195 |
| 78 | 77 | 76 | 308 | 153 | 231 |
| 83 | 82 | 81 | 328 | 163 | 246 |
| 86 | 85 | 84 | 340 | 169 | 255 |
| 87 | 86 | 85 | 344 | 171 | 258 |
| 92 | 91 | 90 | 364 | 181 | 273 |
| 95 | 94 | 93 | 376 | 187 | 282 |
| 96 | 95 | 94 | 380 | 189 | 285 |
| 107 | 106 | 105 | 424 | 211 | 318 |
| 116 | 115 | 114 | 460 | 229 | 345 |
| 119 | 118 | 117 | 472 | 235 | 354 |
| 120 | 119 | 118 | 476 | 237 | 357 |
| 123 | 122 | 121 | 488 | 243 | 366 |
| 134 | 133 | 132 | 532 | 265 | 399 |
| 135 | 134 | 133 | 536 | 267 | 402 |
| 143 | 142 | 141 | 568 | 283 | 426 |
| 144 | 143 | 142 | 572 | 285 | 429 |
| 146 | 145 | 144 | 580 | 289 | 435 |
| 147 | 146 | 145 | 584 | 291 | 438 |
| 156 | 155 | 154 | 620 | 309 | 465 |
| 159 | 158 | 157 | 632 | 315 | 474 |
| 162 | 161 | 160 | 644 | 321 | 483 |
| 167 | 166 | 165 | 664 | 331 | 498 |
| 179 | 178 | 177 | 712 | 355 | 534 |
| 195 | 194 | 193 | 776 | 387 | 582 |
| 203 | 202 | 201 | 808 | 403 | 606 |
| 206 | 205 | 204 | 820 | 409 | 615 |
| 207 | 206 | 205 | 824 | 411 | 618 |
| 210 | 209 | 208 | 836 | 417 | 627 |
| 215 | 214 | 213 | 856 | 427 | 642 |
| 216 | 215 | 214 | 860 | 429 | 645 |
| 218 | 217 | 216 | 868 | 433 | 651 |
| 219 | 218 | 217 | 872 | 435 | 654 |
| 227 | 226 | 225 | 904 | 451 | 678 |
| 236 | 235 | 234 | 940 | 469 | 705 |
| 255 | 254 | 253 | 1016 | 507 | 762 |
| 260 | 259 | 258 | 1036 | 517 | 777 |
| 263 | 262 | 261 | 1048 | 523 | 786 |

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

| | | | | | |
|-----|-----|-----|------|-----|-----|
| 266 | 265 | 264 | 1060 | 529 | 795 |
| 275 | 274 | 273 | 1096 | 547 | 822 |
| 279 | 278 | 277 | 1112 | 555 | 834 |
| 288 | 287 | 286 | 1148 | 573 | 861 |
| 296 | 295 | 294 | 1180 | 589 | 885 |
| 299 | 298 | 297 | 1192 | 595 | 895 |
| 300 | 299 | 298 | 1196 | 597 | 897 |
| 302 | 301 | 300 | 1204 | 601 | 903 |
| 303 | 302 | 301 | 1208 | 603 | 906 |
| 306 | 305 | 304 | 1220 | 609 | 915 |
| 315 | 314 | 313 | 1256 | 627 | 942 |
| 320 | 319 | 318 | 1276 | 637 | 957 |
| 324 | 323 | 322 | 1292 | 645 | 969 |
| 327 | 326 | 325 | 1304 | 651 | 978 |
| 330 | 329 | 328 | 1316 | 657 | 987 |

Thus it is seen that for each pair, the sum of the areas of a Rectangle is equal to twice the sphenic number minus one. From the table 1, all the pairs of Rectangles are primitive. In each of the primitive pairs, one of the Rectangles is primitive and the other is non-primitive Rectangle.

IV. CONCLUSION

One may search for the connections between the pairs of Rectangles and the other sphenic numbers of higher order.

REFERENCES

- [1] Carmichael.R.D, The Theory of Numbers and Diophantine Analysis, Dover Publication, New York (1959).
- [2] Dickson.L.E, History of the theory of Numbers, Vol II, Diophantine Analysis, New York, Dover 2005.
- [3] John.H. Conway and Richard K.Guy, the Book of Numbers, Springer Verlag, New York, 1995.
- [4] Mordell.L.J, Diophantine Equations, Academic press, London (1969).
- [5] Kapoor.J.N, Dhuruva numbers, Fascinating world of Mathematics and mathematical Sciences, Trust Society, Vol.17, 1997.
- [6] Bert Miller, Nasty numbers, The Mathematics Teacher, No.9, Vol.73,649, 1980.
- [7] Charles Boum.K, Nasties are primitives, The Mathematics Teacher, No.9, Vol.74,502-504, 1981
- [8] Sastry,P,S,N, Jarasandha numbers, The Mathematics Teacher, N0.9, Vol.37, issue 3 and 4200.,
- [9] Gopalan.M.A, Sangeetha. V and Manju Somanath, "Pythagorean triangle and Polygonal numbers", Cayley J. Math, Vol.2 (2), 151-156, 2013.
- [10] Gopalan.M.A, Janaki.G, "Pythagorean triangle with Nasty number as a leg", Journal of Applied Mathematical Analysis and Applications, Vol.4, No. 1-2, 13-17, 2008.
- [11] Gopalan.M.A, Devibala.S "Pythagorean triangle with triangular number as a leg,
- [12] Dr. Mita Darbari, A connection between Hardy-Ramanujan number and Special Pythagorean triangle", Bulletin of Society for Mathematical Services and Standards, Vol.3, No.2, 71-73, 2014.
- [13] Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pythagorean triangles and 5-digit Dhuruva numbers, IRJMEIT, Vol.1 (4), 29-33, Aug 2004.
- [14] Gopalan.M.A, Vidhyalakshmi.S, Premalatha.E and Presenna.R, "Special Pairs Pythagorean triangles and Dhuruva number, Global Journal of Science Frontier Research, Vol.15, p.33-38, 2015..



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)