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(1, 2) – Double Domination in Graphs

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Abstract - In this paper , we introduced the (1, 2) – double domination number and also we discussed about its properties .
Keywords - (1, 2) – double domination number , independent double dominating set , independent triple dominating set .
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I. INTRODUCTION

Dominating queens is the origin of the study of dominating set in graphs . Berge [1] and Ore [7] were the first to define dominating sets. A new type of dominating set, (1, 2) – dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [8] . In this paper , we introduced the (1, 2) – double domination number and also we discussed about its properties .

II. PRELIMINARIES

Definition 2.1 : A graph is said to be complete if each of its vertices is adjacent to every other vertex.
Definition 2.2 : A graph is said to be regular if each of its vertices has the same degree.
Definition 2.3 : A graph is said to be cubic graph if each of its vertices is of degree three.
Definition 2.4 : A bipartite graph is a graph in which vertices can be divided into two disjoint sets A and B such that every edge connects a vertex in A to one in B.
Definition 2.5 : A (1, 2) – dominating set in a graph G = (V,E) is a set S having the property that for every vertex v in V – S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v.
Definition 2.6 : The order of the smallest (1,2)- dominating set of G is called the (1,2) – domination number of G and we denote it by γ (1,2) .
Remark 2.1 : From the definition of 2.1 , we see that a (1,2) – dominating set contains atleast 2 vertices, (1,2) – domination number of a graph will be always $\geq$ 2 and (1,2) – dominating sets occur in graphs of order atleast 3.
Definition 2.7 : For each vertex x in a graph G, we introduce a new vertex x' and join x and x' by an edge. The resulting graph is called the ***corona*** of G.

III. (1, 2) – DOUBLE DOMINATING SET

Definition 3.1 **:** A (1, 2) – double dominating set in a graph G = (V, E) is a set S having the property that for every vertex v in V – S there is atleast two vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v.
Definition 3.2 : The order of the smallest (1, 2) - double dominating set of G is called the (1,2) – double domination number of G and we denote it by $\gamma_{d(1,2)}$
From the definition of (1,2) – double dominating sets, we see that a (1,2) – double dominating set contains atleast 2 vertices, (1,2) – double domination number of a graph will be always $\geq$ 3 and (1,2) – double dominating sets occur in graphs of order atleast 3.
Example 3.1 : Consider the graph

Fig. 1

In Fig. 1, { 1 , 4, 3 } , { 1, 4, 2 } is a (1,2) – double dominating set.
Definition 3.3 : A dominating set S is an independent double dominating set if no two vertices in S are adjacent, that is, S is an independent set. The independent double domination number $i_2(G)$ of a graph G is the minimum cardinality of an independent double dominating set. Thus, $i_2(G) = \min\{ |S| \text{ dominates and } \Delta(\langle S \rangle)\}$.
Example 3.2 : In example 3.1 [9] , $i_2(G) = 3$.
Definition 3.4 : A double dominating set S is called a perfect double dominating set if for every vertex , The perfect double

domination number is denoted as $u \in V$, $|N[u] \cap S| = 1$. The perfect double domination number is denoted as $\gamma_{pd}(G)$.

Definition 3.5 : A double dominating set S is called an efficient double dominating set if for every vertex, $u \in V - S$, $|N(u) \cap S| = 1$. Equivalently, a dominating set is efficient if the distance between any two vertices in S is at least three, that is, S is a packing.

We note that, if a graph has an efficient double dominating set, then all efficient double dominating sets in G have the same cardinality namely $\gamma(G)$.

Theorem 3.1 : All (1,2) – double dominating sets are dominating sets.

Proof : The result is trivial from the definition of (1,2) – double dominating sets.

But the converse need not be true.

Example 3.2 : In example 3.1 , { 1,4} is a dominating set .

But it is not a (1, 2) – dominating set.

{ 2, 3, 4 } is a (1, 2) – dominating set .

{ 1 , 4, 3 } is a (1, 2) – double dominating set and it is a dominating set also.

IV. (1, 2) – DOUBLE DOMINATION IN COMPLETE GRAPHS

Theorem 3.2.1 : (1, 2) – double domination is not possible in complete graphs.

Proof: In a complete graph, each vertex is adjacent to every other vertices. So we cannot find a (1, 2) – double dominating set. No vertex can be found at a distance atmost 2 from any other vertex. Let G be a complete graph with n vertices. Then it will have nC2 edges and each vertex is of degree n – 1.The minimum number of edges to be deleted so as to become the resulting graph (1, 2) – double dominating is n - 2 . If we delete n – 2 edges from a complete graph, then in the resulting graph , we can find a (1, 2) – double dominating set.

Lemma 3.2.1 : If a graph G with n vertices, has a vertex of degree n – 1 ,we cannot find a (1, 2) – dominating set.

Example 3.2.1 : In this graph, we cannot find a (1, 2) – double dominating set since each vertex is adjacent to all other vertices.

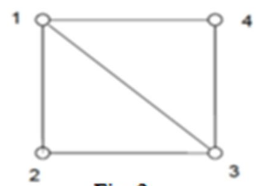

Fig. 2

In graph Fig. 2 , we cannot find a (1, 2) – double dominating set since each vertex is adjacent to all other vertices.

V. RELATION BETWEEN DOMINATION NUMBER AND (1, 2) – DOUBLE DOMINATION NUMBER

In this section we consider different types of graphs and find out their domination number , (1, 2) - domination number and (1, 2) – double domination number and check the relation between them.

Example 3.3.1 :

Fig. 3

In Fig. 3,

{1,3}, {1,4}, {2,4}, {2,3} are all dominating sets. γ (G) = 2.

{1,4} is a (1, 2) – dominating set.

γ (1,2) = 2.

{ 1, 3, 4 } is a (1, 2) – double dominating set and double dominating set.

$\gamma_{d(1,2)} = \gamma_d = 3$.

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

$\gamma_{d(1,2)} = \gamma_d$.

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Example 3.3.2 :

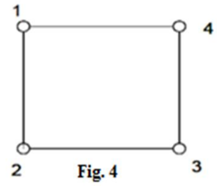

Fig. 4

In Fig. 4,

{1,3}, {1,4}, {2,4}, {2,3} are all dominating sets. γ (G) = 2.
{2,3} is a (1, 2) – dominating set.
γ (1,2) = 2.
{ 2, 4 } is a double dominating set.
{ 2, 3, 4 } is a (1, 2) – double dominating set.
$\gamma_{d(1,2)} = 3$.

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.
$\gamma < \gamma_{d(1,2)}$.
$\gamma_d < \gamma_{d(1,2)}$.

Example 3.3.3 :

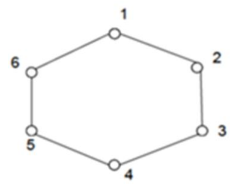

Fig. 5

In Fig. 5 ,

{ 1, 3, 5 } , { 2, 4, 6 } are dominating sets.
γ (G) = 3.
{ 1, 4, 6 } is a (1, 2) – dominating set.
γ (1,2) = 3.
{ 1, 5, 3 } is a double dominating set.
{ 1, 3, 4, 6 } is a (1, 2) – double dominating set.
$\gamma_{d(1,2)} = 4$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.
$\gamma < \gamma_{d(1,2)}$.
$\gamma_d < \gamma_{d(1,2)}$.

Example 3.3.4 :

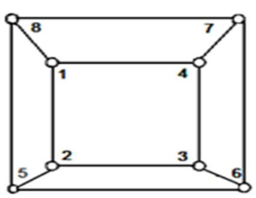

Fig. 6

In Fig. 6 ,

{1,2,3,4} , {5,6,7,8} are dominating.
γ (G) = 4 .
{1,2,3,4} is a (1,2) – dominating set.
γ (1,2) = 3.
{ 4, 8, 2 , 6 } is a double dominating set.
{ 1, 3, 5, 6, 7, 8 } is a (1, 2) – double dominating set.
$\gamma_{d(1,2)} = 6$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

$\gamma_d < \gamma_{d(1,2)}$.

Example 3.3.5 : Consider the bipartite graph G

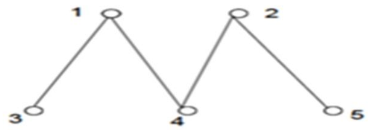

Fig. 7

In Fig. 7 ,

{1,2} is a dominating set.

γ (G) = 2.

{1,4,5} is a (1,2) – dominating set.

γ (1,2) = 3.

{3, 4, 5 } is a double dominating set.

{ 2, 3, 4, 5 } is a (1, 2) – double dominating set.

$\gamma_{d(1,2)} = 4$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

$\gamma_d < \gamma_{d(1,2)}$.

Example 3.3.6 : Consider the cubic bipartite graphs G ,

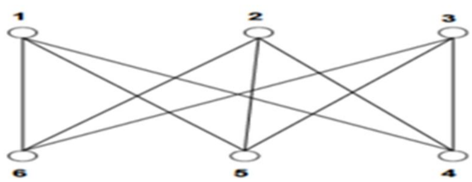

Fig. 8

In Fig. 8 ,

{ 1, 5 } , { 2, 6 } is a dominating set.

γ (G) = 2.

{ 1, 5 } is a (1,2) – dominating set.

γ (1,2) = 2.

{ 2, 4, 6, 5 } is a (1, 2) – double dominating set.

$\gamma_{d(1,2)} = 4$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

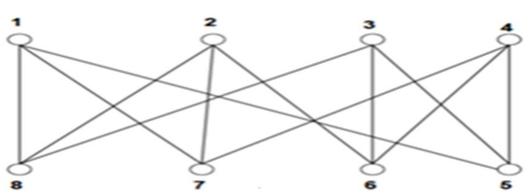

Fig. 9

In Fig. 9,

{ 1, 6 } is a dominating set.

γ (G) = 2.

{ 1, 6 } is a (1,2) – dominating set.

γ (1,2) = 2.

{ 1, 3, 6, 7, 8 } is a (1, 2) – double dominating set.

$\gamma_{d(1,2)} = 5$.

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

Remark 3.3.1 : In all the above examples , we conclude the following

i) domination number is less than (1, 2) – double domination number .

ii) double domination number is less than (1, 2) – double domination number .

iii) (1, 2) – domination number is less than (1, 2) – double domination number .

From the above examples we have the following theorem.

Theorem 3.3.1 : In a graph G, domination number is less than or equal to (1, 2) – double domination number.

Proof : Let G be a graph and D be its double dominating set. Then every vertex in V – D is adjacent to a vertex in D. That is, in D , for every vertex u, there is a 2 vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1, 2) – dominating set, it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to (1,2) – domination number.

Theorem 3.3.2 : In a graph G, (1, 2) – domination number is less than or equal to (1, 2) – double domination number.

Proof : Similar to theorem 3.3.1.

Theorem 3.3.3 : In a graph G, double domination number is less than or equal to (1, 2) – double domination number.

Proof : Similar to theorem 3.3.1.

Theorem 3.3.4 : If G is a 2-regular graph, then the (1, 2) – double domination number of the corona of G is equal to the number of vertices of G.

Proof : Let G be a 2- regular graph. Then each of its vertices will be of degree 2. In the corona of G, for each vertex x, we introduce a new vertex and join them. Consequently, an edge is added to each of its vertices. By the definition of (1,2) – double dominating set each vertex v in V – S has atleast two vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v. Hence (1,2) – double dominating set of the corona of G will consist of all the vertices of G.

Theorem 3.3.5 : If in a graph G, an edge e is added, $\gamma_{d(1,2)}(G + e) \geq \gamma_{d(1,2)}(G)$.

Proof: Let G be a graph. Let S be the (1, 2) – double dominating set of G. If we add an edge to a vertex in S, that will not affect the cardinality of S. If we add an edge to a vertex in V- S , the cardinality of (1, 2) – double dominating set will increase. Therefore, $\gamma_{d(1,2)}(G + e) \geq \gamma_{d(1,2)}(G)$.

Theorem 3.3.6 : If G is a complete bipartite graph, then the (1, 2) – double domination number $\gamma_{d(1,2)}$ is 3.

Proof: Let G be a complete bipartite graph. Then V (G) can be partitioned in to 2 disjoint sets X and Y and each edge has one end in X and other end in Y. Since G is complete bipartite, each vertex of X is joined to every vertex in Y. A set of 2 vertices, one from X and another from Y will constitute a (1, 2) – double dominating set. Therefore, $\gamma_{d(1,2)} = 3$.

VI. CONCLUSIONS

We considered the problem of finding a (1, 2) – double dominating set in graphs and compared them with the domination number. Also some preliminary theorems on (1, 2) - dominating sets are proved.

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