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γ - Splitting Graphs

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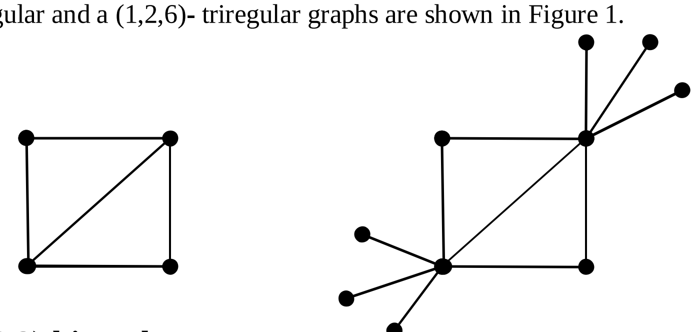
Abstract--Let G(V,E) be a graph. A dominating set is a subset S of V such that every vertex not in S is adjacent to at least one vertex in S. The cardinality of a minimum dominating set is called the domination number, $\gamma(G)$. A dominating set with γ vertices is called a γ-set. Let η denote the number of γ-sets in G. For a graph G, the splitting graph S(G), is obtained by adding a new vertex v′ corresponding to each vertex v of G and joining v′ to all vertices which are adjacent to v in G. Here we introduce a new type of graphs called minimum domination splitting graphs or simply γ-splitting graphs. Let G be a graph and let $S_1, S_2, \ldots, S_\eta$ be the γ-sets in G. The γ-splitting graph, $S_\gamma(G)$, of a graph G is the graph obtained from G by adding new vertices $w_1, w_2, \ldots, w_\eta$ and joining w_i to each vertex in S_i where $1 \le i \le \eta$. In this paper, we establish some results on γ-splitting graphs.
Keywords: Dominating set, domination number, splitting graph, γ-splitting graph.
AMS Subject Classification Code(2010): 05C(Primary)

I. INTRODUCTION

Throughout this paper, we consider only finite, simple, undirected graphs. For notations and terminology we follow [3]. Let G(V,E) be a graph of order n. We denote the *cycle* on n vertices by C_n, the *path* of n vertices by P_n, and the *complete graph* on n vertices by K_n. The *complete bipartite* graph is denoted by $K_{m,n}$. In a graph G, *degree* of a vertex v is denoted by d(v). If S is a subset of V, then <S> denotes the *vertex induced subgraph* of G induced by S. For any vertex $v \in V(G)$, the *open neighbourhood* N(v) of V(G) is the set of all vertices adjacent to v, that is, $N(v) = \{u \in V(G) \,/\, uv \in E(G)\}$, and the *closed neighbourhood* of v is defined by $N[v] = N(v) \bigcup \{v\}$. $N^c(v) = V - N(v)$ is called the *neighbourhood complement*. For any set S, $N(S) = \bigcup_{v \in S} N(v)$.

A *full vertex* of G is a vertex in G which is adjacent to all other vertices of G. A graph G is said to be *r-regular* if every vertex in G is of degree r. For any two integers k and d, $k \neq d$, a *(k,d)- biregular* graph is a graph in which every vertex is of degree either k or d. For any three integers x, a, and b, $x \neq a \neq b$, a *(x,a,b)- triregular* graph is a graph in which every vertex is of degree either x or a or b. For example, a (2,3)-biregular and a (1,2,6)- triregular graphs are shown in Figure 1.



(2,3)-biregular **(1,2,6)- triregular**

Figure 1

The *distance* d(u,v) in G between two vertices u and v is the length of a shortest u-v path in G. The *eccentricity* e(u), of a vertex u is the distance of a farthest vertex from u, and *radius* rad(G) of G is the minimum eccentricity. The maximum distance between any two vertices in G is the *diameter* of G, denoted by diam(G), that is, $\text{diam}(G) = \max_{u,v \in V(G)} \{d(u,v)\}$. A vertex u with e(u) = rad(G) is called a *central* vertex. A graph G for which rad(G) = diam(G) is called a *self-centered graph* of radius rad(G). Or equivalently, a graph is *self-centered* if all of its vertices are central vertices. For further basic definitions on distance in graphs one can refer [4].

Let $H_{n,n}$ denote the graph with vertex set $\{v_1, v_2, \ldots, v_n ; u_1, u_2, \ldots, u_n\}$ and edge set $\{v_i u_j \,/\, 1 \le i \le n, n-i+1 \le j \le n\}$. The graph $B_{m,n}$ is the *bistar* obtained from the stars $K_{1,m}$ and $K_{1,n}$ by joining their central vertices by means of an edge. For example, the graph $H_{4,4}$

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and the bistar $B_{4,5}$ are shown in Figure 2.

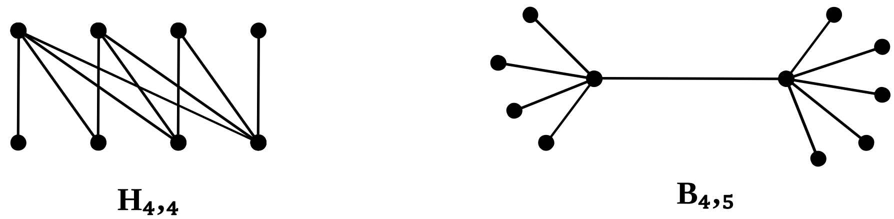


Figure 2

The *join* $G \vee H$ of the graph G and H is the graph obtained from $G \cup H$ by joining every vertex of G to each vertex of H by means of an edge. The graph $W_n = C_{n-1} \vee K_1$ is called the *wheel* graph on n vertices. The *corona* $G \circ H$ of two graphs G and H is obtained by taking one copy of G and $|V(G)|$ copies of H, and by joining each vertex in the i^{th} copy of H to the i^{th} vertex of G, where $1 \leq i \leq |V(G)|$. The corona graph $C_5 \circ K_2$ is depicted in Figure 3, for reference,

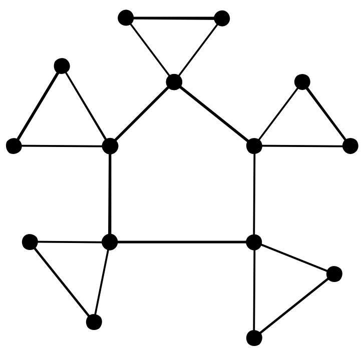
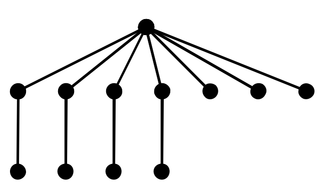


Figure 3

In a graph G, the process of deleting an edge uv and introducing a new vertex w and the edges uw and vw is called the *subdivision of the edge* uv. A *spider* is a tree on 2n + 1 vertices obtained by subdividing each edge of a star $K_{1,n}$. In other words, spider is nothing but $K_{1,n} \circ K_1$. A *wounded spider* is a graph obtained from subdividing at most n – 1 edges of a star $K_{1,n}$. The wounded spider includes K_1, the star $K_{1,n-1}$. For example, a wounded spider G the graph shown in Figure 4. The *cartesian product* of two graphs G_1 and G_2 is denoted by $G_1 \times G_2$. The graph $K_{1,m} \times P_2$ is called the *m-book* graph and it is denoted by B_m. For example, the book graph B_4 is shown in Figure 5.



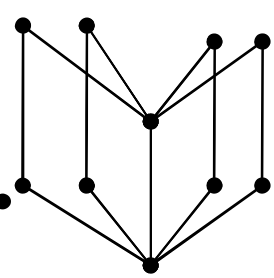


Figure 4 **Figure 5**

A *dominating set* is a subset S of the vertex set V such that every vertex is either in S or adjacent to a vertex in S, that is, such that every vertex in V-S is adjacent to at least one vertex in S. The *domination number* is the number of vertices in a smallest dominating set of G, it is denoted by $\gamma(G)$. A dominating set with γ elements is called a γ-set. For example, $S_1 = \{b,d\}$ and $S_2 = \{a,c\}$ are the minimum dominating sets of the graph G can be verified in Figure 6. For further results on domination in graphs, one can refer [5].

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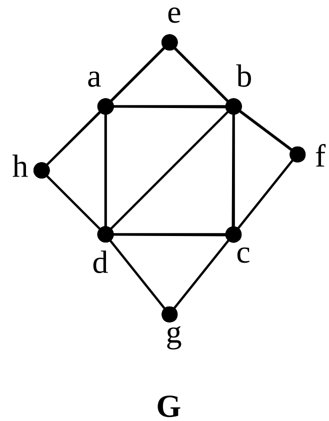

Figure 6

Note that $S_3 = \{a,b,c,d,e,f,g,h\}$ and $S_4 = \{a,b,c,d\}$, etc., are also dominating sets in G. The concept of splitting graph was introduced by Sampath Kumar and Walikar [6]. The *splitting graph* S(G), is the graph obtained from G, by adding a new vertex w for every vertex v $\in$ V(G), and joining w to all vertices of G adjacent to v. For example, a graph G and its splitting graph S(G) are shown in Figure 7.

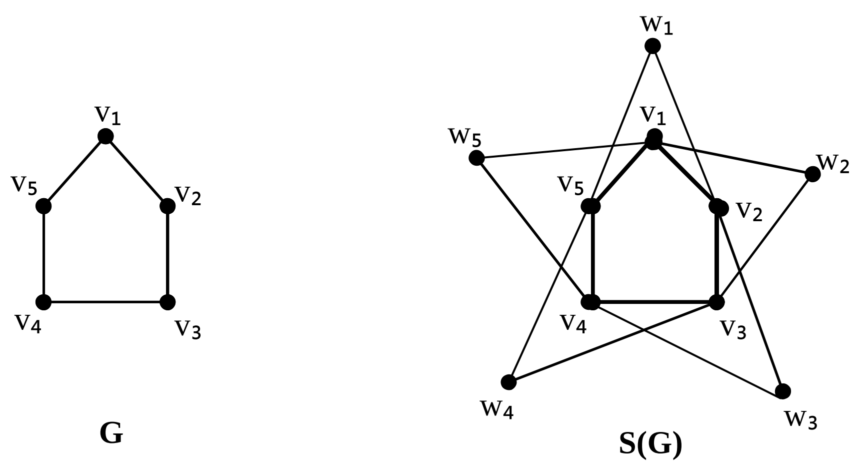

Figure 7

The concept of cosplitting graphs has been recently introduced by Selvam Avadayappan and M. Bhuvaneshwari [1]. Let G be a graph with vertex set $\{v_1, v_2,\ldots,v_n\}$. The *cosplitting graph* CS(G) is the graph obtained from G, by adding a new vertex w_i for each vertex v_i and joining w_i to all vertices which are not adjacent to v_i in G. As an illustration, a graph G and its cosplitting graph CS(G) are shown in Figure 8.

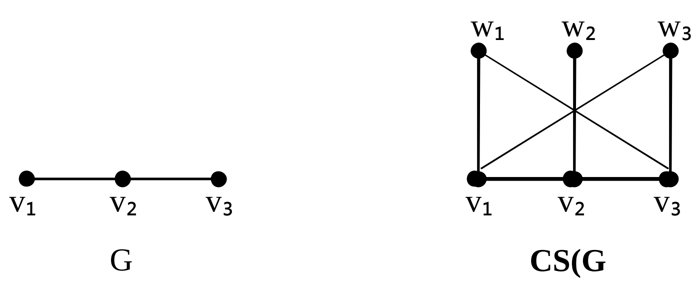

Figure 8

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The concept of β-splitting graph has been introduced by Selvam Avadayappan, M. Bhuvaneshwari and B. Vijaya Lakshmi [2]. Let $S_1, S_2, \ldots, S_\rho$ be the maximum independent sets of G. The β-splitting graph $S_\beta(G)$ of a graph G is a graph obtained from G by adding new vertices $w_1, w_2, \ldots, w_\rho$ such that each w_i is adjacent to each vertex in S_i, for $1 \leq i \leq \rho$. For example, a graph G and its β-splitting graph $S_\beta(G)$ are shown in Figure 9.

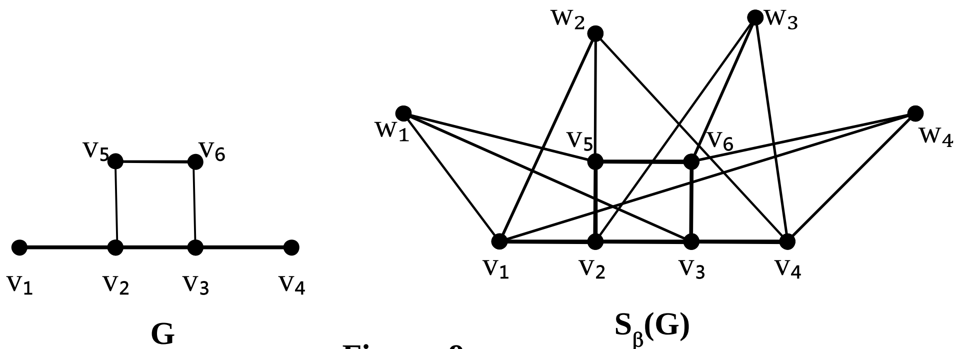

Figure 9

In this paper, we introduce a new type of splitting graphs called γ- splitting graphs. Let G be a graph and let η be the number of γ-sets in G. Let $S_1, S_2, \ldots, S_\eta$ be the minimum dominating sets in G. The γ-splitting graph, $S_\gamma(G)$, of a graph G is the graph obtained from G by adding new vertices $w_1, w_2, \ldots, w_\eta$ and joining w_i to each vertex in S_i where $1 \leq i \leq \eta$. For example, the γ- splitting graph of P_4 is shown in Figure 10.

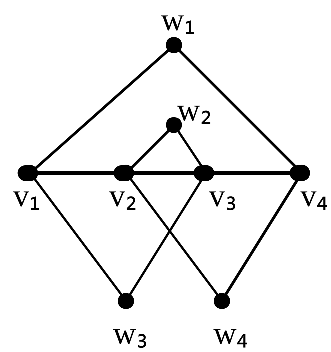

Figure 10

Clearly, $S_1 = \{v_1,v_4\}$, $S_2 = \{v_2,v_3\}$, $S_3 = \{v_1,v_3\}$, $S_4 = \{v_2,v_4\}$ are the γ-sets in P_4, also w_1,w_2,w_3,w_4 are newly added vertices in $S_\gamma(P_4)$. Here, we discuss a few results on γ-splitting graphs. In this paper, we independently characterise graphs for which $S_\gamma(G)$ is a regular, biregular, tree, unicyclic graph. We attain bounds for the maximum and minimum degree of a vertex in $S_\gamma(G)$. Finally we study the distance properties of γ-splitting graphs.

II. CHARACTERISATION OF γ-SPLITTING GRAPHS

The following facts can be easily verified for γ-splitting graphs. For a vertex v in $S_\gamma(G)$, let $d^*(v)$ denote the degree of v in $S_\gamma(G)$.

Fact 2.1 The newly added vertices $\{w_1,w_2,\ldots,w_\eta\}$ are independent in $S_\gamma(G)$, that is, $d(w_i,w_j) \geq 2$, for any i, j, $1 \leq i, j \leq \eta$.

Fact 2.2 $d^*(w_i) = \gamma(G)$, for i, $1 \leq i \leq \eta$.

Fact 2.3 For any vertex $v \in V(G)$, $d(v) \leq d^*(v)$.

Fact 2.4 Every graph G is an induced subgraph of $S_\gamma(G)$. Even more G is a proper subgraph of $S_\gamma(G)$, since every graph contains at least one γ-set.

Fact 2.5 The graph having only one full vertex, bistar graph, the graph $H_{n,n}$, the path P_{3k}, $k \geq 1$ and the book graph B_m are some

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graphs whose γ-splitting graphs contain exactly one newly added vertex.

Fact 2.6 $S_\gamma(K_n) \cong K_n \circ K_1$ for any $n \geq 1$.
Fact 2.7 $S_\gamma(K_{1,n}) \cong K_{1,n+1}$ for any $n \geq 2$.
Fact 2.8 $S_\gamma(K_n^c) \cong K_{1,n}$ for any $n \geq 1$.

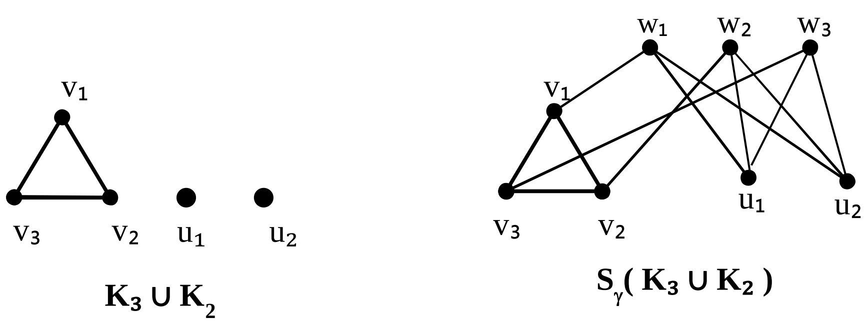
The following theorems establish some properties of γ-splitting graphs.

Proposition 2.9 For any $m \geq 1$ and $n \geq 1$, $\eta(K_{m,n}) = \begin{cases} 1 & \text{if } m = 1, n \geq 2 \\ 2 & \text{if } m = n = 1 \\ 6 & \text{if } m = n = 2 \\ mn+1 & \text{if } m = 2, n > 2 \\ mn & \text{if } m \geq 3, n \geq 3. \end{cases}$

Proof Let $V = \{u_1,u_2,\ldots,u_m ; v_1,v_2,\ldots,v_n\}$ be the vertex set of $K_{m,n}$.
Case (i) Suppose $m = n = 1$, then clearly $\{u_1\}$ and $\{v_1\}$ are only the γ-sets and hence $\eta(K_{m,n}) = 2$.
Case (ii) If $m = n = 2$, then clearly $\{u_1,v_1\},\{u_2,v_2\},\{u_1,v_2\},\{u_2,v_1\},\{u_1,u_2\}$and $\{v_1,v_2\}$ are the only γ-sets in $K_{2,2}$ and hence $\eta(K_{m,n}) = 6$.
Case (iii) If $m = 1$ and $n \geq 2$, then $G \cong K_{1,n}$, and therefore $\{u_1\}$ is the only γ-set. That is, $\eta(G) = 1$.
Case (iv) Suppose $m = 2$ and $n > 2$. Then $\{u_1,u_2\}$ and $\{u_j,v_k\}$ $1 \leq j \leq 2$, $1 \leq k \leq n$ are the γ-sets of G. Thus $\eta(K_{m,n}) = mn+1$.
Case (v) If $m \geq 3$ and $n \geq 3$, then clearly $\{u_i,v_k\}$ $1 \leq i \leq m$, $1 \leq k \leq n$. Thus $\eta(K_{m,n}) = mn$. ■

Theorem 2.10 For any $n \geq 1$, there exists a graph G of order n, such that $S_\gamma(G)$ is n-regular.

Proof When $n = 1$, $G \cong K_1$, for which $S_\gamma(G) \cong K_2$ is the required graph. Therefore assume that $n \geq 2$, consider the graph $G \cong K_n \cup K_{n-1}^c$ with vertex set $\{v_1,v_2,\ldots,v_n; u_1,u_2,\ldots,u_{n-1}\}$ with edge set $\{v_iv_j / 1 \leq i, j \leq n\}$. For any i, $1 \leq i \leq n$, clearly $\{v_i, u_1,u_2,\ldots, u_{n-1}\}$ is a γ-set of G, that is, $\gamma(G) = n$. Hence there are n such γ-sets in G. Let $w_1,w_2,\ldots,w_n$ be the newly added vertices in $S_\gamma(G)$. Now for any i, j, $1 \leq i \leq n$, $1 \leq j \leq n-1$. Thus $d^*(v_i) = d^*(w_i) = d^*(u_j) = n$. Hence $S_\gamma(G)$ is n-regular. Thus G is the required graph. For example, the graph $K_3 \cup K_2^c$ and $S_\gamma(K_3 \cup K_2^c)$ which is a 3-regular graph are shown in Figure 11.



Figure

Now, consider the star graph $K_{1,n-1}$, $n \geq 3$, which is biregular. In addition $S_\gamma(K_{1,n-1})$ is also biregular. This shows that there are biregular graphs G whose $S_\gamma(G)$ are also biregular. Some examples are listed below:

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Graph G	Degree set of G	$S_\gamma(G)$	Degree set of $S_\gamma(G)$
$K_{1,n-1}$, n ≥3	{1, n-2}	$K_{1,n}$	{1, Δ(G)+1}
P_5	{1, 2}	$S_\gamma(P_5)$	{2, Δ(G)+2}
B_m	{2, m+1}	$S_\gamma(B_m)$	{2, Δ(G)+1}

Theorem 2.11 The graph $S_\gamma(K_{m,n})$ is biregular if m = n and $S_\gamma(K_{m,n})$ is triregular if m ≠ n for m ≥ 2.

Proof Let V = $\{v_1,v_2,\ldots,v_m; u_1,u_2,\ldots,u_n\}$ be the vertex set of $K_{m,n}$.

Case (i) Suppose m = n, and m ≥ 3. The graph $S_\gamma(K_{m,m})$, then $d^*(w_i) = 2$. Also, by Proposition 1, $\eta = m^2$. Each u_i or v_i belongs to exactly m γ-sets. Hence $d^*(u_i) = d^*(v_i) = 2m$. Then $S_\gamma(K_{m,m})$ is a (2m, 2)-biregular graph when m = n.

Case (ii) Let m ≠ n. The graph $S_\gamma(K_{m,n})$, then $d^*(w_i) = 2$, and η = mn. Each u_i belongs to n γ- sets and each v_i belongs to m γ-sets. Then $d^*(u_i) = 2n$ and $d^*(v_i) = 2m$. Hence $S_\gamma(K_{m,n})$ is a (2m, 2n, 2)-triregular graph when m ≠ n. Hence the proof. For example, the graph $K_{2,2}$ and $S_\gamma(K_{2,2})$ are shown in Figure 12.

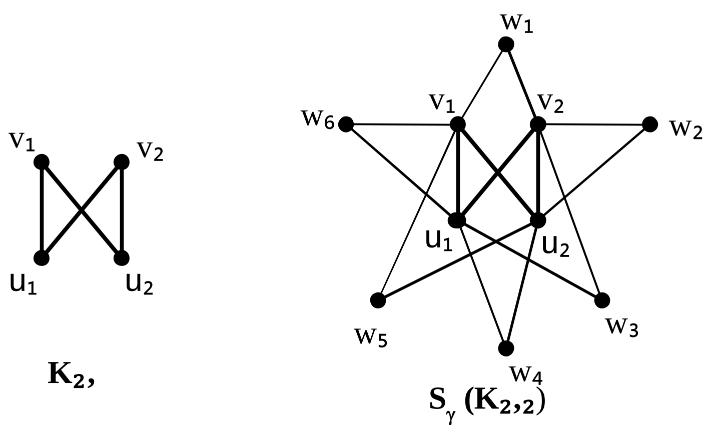


Figure 12

Theorem 2.12 The graph $S_\gamma(G)$ is a tree if and only if G is one among the following graphs K_n^c, P_2, $\left(\bigcup_{i=1}^{k} K_{1,n_i}\right) \cup K_m^c$, k ≥ 1, n_i ≥ 2, m ≥ 1, or $\bigcup_{i=1}^{k} K_{1,n_i}$, k ≥ 1, n_i ≥ 2.

Proof Consider a graph G for which $S_\gamma(G)$ is a tree. Since G is an induced subgraph of $S_\gamma(G)$, G is acyclic. If G contains only two vertices, then obviously G ≅ K_2 or K_2^c for which $S_\gamma(G)$ ≅ P_4 or P_3 respectively. So we assume that G contains at least three vertices.

Case (i) Suppose G is a tree. Then G contains at most one full vertex. If G contains only one full vertex, then G ≅ $K_{1,n}$ for which $S_\gamma(G)$ ≅ $K_{1,n+1}$. If G contains no full vertex, then γ(G) > 1 and thus G contains at least two vertices u and v in any γ-set S of G. Let w be the newly added vertex in $S_\gamma(G)$, corresponding to S. Now the u-v path together with the edges uw and wv forms a cycle in $S_\gamma(G)$, which is a contradiction to our assumption that $S_\gamma(G)$ is a tree. Therefore, this case does not arise.

Case (ii) Let G be a forest. If a γ-set contains at least two vertices in the same component, then $S_\gamma(G)$ contains a cycle, which is a contradiction. Therefore every component must contain exactly one vertex of each γ-set of G, which is possible when each component is a star or a trivial graph and hence $G \cong \left(\bigcup_{i=1}^{k} K_{1,n_i}\right) \cup K_m^c$, $k \geq 1$, $n_i \geq 2$ and $m \geq 1$ or $G \cong \bigcup_{i=1}^{k} K_{1,n_i}$, $k \geq 1$, $n_i \geq 2$.

And the converse is obvious. ■

For example, the graph $S_\gamma\left(\bigcup_{i=1}^{3} K_{1,3}\right)$ and $S_\gamma\left(\left(\bigcup_{i=1}^{2} K_{1,3}\right) \cup K_3^c\right)$ are shown in Figure 13.

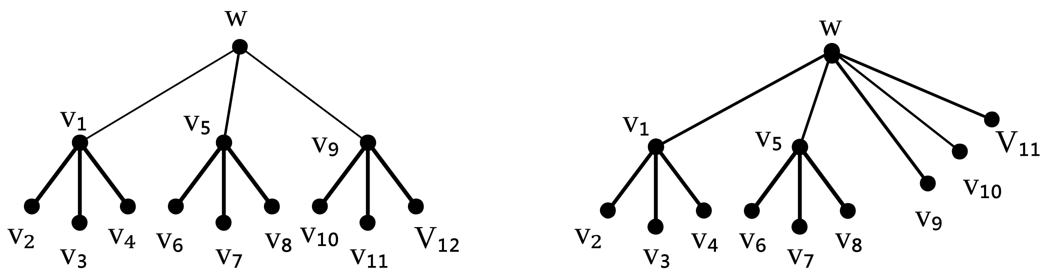
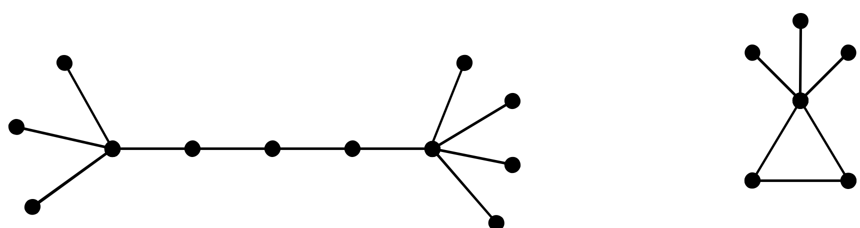


Figure 13

Let $P_k(m,n)$, where $k \geq 2$ and $m,n \geq 1$, be the graph obtained by identifying the centre vertices of the stars $K_{1,m}$ and $K_{1,n}$ at the ends of P_k respectively. The graph $C_3(m_1,m_2,m_3)$, where $m_i \geq 0$, is obtained from the cycle $C_3 = v_1v_2v_3v_1$ by identifying the centre of the star K_{1,m_i}, at v_i of C_3, for $1 \leq i \leq 3$. For example, the graph $P_5(3, 4)$ and $C_3(3, 0, 0)$ are shown in Figure 14.



$P_5(3, 4)$ **$C_3(3, 0, 0)$**

Figuer 14

Theorem 2.13 The graph $S_\gamma(G)$ is unicyclic if and only if G is isomorphic to any one of the following graphs: (i) $P_2 \cup K_1$, (ii) K_3, (iii) $B_{m,n}$, $m > 1$, $n > 1$, (iv) $P_k(m,n)$, $k = 3, 4$ and $m,n \geq 1$, (v) $B_{m,n} \cup K_t^c$, $m > 1$, $n > 1$, $t \geq 1$, (vi) $P_k(m,n) \cup K_t$, $k = 3, 4$ and $m, n \geq 1$, $t \geq 1$, (vii) $C_3(m_1,0,0) \bigcup_{p=0}^{r} pK_{1,n} \bigcup_{q=0}^{s} qK_n^c$ where $m_1 \geq 1$.

Proof Consider the graph G for which $S_\gamma(G)$ is unicyclic. Then there arise two cases.

Case (i) Suppose G is acyclic. Then clearly the cycle contains a newly added vertex w in $S_\gamma(G)$. Therefore, $\gamma(G) \neq 1$. Let G be a connected graph. Then $\eta = 1$, that is, G contains exactly one γ-set, since every newly added vertex forms a new cycle. In particular, $\gamma(G) = 2$ with the γ-set {u,v}. Let w be the newly added vertex in $S_\gamma(G)$. Then the (u,v)-path in G together with the newly added edges wu and vw forms the unique cycle in $S_\gamma(G)$, this is possible only when $G \cong B_{m,n}$, $m > 1$, $n > 1$, $P_k(m,n)$, $k = 3, 4$ and $m, n \geq 1$.

Let G be disconnected. If G has more than one component, with at least one edge, then $S_\gamma(G)$ has more cycles, which is a contradiction to our assumption that $S_\gamma(G)$ is unicyclic. Hence only one component G_1 of G can contain edges and the others are isolated vertices. If G_1 contains only one edge, then G must be $P_2 \cup K_1$. If G_1 contains more than one edge, then G_1 is isomorphic to $B_{m,n}$, $m > 1$, $n > 1$, $P_k(m,n)$, $k = 3,4$ and $m,n \geq 1$ and hence $G \cong B_{m,n} \cup K_t^c$, $m > 1$, $n > 1$, $t \geq 1$, $P_k(m,n) \cup K_t^c$, $k = 3, 4$ and $m, n \geq 1$, $t \geq 1$.

Case (ii) Suppose G is unicyclic. Let G be a connected graph. Then newly added edges cannot be in a cycle. This is possible only

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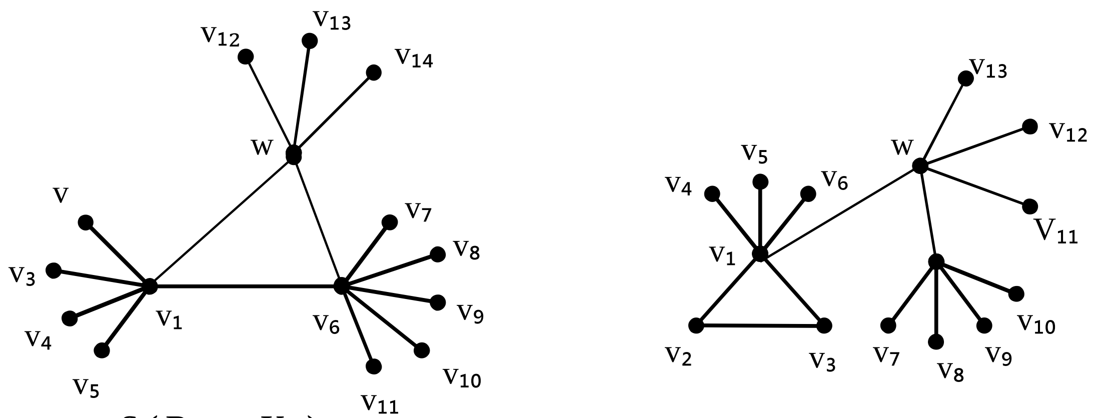
when $\gamma(G) = 1$. This forces that $G \cong K_3$ or $C_3(m_1,0,0)$ where $m_1 \geq 1$.

Let G be disconnected graph. Then $\omega(G) \geq 2$. Clearly, one of the component of G is unicyclic and the remaining are trees. Since every component is connected, by the above argument exactly one vertex of each component belongs to γ-set of G. Also, the γ-set must be unique to avoid cycles formed by newly added vertices. Such a graph is isomorphic to $C_3(m_1,0,0) \bigcup_{p=0}^{r} pK_{1,n} \bigcup_{q=0}^{s} qK_n^c$ where $m_1 \geq 1$. And the converse is obvious. ■

For example, the graphs $S_\gamma(B_{4,5} \cup K_3^c)$ and $S_\gamma(C_3(3, 0, 0) \cup K_{1,4} \cup K_3^c)$ are shown in Figure 15.



$S_\gamma(B_{4,5} \cup K_3)$ $S_\gamma(C_3(3, 0, 0) \cup K_{1,4} \cup K_3)$

Figure

Theorem 2.14 Let G be a graph. Then $S_\gamma(G)$ has a full vertex if and only if $G \cong K_n^c$ or $H \vee K_1$ where H is a graph without a full vertex.

Proof Let w_i be the newly added vertices in $S_\gamma(G)$ for $1 \leq i \leq \eta$. Let v be a full vertex in $S_\gamma(G)$.

Case (i) Suppose v is a newly added vertex. Since w_i's are all independent in $S_\gamma(G)$, v is the only newly added vertex. And hence V(G) is the only dominating set of G. This is possible only when $G \cong K_n^c$.

Case (ii) Let $v \in V(G)$. Then v is a full vertex of G. If G has a full vertex u other then v, then there are w_1 and w_2 corresponding to the γ-sets {u} and {v}. But w_1 and w_2 are not adjacent. In addition uw_2 and vw_1 are not the edges in $S_\gamma(G)$. Thus $S_\gamma(G)$ contains no full vertices, a contradiction. Therefore, G has exactly one full vertex. In other words, $G \cong H \vee K_1$ where H has no full vertex.

Conversely, assume that $G \cong H \vee K_1$. The graph $S_\gamma(G)$ is nothing but a graph obtained from $H \vee K_1$ by adding a new vertex and join it to the vertex of K_1. Also $S_\gamma(K_n^c) \cong K_{1,n}$. In both the cases, $S_\gamma(G)$ has a full vertex. Hence the proof.

Proposition 2.15 For any connected graph G, $\Delta(G) \leq \Delta(S_\gamma(G)) \leq \max\{\Delta(G) + \eta, \gamma\}$.

Proof Let v be a vertex of maximum degree in $S_\gamma(G)$. If v is a newly added vertex, then $\Delta(S_\gamma(G)) = \gamma$. Otherwise, if $v \in V(G)$, then there arise two cases. When $v \notin \bigcup S_i$, $1 \leq i \leq \eta$, then $\Delta(S_\gamma(G)) = \Delta(G)$. When $v \in \bigcap S_i$, $1 \leq i \leq \eta$, $\Delta(S_\gamma(G)) = \Delta(G) + \eta$. Hence the maximum degree of the graph $S_\gamma(G)$ varies as, $\Delta(G) \leq \Delta(S_\gamma(G)) \leq \max\{\Delta(G) + \eta, \gamma\}$. Hence the proof.■

For any $n \geq 6$, there exists a graph of order n with $\Delta(S_\gamma(G)) = \gamma(G)$, P_{3k}, $k \geq 2$ is one such a graph. Also the spider graph proves the existence of graphs with $\Delta(S_\gamma(G)) = \Delta(G)$. The wounded spider graph stands as an example of graphs with $\Delta(S_\gamma(G)) = \Delta(G) + \eta$. For example the graphs G_1, G_2, G_3 with $\Delta(S_\gamma(G_1)) = \Delta(G_1)$, $\Delta(S_\gamma(G_2)) = \Delta(G_2) + \eta$ and $\Delta(S_\gamma(G_3)) = \gamma(G)$ respectively are shown in Figure 16. Here G_1 is the spider graph on 9 vertices, G_2 is the wounded spider graph on 5 vertices and G_3 is the path graph on 12 vertices.

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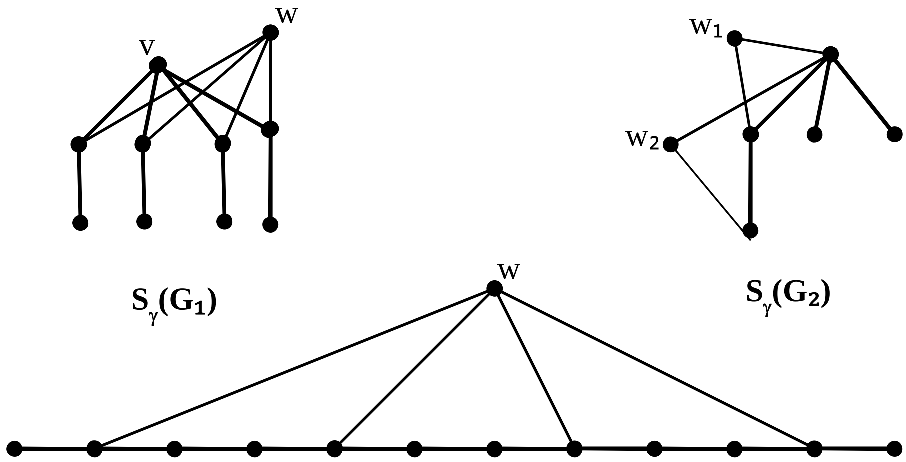


$S_\gamma(G_3)$.

Figure 16

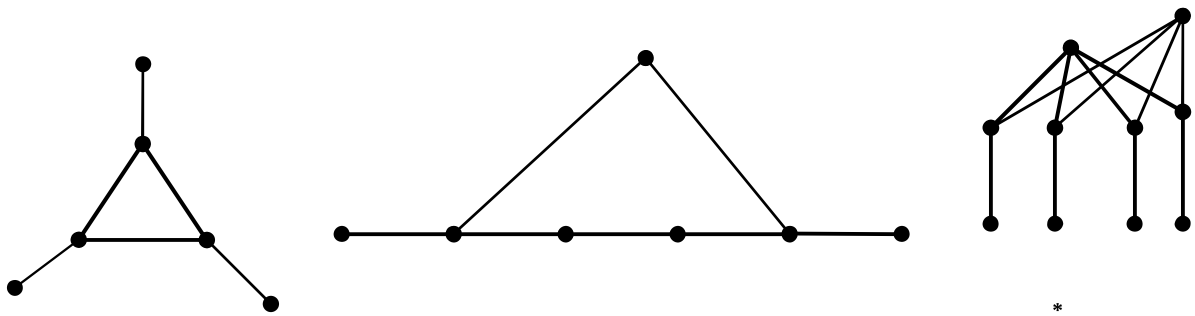
III. DISTANCE PROPERTIES OF γ-SPLITTING GRAPHS

Here we are interested in studying about the distance properties in S_γ-graphs. Also normally we expect diam(S_γ(G)) < diam(G). But there are graphs with diam(S_γ(G)) ≥ diam(G). This behaviour gives rise to following three definitions S_γ^+-graphs, S_γ^--graphs, and S_γ^*-graphs as given below:

A graph G is called a *S_γ^+-graph* if diam(G) < diam(S_γ(G)).

It is called a *S_γ^--graph* if diam(G) > diam(S_γ(G)).

Finally, it is said to be a *S_γ^*-graph* if diam(G) = diam(S_γ(G)). For example, S_γ^+, S_γ^-, and S_γ^*-graphs are shown in Figure 17.



S_γ^+-graph **S_γ^--** **S_γ^*-graph**

Figure 17

Some standard graphs with their diameters and corresponding families are listed below:

Graph G	diam(G)	diam($S_\gamma(G)$)	Type
K_n	1	3	S_γ^+-graph
$K_{m,n}$	2	3	S_γ^+-graph
W_3	1	2	S_γ^+-graph
Graph with exactly one full vertex	2	2	S_γ^*-graph
$H_{n,n}$	3	3	S_γ^*-graph
B_m	3	3	S_γ^*-graph
$B_{m,n}$	3	3	S_γ^*-graph
Spider	4	4	S_γ^*-graph

Theorem 3.1 For any graph G, the distance between newly added vertices in $S_\gamma(G)$ is 2 or 3.

Proof Let G be any graph of order n, and w_1 and w_2 be any two newly added vertices in $S_\gamma(G)$. We know that $d^*(w_i) = \gamma(G)$, $1 \leq i \leq \eta$ and $d(w_1,w_2) \geq 2$ (Fact 2.1).

Case (i) Suppose $N(w_1) \bigcap N(w_2) \neq \varphi$. Let $x \in N(w_1) \cap N(w_2)$. Then x is the common neighbour of w_1 and w_2, and so $d(w_1,w_2) = 2$.

Case (ii) Suppose $N(w_1) \bigcap N(w_2) = \varphi$. Then let $x \in N(w_1)$. Since $N(w_1)$ is a γ-set, every vertex in $N^c(w_1)$ is adjacent to at least one vertex in $N(w_1)$. But $N(w_2) \subseteq N^c(w_1)$. Therefore, there exists a vertex $y \in N(w_2)$ such that y is adjacent to a vertex x in $N(w_1)$. Then $d(w_1,w_2) = 3$. ■

Theorem 3.2 For any graph G, $diam(S_\gamma(G)) \leq 4$.

Proof Let G be any graph and $S_\gamma(G)$ be its corresponding γ-splitting graph. Let u and v be any two vertices in $S_\gamma(G)$. We claim that $d(u,v) \leq 4$ for every $u,v \in V(G)$.

Case (i) If u and v are newly added vertices in $S_\gamma(G)$. By Theorem 3.1, $d(u,v) \leq 3$.

Case (ii) If u is a newly added vertex and $v \in V(G)$. Then N(u) is a dominating set, and therefore $v \in N(u)$ or v is adjacent to a vertex in N(u) in $S_\gamma(G)$. This forces that $d(u,v) \leq 2$.

Case (iii) Suppose $u, v \in V(G)$. Then there arise two subcases.

Subcase (i) Let u belong to a γ-set S. Then there is a newly added vertex w corresponding to S. If $v \in S$, then uwv is a u-v path of length 2 in $S_\gamma(G)$. Therefore $d(u,v) \leq 2$. If $v \notin S$, then there is a vertex v_1 in S, adjacent to v. Therefore uwv_1v is a u-v path of length 3, and so $d(u,v) \leq 3$. If v belongs to any other γ-set, then in a similar way we can show that $d(u,v) \leq 3$.

Subcase (ii) Neither u nor v belongs to any γ-set. Fix a newly added vertex w. Clearly, N(w) is a γ-set. So $V(G) \subseteq N(N(w))$ in $S_\gamma(G)$. Therefore, $d(u,v) \leq 4$. Hence $diam(S_\gamma(G)) \leq 4$. ■

The inequality stated above is strict. For example, $diam(S_\gamma(P_{3k})) = 4$, for any $k \geq 2$. For example, $diam(S_\gamma(P_6)) = 4$ can be verified in Figure 18.

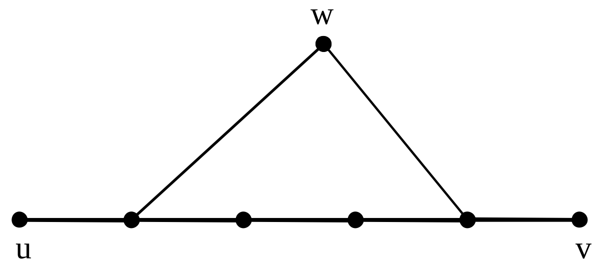


Figure 18

The following corollary gives a characterisation of S_γ^--graphs.

Corollary 3.3 Any connected graph G with diam(G) > 4, is a S_γ^--graph.

Proof Suppose G is a connected graph and diam(G) > 4. Let $S_\gamma(G)$ be its corresponding γ-splitting graph. By Theorem 3.2, diam($S_\gamma(G)$) ≤ 4 and the result follows. ■

It has been prove in [7], that

$$\eta(P_n) = \begin{cases} 1 & \text{if } n = 3k,\ k \geq 1 \\ \dfrac{k^2+5k+2}{2} & \text{if } n = 3k+1,\ k \geq 0 \\ k+2 & \text{if } n = 3k+2,\ k \geq 0 \end{cases}$$

and

$$\eta(C_n) = \begin{cases} 3 & \text{if } n = 3k,\ k \geq 1 \\ \dfrac{(3k+1)(k+2)}{2} & \text{if } n = 3k+1,\ k \geq 1 \\ 3k+2 & \text{if } n = 3k+2,\ k \geq 1 \end{cases}$$

Proposition 3.4 The path graph P_n is S_γ^+-graph if n ≤ 2, S_γ^*-graph if n = 3, 4, and S_γ^--graph if n ≥ 5.

Proposition 3.5 The cycle graph C_n is S_γ^+-graph if n ≤ 5, S_γ^*-graph if n = 6, 7, and S_γ^--graph if n ≥ 8.

REFERENCES

[1] Selvam Avadayappan and M. Bhuvaneshwari, Cosplitting and coregular graphs, International Journal of Mathematics and Soft Computing Vol.5, (2015), 57-64.
[2] Selvam Avadayappan, M. Bhuvaneshwari and B. Vijaya Lakshmi, β-splitting graphs, Preprint.
[3] R. Balakrishnan and K. Ranganathan, A Text Book of graph Theory, Springer-Verlag, New York, Inc(1999).
[4] F. Bunkley and F.Harary, Distance in Graph, Addison-Wesley Reading, 1990.
[5] T.W. Haynes, S.T. Hedetneimi and P.J. Slater, Fundamentals of domination in graphs, Marcel Dekker Inc., New York, (1998).
[6] Sampath Kumar. E, Walikar. H.B, On the Splitting graph of a graph, (1980), J. Karnatak Uni. Sci 25 : 13.
[7] H. B. Walikar, H. S. Ramane, B. D. Acharya, H.S. Shekhareppa, S. Arumugum, Partially balanced incomplete block design arising from minimum dominating sets of path and cycles, AKCE J. Graph. combin., 4(2) (2007), 223-232.