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Dynamic analysis of laminated composite plates and shells by using a new higher-order shear deformation theory and ABAQUS software

Bavane. N. U.[1], Patil. S. B.[2], Shinde. S. S.[3], Kore. S. B.[4], Kamble. R. P.[5]
Sanjay Ghodawat Institute, Atigre- Civil Department, Shivaji University

Abstract— A new higher order shear deformation theory for elastic composite/sandwich plates and shells is developed. The new displacement field depends on a parameter ''m'', whose value is determined so as to give results. Static and dynamic results are presented for cylindrical and spherical shells and plates for simply supported boundary conditions. Results are provided for thick to thin as well as shallow and deep shells. The accuracy of ABAQUS software is verified by comparing it with various available results in the literature.
Keywords— FSDT - First order shear deformation theory, HSDT - Higher order shear deformation theory, Dynamic, Doubly-curved shells.

I. INTRODUCTION

An increasing number of structural designs, especially in the aerospace, automobile, and petrochemical industries are extensively utilizing fiber composite laminated plates and shells as structural elements. The laminated orthotropic shell belongs to the composite shell category. One of the important factors in the analysis of the layered shells is its individual layer properties, which may be anisotropic, orthotropic or isotropic.

A shell is a curved, thin walled structure. Two important classes of shells are plates (shells which are flat when un-deformed) and membranes (shells whose walls offer no resistance to bending). Shells may be made of a single homogeneous or anisotropic material or may be made of layers of different materials.

The primary function of a shell may be to transfer loads from one of its edges to another, to support a surface load, to provide a covering, to contain a fluid, to please the eye or a combination of these.

A thin shell is defined as a shell with a thickness which is small compared to its other dimensions and in which deformations are not large compared to thickness. A primary difference between a shell structure and a plate structure is that, in the unstressed state, the shell structure has curvature as opposed to plates. Membrane action in a shell is primarily caused by in-plane forces (plane stress), though there may be secondary forces resulting from flexural deformations. Where a flat plate acts similar to a beam with bending and shear stresses, shells are analogous to a cable which resists loads through tensile stresses. The ideal thin shell must be capable of developing both tension and compression.

First order shear deformation theory (FSDT) is based on the kinematic field assumption postulated by Mindlin, in which the constant transverse shear strain components along the thickness is accounted. Higher order shear deformation theories (HSDTs) were developed to improve the analysis of shell responses and extensively used by many researchers. Normally, these well-known theories comply with the free surface boundary conditions and account for approximately parabolic distribution of shear stresses through the thickness of the shell.

Many higher order theories were proposed by Kant and Swaminathan, Reddy and Liu, Touratier, Soldatos. The present HSDT is simple in the sense that it contains the same dependent unknowns as in the first order shear deformation theory.

The present theory is based on a displacement field in which the displacements of the middle surface are expanded as a combination of exponential and polynomial functions of the thickness coordinate, and the transverse displacement is assumed to be constant through the shell thickness. The theory is constructed from 3D elasticity bending solutions of plates, by performing several computations of the present shell/plate governing equations, which has ''m'' parameter dependent.

A. Application Of Shells

Thin shell structures are light weight constructions using shell elements. These elements are typically curved and are assembled to large structures. Typical applications are fuselages of aero planes, boat hulls and roof structures in some buildings. Shell structures are mainly used in industrial applications such as automobile, civil, aerospace and petrochemical engineering. Various types of

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shells are used in civil field such as conoid, hyperbolic paraboloid and elliptical paraboloid shell. All are used for roofing to cover large column-free areas. Laminated composites are such type of material which has high strength to weight and strength to stiffness ratios. The mechanical properties of the laminated composites depend on the degree of orthotropy of the layers, ratio of the transverse shear modulus to the in-plane shear modulus and stacking sequence of laminates. Many of the classical theories developed for thin elastic shells are based on the Love-Kirchhoff assumptions in which thenormal to the mid-plane before deformation is considered to be normal and straight after the deformation.

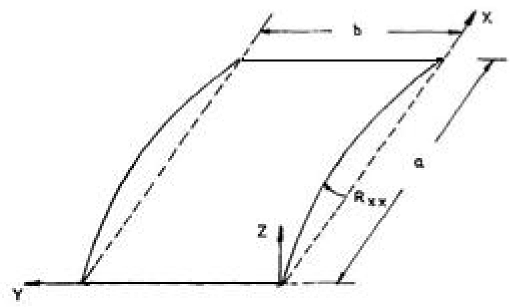

Figure: 1.1 Elliptical Paraboloidal Shell

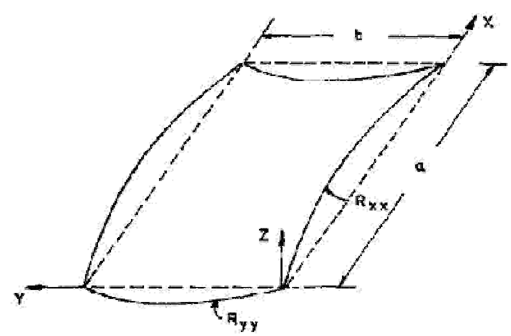

Figure: 1.2 Hyperbolic Paraboloidal Shell

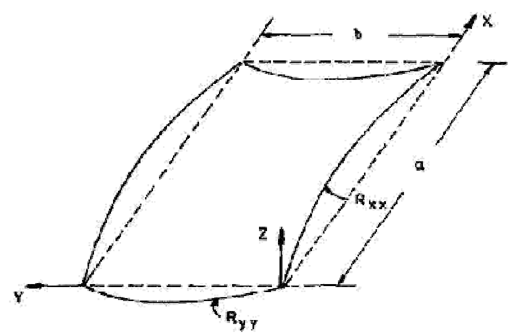

Figure: 1.3 Cylindrical Shell

II. PROBLEM DEFINITION

A. Introduction

In this chapter we are going to discuss about the problem taken for Paper.

B. Details of Problem

Type of Shell - Laminated Composite Cylindrical Shell Lamination Scheme – [$0^o/90^o$]

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Modulus of elasticity - $E_1 = 2*10^5$, $E_2 = 2*10^4$, $E_1/E_2 = 10$, Modulus of rigidity - $G_{12} = G_{13} = 0.6 * E_2 = 1.2*10^4$
$G_{23} = 0.5 * E_2 = 1*10^4$
Poission's Ratio - $\mu = 0.25$,
Mass Density - $\rho = 1$

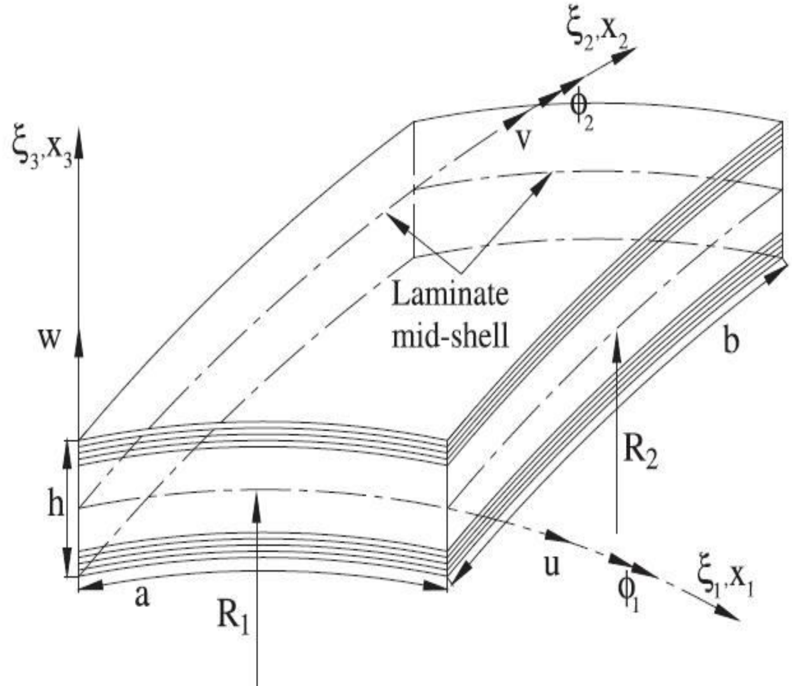

Figure: 2.1 A Cross-ply of laminated shell
Coares C.G., Mantari J.L., and Oktem A.S. (2011)

C. Analytical Method

The equations of motion of first-order shear deformation shell theory (FSDT) of a laminated cylindrical shell are

$$\frac{\partial N_1}{\partial x_1} + \frac{\partial N_6}{\partial x_2} = I_0 \ddot{u}_0 + I_1 \ddot{\phi}_1$$
$$\frac{\partial N_6}{\partial x_1} + \frac{\partial N_2}{\partial x_2} + \frac{Q_2}{R} = I_0 \ddot{u}_0 + I_1 \ddot{\phi}_2$$
$$\frac{\partial M_1}{\partial x_1} + \frac{\partial M_6}{\partial x_2} - Q_1 = I_1 \ddot{u}_0 + I_2 \ddot{\phi}_1$$
$$\frac{\partial M_6}{\partial x_1} + \frac{\partial M_2}{\partial x_2} - Q_2 = I_1 \ddot{v}_0 + I_2 \ddot{\phi}_2$$
$$\frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2} - \frac{N_2}{R} - \hat{N} \frac{\partial^2 w_0}{\partial x_1^2} = I_0 \ddot{w}_0$$

Analytical solutions of these equations can be obtained for simply supported cross-ply laminated shells. Towards using the Navier type solution, first we write the equations of motion in terms of displacements (u0, v0, w0, Ø1, Ø2) by substituting for the force and moment resultants. Then this simply supported boundary conditions are satisfied by the following expansions of the generalized displacement field:

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$$u_0(x_1,x_2,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{mn}(t)\cos\alpha x_1\sin\beta x_2$$
$$v_0(x_1,x_2,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}V_{mn}(t)\sin\alpha x_1\cos\beta x_2$$
$$w_0(x_1,x_2,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{mn}(t)\sin\alpha x_1\sin\beta x_2$$
$$\phi_1(x_1,x_2,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}X_{mn}(t)\cos\alpha x_1\sin\beta x_2$$
$$\phi_2(x_1,x_2,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}Y_{mn}(t)\sin\alpha x_1\cos\beta x_2$$

Substituting the expansions into equations of motion yields the equations,

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & C_{13} & 0 & 0 \\ 0 & 0 & C_{23} & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & C_{43} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot U_{mn} \\ \dot V_{mn} \\ \dot W_{mn} \\ \dot X_{mn} \\ \dot Y_{mn} \end{Bmatrix}$$

$$+ \begin{bmatrix} M_{11} & 0 & 0 & M_{14} & 0 \\ 0 & M_{22} & 0 & 0 & M_{25} \\ 0 & 0 & M_{33} & 0 & 0 \\ M_{41} & 0 & 0 & M_{44} & 0 \\ 0 & M_{52} & 0 & 0 & M_{55} \end{bmatrix} \begin{Bmatrix} \ddot U_{mn} \\ \ddot V_{mn} \\ \ddot W_{mn} \\ \ddot X_{mn} \\ \ddot Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix}$$

For natural vibrations without structural damping, we set all Cij=0 and assume solution of the form

$$U_{mn}(t)=U_{mn}^0 e^{i\omega t},\quad V_{mn}(t)=V_{mn}^0 e^{i\omega t},\quad W_{mn}(t)=W_{mn}^0 e^{i\omega t}$$

$$X_{mn}(t)=X_{mn}^0 e^{i\omega t},\quad Y_{mn}(t)=Y_{mn}^0 e^{i\omega t}$$

Sustituting all these solutions in the equations of motion we get,

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & 0 & 0 & M_{14} & 0 \\ 0 & M_{22} & 0 & 0 & M_{25} \\ 0 & 0 & M_{33} & 0 & 0 \\ M_{41} & 0 & 0 & M_{44} & 0 \\ 0 & M_{52} & 0 & 0 & M_{55} \end{bmatrix} \right) \begin{Bmatrix} U_{mn}^0 \\ V_{mn}^0 \\ W_{mn}^0 \\ X_{mn}^0 \\ Y_{mn}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Which is an eigen value problem. For nontrivial solution, the determinant of matrix in the paranthesis is set to be zero. This gives values of ω2. ω is the fundamental natural freaquency of the shell under the given support condition using first order shear deformation theory (FSDT).

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$$\frac{\partial N_1}{\partial x_1}+\frac{\partial N_6}{\partial x_2}=I_0\ddot{u}_0+I_1\ddot{\phi}_1$$
$$\frac{\partial N_6}{\partial x_1}+\frac{\partial N_2}{\partial x_2}+\frac{Q_2}{R}=I_0\ddot{u}_0+I_1\ddot{\phi}_2$$
$$\frac{\partial M_1}{\partial x_1}+\frac{\partial M_6}{\partial x_2}-Q_1=I_1\ddot{u}_0+I_2\ddot{\phi}_1$$
$$\frac{\partial M_6}{\partial x_1}+\frac{\partial M_2}{\partial x_2}-Q_2=I_1\ddot{v}_0+I_2\ddot{\phi}_2$$
$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial x_2}-\frac{N_2}{R}-\hat{N}\frac{\partial^2 w_0}{\partial x_1^2}=I_0\ddot{w}_0$$

III. NUMERICAL TECHNIQUE AND COMPUTATIONAL TOOL

A. Introduction

ABAQUS software has been used to study the vibrational analysis of laminated cylindrical shell with lamination scheme [$0^O/90^O$] with various

1) *Radius-arc length ratio i.e. R/a ratio*
2) Arc-length to thickness i.e. a/h ratio

B. Computational Tool

1) ABAQUS modelling

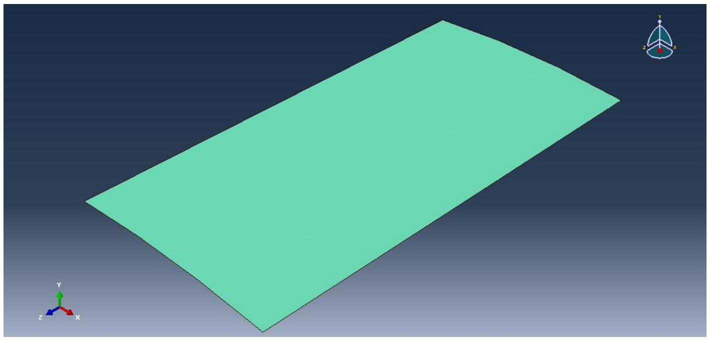

Figure: 3.1 a model of Cylindrical Shell in ABAQUS software

2) Meshing of model

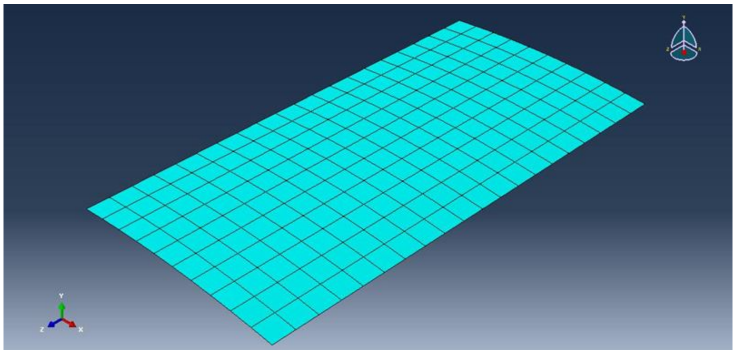

Figure: 3.2 Meshing of Cylindrical Shell in ABAQUS software with mesh size = 100*100

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3) Support Condition

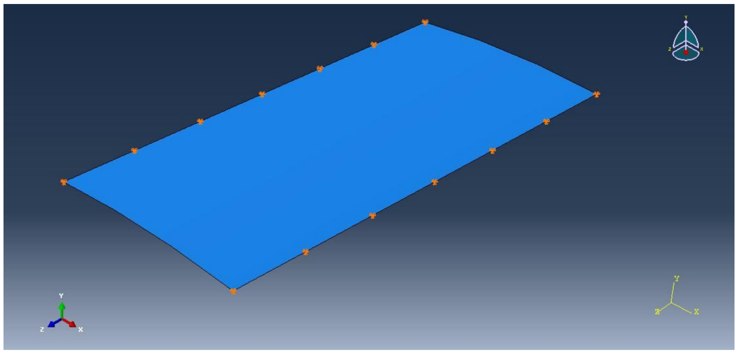

Figure: 3.3 Support condition of Cylindrical Shell in ABAQUS software

IV. RESULTS AND DISCUSSIONS

A. Introduction

In this chapter we are going to discuss the result of the problem with analytical solution as well as the results obtained by ABAQUS. The comparisons of both the analytical and software results are tabulated along with results which are presented in the literature.

B. Analytical Solution

Mantari J.L., Oktem A.S., and Coares C.G. (2011) calculated the fundamental natural frequency for various types of composite laminated shells such as spherical, cylindrical and also for composite laminated plates. First they have checked their results with literature available on first order shear deformation theory (FSDT). They have used new higher order shear deformation theory. They worked on the fundamental natural frequency for various laminates for different ratios of

1) Radius to arc-length (i.e. R/a ratio).
2) Arc-length to thickness of shells (i.e. a/h ratio

The results obtained by him for non-dimensional parameter are tabulated below.

Table No. 4.1 Comparison of Nondimensionalized fundamental frequencies of cross-ply cylindrical shells

R/a	Method	Lamination Scheme [$0^o/90^o$]	
		a/h = 10	a/h = 100
5	FSDT	8.9082	16.6680
	HSDT	9.0230	16.6900
10	FSDT	8.8879	11.8310
	HSDT	8.9790	11.8400
	FSDT	8.8900	10.2650

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20			
	HSDT	8.9720	10.2700
50	FSDT	8.8951	9.7816
	HSDT	8.9730	9.7830
100	FSDT	8.8974	9.7108
	HSDT	8.9750	9.7120

Coares C.G., Mantari J.L., and Oktem A.S. (2011)

C. Results from ABAQUS

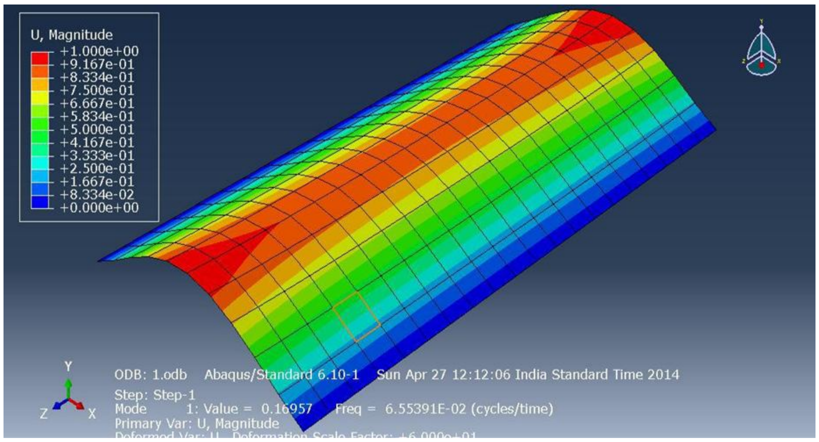

Figure: 4.1 Result of Cylindrical Shell in ABAQUS software with R/a = 5 & a/h = 10

1) *For R/a = 5, a/h = 10:ω = 0.0655391*------ (From ABAQUS results) putting all other values we get, ω = 8.7341
   Value of ωfor given type of problem in literature = 8.9082 (FSDT)
2) *For R/a = 5, a/h = 100: ω = 0.0147773*------ (From ABAQUS results) putting all other values we get, ω = 18.7203
   Value of ωfor given type of problem in literature = 16.6688 (FSDT).

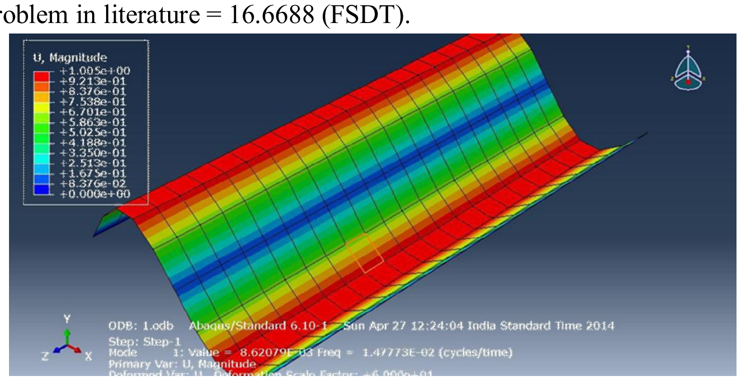

Figure: 4.2 Result of Cylindrical Shell in ABAQUS software with R/a = 5 & a/h = 10

D. Comparison of analytical and ABAQUS results

In this section we will compare the results obtained by ABAQUS and results in literature for non-dimensional fundamental natural

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frequency ω.

Table No. 4.2 Comparison of literature results of non-dimensional fundamental natural frequency (ω) of cross-ply cylindrical shells

R/a	Method	Lamination Scheme [$0^o/90^o$]	
		a/h = 10	a/h = 100
5	FSDT	8.9082	16.6680
	ABAQUS Results	8.7341 (1.95%)	18.7203 (12.31%)
10	FSDT	8.8879	11.8310
	ABAQUS Results	9.0354 (1.65%)	13.4568 (13.74%)
20	FSDT	8.8900	10.2650
	ABAQUS Results	9.0445 (1.74%)	11.5545 (12.56%)
50	FSDT	8.8951	9.7816
	ABAQUS Results	9.0996 (2.3%)	11.1220 (13.7%)
100	FSDT	8.8974	9.7108
	ABAQUS Results	9.0812 (2.07%)	10.8891 (12.13%)

Coares C.G., Mantari J.L., and Oktem A.S. (2011)

V. CONCLUSION

A. Introduction

In this chapter we are going to conclude the all our remarks based on the literature results and ABAQUS results obtained.

B. Concluding Remarks

1) Results obtained from ABAQUS are fairly close to the results that are obtained by analytical solution.

2) Analytical solution which is used in the literature gives the results very near to the results obtained by FSDT method for laminated cylindrical shell.

3) As we increase the arc side to thickness ratio i.e. a/h, the results from ABAQUS software gives somewhat higher % of error.
4) If we try to obtain the solution by analytical method (Navier), then solution leads to very complex algebraic equations.

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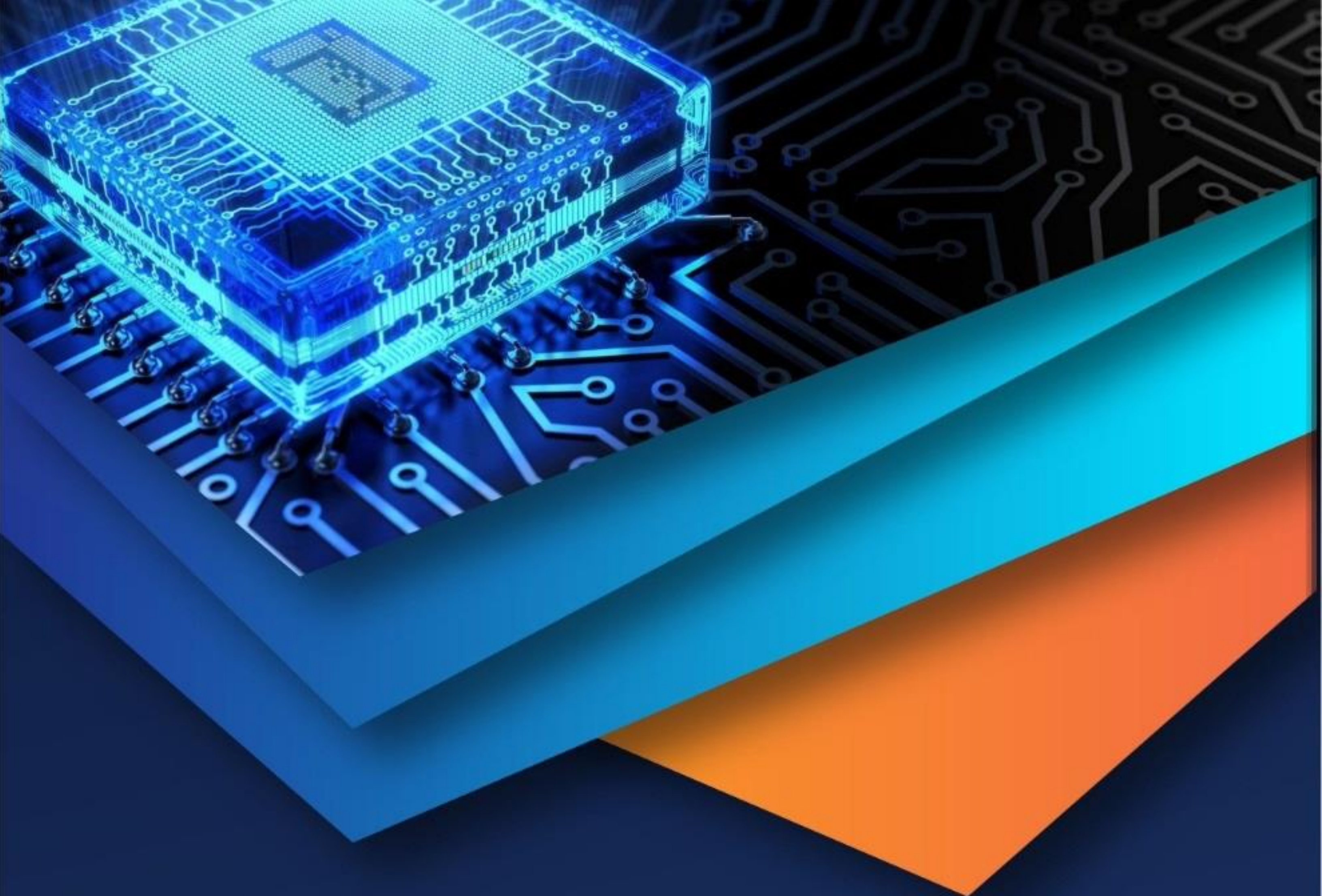

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