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Intuitionistic Fuzzy Queues with Priority Discipline

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Abstract: Queuing theory is popularly known to applicable in real life situation where frequently used in communication and computer network system. The present work, we use the Intuitionistic fuzzy set theory to describe queuing models of priority discipline. In this paper Intuitionistic fuzzy priority queuing models are optimized and the priority cases are preemptive and non-preemptive. To define the membership and non-membership function of the performance measures of priority queuing model we use the DSW (Dong, Shah and Wong) algorithm of approximate method of Extension. To transform a Intuitionistic fuzzy queue the α-cut and β–cut approach is used.
Keywords: Intuitionistic fuzzy set theory, Queuing models, Priority discipline, DSW algorithm.

I. INTRODUCTION

Queueing models have been proved to applicable in many practical applications in areas such as, e.g., production systems, inventory systems and communication systems**.** The service can be enter in many of the manner like first come first served, last come first served , random order, processor sharing (in computers that equally divide their processing power over all jobs in the system) and in priorities based.

The customer having highest priority is selected for the service in the priority discipline manner. In the priority discipline, preemptive and non preemptive are two further possible refinements arised. In aspect-of preemptive cases the customer with the highest priority is allowed to enter service immediately even if another with lower priority is already present in service when the higher customer arrives to the system. In addition to that a decision has to be made whether to continue the preempted customer's service from point of preemption when resumed or to start a new.The priority discipline is said to be non-preemptive if there is no interruption and the highest priority customer just goes to the head of the queue to wait his turn. In practical, the priority queuing model, the input data arrival rate, service rate are uncertainly known. Uncertainty is resolved by using fuzzy set theory. Hence the classical queuing model with non-preemptive priority discipline will have more application if it is expanded using fuzzy models.

Fuzzy queuing models have been described by such researchers like Li and Lee [12], Buckley[2, 3], Negi and Lee [13], Kao et al [11], Chen [4, 5] are analyzed fuzzy queues using Zadeh's extension principle [16]. Kao et al [11] constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming. Recently, Chen [4] developed (FM/FM/1) : (∞ / FCFS) and (*FM*/*FM*[*k*]/1) : (∞ / FCFS). Also fuzzy priority disciplined models are described by Groenevelt, and Altman [8], Harrison and Zhang [3], Kao, Li and Chen[11].

II. MODELS WITH THE PRIORITY

To define the queuing model with Intuitionistic fuzzy theory we consider a queuing system which is based on priority with single server, infinite calling population, arrival rate λ and rate of service μ. To establish the priority discipline queuing model, we must compare the average total cost of inactivity for the three cases: no priority discipline, Preemption priority and non-preemptive priority discipline, which are denoted respectively by C, C^1 and C^2.

(a) No priority queuing model :

Average total cost of inactivity when there is no priority discipline, C

$C = (C_1 \lambda_1 + C_2 \lambda_2)\, W$, with $W = \frac{1}{\mu-\lambda}$

(b) Preemption priority queuing model :

Average total cost of inactivity when there is Preemption priority C^1

$C^1 = C_1 \lambda_1 W_1 + C_2 \lambda_2 W_2$, with $W_{q,i} = \frac{\lambda}{\mu^2}\frac{1}{(1-\sigma_i)(1-\sigma_{i+1})}$

$\sigma_1 = \frac{\lambda_1}{\mu}$, $\sigma_2 = \frac{\lambda_2}{\mu}$ and $\sigma_3 = 0$.

(c) Average total cost of inactivity when there is a non preemptive priority discipline:

Average total cost of inactivity when there is non Preemption priority C^2

$C^2 = C_1 \lambda_1 W_1 + C_2 \lambda_2 W_2$

Where $W_i = W_{q,i} + \frac{1}{\mu}$, $W_{q,i} = \frac{1}{\mu}\frac{1}{(1-\sigma_i)(1-\sigma_{i+1})} - \frac{1}{\mu}$

$\sigma_1 = \frac{\lambda_1}{\mu}$, $\sigma_2 = \frac{\lambda_2}{\mu}$ and $\sigma_3 = 0$.

$$W_1 = \frac{1}{\mu}\frac{1}{(1-(\lambda_1/\mu))(1-(\lambda_2/\mu))}, W_2 = \frac{1}{\mu}\frac{1}{(1-(\lambda_2/\mu))}; L_1 = \lambda_1 W_1; L_2 = \lambda_2 W_2$$

By comparing the three total costs shows which of the priority discipline minimizes the average total cost function of inactivity.

III. FUZZY PRIORITY QUEUING MODELS

Intuitionistic fuzzy priority queues are described by Intuitionistic fuzzy set theory. This paper develops Intuitionistic fuzzy priority queuing model in which the input source arrival rate and service rate are uncertain parameters. To establish the priority discipline Intuitionistic fuzzy queuing model, we must compare the average total cost of inactivity for the three cases: no priority discipline, Preemption priority, non-preemptive priority discipline, which are denoted respectively by C, C^1, and C^2. Approximate methods of extension are propagating fuzziness for continuous valued mapping determined the membership and non-membership functions for the output variable. We followed the following interval analysis arithmetic for Intuitionistic fuzzy operations.

A. Interval analysis arithmetic

Let $I_1$and I_2be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

$I_1 = [a, b], a \le b$; $I_2 = [c, d], c \le d$.

Define a general arithmetic property with the symbol *, where * = [+,-, ×, ÷] symbolically the operation.

$I_1 * I_2 = [a, b] * [c, d]$ represents another interval. The interval calculation depends on the magnitudes and signs of the element *a, b, c, d*.

$[a, b] + [c, d] = [a + c, b + d]$

$[a, b] - [c, d] = [a - d, b - c]$

$[a, b] \bullet [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$

$[a, b] \div [c, d] = [a, b] \bullet [1/d, 1/c]$ provided that $0 \notin [c, d]$

$$\alpha [a, b] = \begin{cases} [\alpha a, \alpha b] & for \alpha > 0 \\ [\alpha b, \alpha a] & for \alpha < 0 \end{cases}$$

where ac, ad, bc, bd, are arithmetic products and are quotients.

B. Algorithm

Any continuous membership function can be represented by a continuous sweep of α-cut interm from α = 0 to α = 1 and similarly non-membership function can be represented by a continuous sweep of β-cut interm from β = 0 to β = 1. We use the full α-cut and β-cut intervals in a standard interval analysis with the help of DSW algorithm [15]. The algorithm consists of the following steps:

Select α-cut and β-cut values where $0 \le \alpha, \beta \le 1, \alpha + \beta \le 1$.

Find the intervals in the input membership functions and non-membership functions that correspond to these α, β respectively.

Using standard binary interval operations, compute the interval for the output membership function and non-membership function for the selected α-cut and β-cut level respectively.

Repeat steps 1 - 3 for different values of α and β to complete the α-cut and β-cut representation of the solution respectively.

IV. SOLUTION PROCEDURE

Decisions relating the priority discipline for a queuing system are mainly based for a cost function.

$$C = \sum_{i=1}^{n} C_i L_i$$

Where C_i is the unit cost of inactivity for units in class i, L_i is the average length in the system for unit of class i. Let us consider a queuing model with two unit classes arrive at α_1 of arrivals belong to one of the classes, and α_2 are in the other class and similar case hold for non-membership with β_1 and β_2. The average arrival rate at the system follows a Poisson process, is approximately known

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and is given by the triangular intuitionistic fuzzy number $\tilde{\lambda}$, the service rate from a single server is the same for both unit classes, follows an exponential pattern and is distributed according to the triangular intuitionistic fuzzy number $\tilde{\mu}$, with membership function $\mu_{\tilde{\lambda}}$, $\mu_{\tilde{\mu}}$ respectively.

$$\mu_{\bar{\lambda}^i} = \begin{cases} \dfrac{\lambda - a}{b - a}, & a \le \lambda \le b \\ \dfrac{c - \lambda}{c - b}, & b \le \lambda \le c \\ 0 & otherwise \end{cases} \qquad \mu_{\bar{\mu}^i} = \begin{cases} \dfrac{\mu - a_1}{b_1 - a_1}, & a_1 \le \mu \le b_1 \\ \dfrac{c_1 - \mu}{c_1 - b_1}, & b_1 \le \mu \le c_1 \\ 0 & otherwise \end{cases}$$

The non-membership functions for $\tilde{\lambda}$ and $\tilde{\mu}$ are denoted by $\nu_{\bar{\lambda}^i}$ and $\nu_{\bar{\mu}^i}$ respectively and are given by as follows.

$$\nu_{\bar{\lambda}^i} = \begin{cases} \dfrac{a' - \lambda}{b - a'}, & a' \le \lambda \le b \\ \dfrac{\lambda - c'}{c' - b}, & b \le \lambda \le c' \\ 1 & otherwise \end{cases} \qquad \nu_{\bar{\mu}^i} = \begin{cases} \dfrac{a_1' - \mu}{b_1 - a_1'}, & a_1' \le \mu \le b_1 \\ \dfrac{\mu - c_1'}{c_1' - b_1}, & b_1 \le \mu \le c_1' \\ 1 & otherwise \end{cases}$$

The possible distribution of unit cost of inactivity for unit in the same class, in established by a triangular intuitionistic fuzzy number $\tilde{C}_A$ and $\tilde{C}_B$ with membership function are given by as follows.

$$\mu_{\bar{C}_A^i} = \begin{cases} \dfrac{C_A - a_2}{b_2 - a_2}, & a_2 \le C_A \le b_2 \\ \dfrac{c_2 - C_A}{c_2 - b_2}, & b_2 \le C_A \le c_2 \\ 0 & otherwise \end{cases} \qquad \mu_{\bar{C}_B^i} = \begin{cases} \dfrac{C_B - a_3}{b_3 - a_3}, & a_3 \le C_B \le b_3 \\ \dfrac{c_3 - C_B}{c_3 - b_3}, & b_3 \le C_B \le c_3 \\ 0 & otherwise \end{cases}$$

And their non-membership functions respectively are given as follows.

$$\nu_{\bar{C}_A^i} = \begin{cases} \dfrac{a_2' - C_A}{b_2 - a_2'}, & a_2' \le C_A \le b_2 \\ \dfrac{C_A - c_2'}{c_2' - b_2}, & b_2 \le C_A \le c_2' \\ 1 & otherwise \end{cases} \qquad \nu_{\bar{C}_B^i} = \begin{cases} \dfrac{a_3' - C_B}{b_3 - a_3'}, & a_3' \le C_B \le b_3 \\ \dfrac{C_B - c_3'}{c_3' - b_3}, & b_3 \le C_B \le c_3' \\ 1 & otherwise \end{cases}$$

Here we choose three values of α viz, 0, 0.2 and 0.5 and three values of β viz, 0.1, 0.3 and 0.4. For $\alpha = 0$ we get 4 intervals as follows.

$$\tilde{\lambda}_0 = [a, c]\ ;\ \tilde{\mu}_0 = [a_1, c_1]\ ;\ \tilde{C}_{A,0} = [a_2, b_2]\ ;\ \tilde{C}_{B,0} = [a_3, b_3]$$

Similarly when α = 0.2, 0.5, we have 8 intervals which are denoted as $\tilde{\lambda}_{0.2}$, $\tilde{\mu}_{0.2}$, $\tilde{C}_{A,0.2}$, $\tilde{C}_{B,0.2}$, $\tilde{\lambda}_{0.5}$, $\tilde{\mu}_{0.5}$, $\tilde{C}_{A,0.5}$, $\tilde{C}_{B,0.5}$ and there are 12 intervals for β = 0.1, 0.3, 0.4 denoted as $\tilde{\lambda}'_{0.1}$, $\tilde{\mu}'_{0.1}$, $\tilde{C}'_{A,0.1}$, $\tilde{C}'_{B,0.1}$, $\tilde{\lambda}'_{0.3}$, $\tilde{\mu}'_{0.3}$, $\tilde{C}'_{A,0.3}$, $\tilde{C}'_{B,0.3}$, $\tilde{\lambda}'_{0.4}$, $\tilde{\mu}'_{0.4}$, $\tilde{C}'_{A,0.4}$, $\tilde{C}'_{B,0.4}$.

Now the total cost of inactivity in three different disciplines (i) No priority (ii) Preemptive priority (iii) non preemptive priority are

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calculated for different α and β level values. For computational efficiency interval arithmetic is used.

A. Calculation for the membership functions

Average cost of inactivity for no priority discipline:

$$\tilde{C}_0 = \left(\tilde{c}_{1,0}\tilde{\lambda}_{1,0} + \tilde{c}_{2,0}\tilde{\lambda}_{2,0}\right) * \left(\frac{1}{\tilde{\mu}_0 - \tilde{\lambda}_0}\right)$$

$$\tilde{C}_{0.2} = \left(\tilde{c}_{1,0.2}\tilde{\lambda}_{1,0.2} + \tilde{c}_{2,0.2}\tilde{\lambda}_{2,0.2}\right) * \left(\frac{1}{\tilde{\mu}_{0.2} - \tilde{\lambda}_{0.2}}\right)$$

$$\tilde{C}_{0.5} = \left(\tilde{c}_{1,0.5}\tilde{\lambda}_{1,0.5} + \tilde{c}_{2,0.5}\tilde{\lambda}_{2,0.5}\right) * \left(\frac{1}{\tilde{\mu}_{0.5} - \tilde{\lambda}_{0.5}}\right)$$

Average cost of inactivity for preemptive priority discipline:

$$\tilde{C}_0^1 = \tilde{c}_{1,0}\alpha_1\tilde{\lambda}_0\left(\frac{\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0^2}}{\left(1-\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)\left(1-\alpha_2\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)} + \frac{1}{\tilde{\mu}_0}\right) + \tilde{c}_{2,0}\alpha_2\tilde{\lambda}_0\left(\frac{\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0^2}}{\left(1-\alpha_2\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)} + \frac{1}{\tilde{\mu}_0}\right)$$

$$\tilde{C}_{0.2}^1 = \tilde{c}_{1,0.2}\alpha_1\tilde{\lambda}_{0.2}\left(\frac{\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}^2}}{\left(1-\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)} + \frac{1}{\tilde{\mu}_{0.2}}\right) + \tilde{c}_{2,0.2}\alpha_2\tilde{\lambda}_{0.2}\left(\frac{\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}^2}}{\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)} + \frac{1}{\tilde{\mu}_{0.2}}\right)$$

$$\tilde{C}_{0.5}^1 = \tilde{c}_{1,0.5}\alpha_1\tilde{\lambda}_{0.5}\left(\frac{\dfrac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}^2}}{\left(1-\dfrac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)} + \frac{1}{\tilde{\mu}_{0.5}}\right) + \tilde{c}_{2,0.5}\alpha_2\tilde{\lambda}_{0.5}\left(\frac{\dfrac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}^2}}{\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)} + \frac{1}{\tilde{\mu}_{0.5}}\right)$$

Average cost of inactivity for non-preemptive priority discipline:

$$\tilde{C}_0^2 = \tilde{c}_{1,0}\alpha_1\tilde{\lambda}_0\left(\frac{\dfrac{1}{\tilde{\mu}_0}}{\left(1-\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)\left(1-\alpha_2\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)}\right) + \tilde{c}_{2,0}\alpha_2\tilde{\lambda}_0\left(\frac{\dfrac{1}{\tilde{\mu}_0}}{\left(1-\alpha_2\dfrac{\tilde{\lambda}_0}{\tilde{\mu}_0}\right)}\right)$$

$$\tilde{C}_{0.2}^2 = \tilde{c}_{1,0.2}\alpha_1\tilde{\lambda}_{0.2}\left(\frac{\dfrac{1}{\tilde{\mu}_{0.2}}}{\left(1-\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)}\right) + \tilde{c}_{2,0.2}\alpha_2\tilde{\lambda}_{0.2}\left(\frac{\dfrac{1}{\tilde{\mu}_{0.2}}}{\left(1-\alpha_2\dfrac{\tilde{\lambda}_{0.2}}{\tilde{\mu}_{0.2}}\right)}\right)$$

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$$\tilde{C}_{0.5}^{2} = \tilde{c}_{1,0.5}\alpha_1\tilde{\lambda}_{0.5}\left(\frac{\frac{1}{\tilde{\mu}_{0.5}}}{\left(1-\frac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)\left(1-\alpha_2\frac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)}\right)+\tilde{c}_{2,0.5}\alpha_2\tilde{\lambda}_{0.5}\left(\frac{\frac{1}{\tilde{\mu}_{0.5}}}{\left(1-\alpha_2\frac{\tilde{\lambda}_{0.5}}{\tilde{\mu}_{0.5}}\right)}\right)$$

B. Calculation for the non-membership functions

Average cost of inactivity for no priority discipline:

$$\tilde{C}'_{0.1} = \left(\tilde{c}'_{1,0.1}\,\tilde{\lambda}'_{1,0.1}+\tilde{c}'_{2,0.1}\,\tilde{\lambda}'_{2,0.1}\right)*\left(\frac{1}{\tilde{\mu}'_{0.1}-\tilde{\lambda}'_{0.1}}\right)$$

$$\tilde{C}'_{0.3} = \left(\tilde{c}'_{1,0.3}\,\tilde{\lambda}'_{1,0.3}+\tilde{c}'_{2,0.3}\,\tilde{\lambda}'_{2,0.3}\right)*\left(\frac{1}{\tilde{\mu}'_{0.3}-\tilde{\lambda}'_{0.3}}\right)$$

$$\tilde{C}'_{0.4} = \left(\tilde{c}'_{1,0.4}\,\tilde{\lambda}'_{1,0.4}+\tilde{c}'_{2,0.4}\,\tilde{\lambda}'_{2,0.4}\right)*\left(\frac{1}{\tilde{\mu}'_{0.4}-\tilde{\lambda}'_{0.4}}\right)$$

Average cost of inactivity for preemptive priority discipline:

$$\tilde{C}'^{1}_{0.1} = \tilde{c}'_{1,0.1}\,\beta_1\tilde{\lambda}'_{0.1}\left(\frac{\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'^{2}_{0.1}}}{\left(1-\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)}+\frac{1}{\tilde{\mu}'_{0.1}}\right)+\tilde{c}'_{2,0.1}\,\beta_2\tilde{\lambda}'_{0.1}\left(\frac{\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'^{2}_{0.1}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)}+\frac{1}{\tilde{\mu}'_{0.1}}\right)$$

$$\tilde{C}'^{1}_{0.3} = \tilde{c}'_{1,0.3}\,\beta_1\tilde{\lambda}'_{0.3}\left(\frac{\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'^{2}_{0.3}}}{\left(1-\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)}+\frac{1}{\tilde{\mu}'_{0.3}}\right)+\tilde{c}'_{2,0.3}\,\beta_2\tilde{\lambda}'_{0.3}\left(\frac{\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'^{2}_{0.3}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)}+\frac{1}{\tilde{\mu}'_{0.3}}\right)$$

$$\tilde{C}'^{1}_{0.4} = \tilde{c}'_{1,0.4}\,\beta_1\tilde{\lambda}'_{0.4}\left(\frac{\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'^{2}_{0.4}}}{\left(1-\frac{\tilde{\lambda}'_{0.5}}{\tilde{\mu}'_{0.5}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'_{0.4}}\right)}+\frac{1}{\tilde{\mu}'_{0.4}}\right)+\tilde{c}'_{2,0.4}\,\beta_2\tilde{\lambda}'_{0.4}\left(\frac{\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'^{2}_{0.4}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'_{0.4}}\right)}+\frac{1}{\tilde{\mu}'_{0.4}}\right)$$

Average cost of inactivity for non-preemptive priority discipline:

$$\tilde{C}'^{2}_{0.1} = \tilde{c}'_{1,0.1}\,\beta_1\tilde{\lambda}'_{0.1}\left(\frac{\frac{1}{\tilde{\mu}'_{0.1}}}{\left(1-\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)}\right)+\tilde{c}'_{2,0.1}\,\beta_2\tilde{\lambda}'_{0.1}\left(\frac{\frac{1}{\tilde{\mu}'_{0.1}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.1}}{\tilde{\mu}'_{0.1}}\right)}\right)$$

$$\tilde{C}'^{2}_{0.3} = \tilde{c}'_{1,0.3}\,\beta_1 \tilde{\lambda}'_{0.3}\left(\frac{\frac{1}{\tilde{\mu}'_{0.3}}}{\left(1-\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)}\right) + \tilde{c}'_{2,0.3}\,\beta_2 \tilde{\lambda}'_{0.3}\left(\frac{\frac{1}{\tilde{\mu}'_{0.3}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.3}}{\tilde{\mu}'_{0.3}}\right)}\right)$$

$$\tilde{C}'^{2}_{0.4} = \tilde{c}'_{1,0.4}\,\beta_1 \tilde{\lambda}'_{0.4}\left(\frac{\frac{1}{\tilde{\mu}'_{0.4}}}{\left(1-\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'_{0.4}}\right)\left(1-\beta_2\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'_{0.4}}\right)}\right) + \tilde{c}'_{2,0.4}\,\beta_2 \tilde{\lambda}'_{0.4}\left(\frac{\frac{1}{\tilde{\mu}'_{0.4}}}{\left(1-\beta_2\frac{\tilde{\lambda}'_{0.4}}{\tilde{\mu}'_{0.4}}\right)}\right)$$

V. NUMERICAL EXAMPLE

Let us take a centralized parallel processing system in which jobs arrive in two class with utilization of 15% and 85%. Jobs arrive at this system follow Poisson process and the service times in accordance with an exponential distribution. Both the group arrival rate and service rate are triangular fuzzy numbers represented by $\tilde{\lambda}$ = (3, 7, 11; 4, 7, 15) and $\tilde{\mu}$ = (14, 20, 22; 16, 20, 21) per minute, respectively. The possibility distribution of unit cost of inactivity for units of the two classes are triangular intuitionistic fuzzy numbers $\tilde{C}_A$ = (12, 17, 24; 13, 17, 22) and $\tilde{C}_B$ = (11, 19, 28; 13, 19, 21) respectively. The system manager wants to evaluate the total cost of inactivity when there is no priority discipline, preemptive priority discipline, non-preemptive priority discipline in the queue. The numerical calculation of total costs of inactivity for membership functions is given in the table 1.The numerical calculation of total costs of inactivity for non - membership functions is given in the table 2.Comparison of the three total costs shows which of the priority disciplines minimizes the average total cost function of inactivity . Minimum average total cost of inactivity is achieved by the intuitionistic fuzzy queuing model with the non-preemptive discipline for both the membership and non-membership functions. We conclude that the optimum selection of a priority discipline for the intuitionistic fuzzy queuing model that we studied entails with the non-preemptive discipline.

VI. CONCLUSION

Fuzzy set theory has been applied to a number of queuing system to provide broader application in many fields. In this paper we apply measures to uncertainty of the initial information when some of the parameters of the models are intuitionistic fuzzy. The method proposed enables reasonable solution to be for each case, with different level of possibility, ranging from the most pessimistic to the most optimistic scenario. The present work also provides more information to help design intuitionistic fuzzy priority discipline queuing system.

6. Tables

Table 1. The total costs of inactivity for membership functions.

Priority discipline	α = 0	α = 0.2	α = 0.5
No priority discipline : $\tilde{C}_\alpha$	[8.263, 126.667]	[9.995, 70.624]	[12.968, 39.25]
Preemptive priority discipline: $\tilde{C}^1_\alpha$	[0.34, 45.406]	[1.886, 30.007]	[2.264, 11.53]
Non-Preemptive priority discipline: $\tilde{C}^2_\alpha$	[0.308, 5.219]	[1.823, 27.103]	[2.265, 11.53]

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Table 2. The total costs of inactivity for non - membership functions.

Priority discipline	β = 0.1	β = 0.3	β= 0.4
No priority discipline : $\tilde{C}_{\beta}'$	[18.661, 22.627]	[18.173, 31.604]	[17.715, 38.068]
Preemptive priority discipline: $\tilde{C}_{\beta}^{1}'$	[3.268, 4.484]	[4.892, 12.611]	[3.076, 11.5]
Non-Preemptive priority discipline: $\tilde{C}_{\beta}^{2}'$	[2.899, 3.916]	[4.664, 11.837]	[3.003, 11.169]

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