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MHD Effect on Free Convective Flow of a Stratified Fluid through Porous Medium Bounded By a Vertical Plane

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Abstract: In this work, MHD effect on free convective flow of a stratified fluid through a porous medium bounded by a vertical plane is investigated. Considering both viscous and Darcy dissipations, the temperature of the plate is varying linearly along the vertical direction. Analytical solutions of momentum and energy equations are obtained by Perturbation technique. The dimensionless Skin friction co-efficient and Nusselt number are also estimated. The effects of various physical parameters like Prandtl number Pr, Hartmann number Ha, Darcy resistance parameter σ, buoyancy force parameter N and equilibrium temperature gradient parameter A_T on velocity and temperature distribution are analyzed through graphs.
Keywords : Free convection, porous medium, viscous dissipation, MHD and stratified fluid.

I. INTRODUCTION

Free and forced convection flows in a saturated porous media are of great interest because of their various engineering, scientific and industrial applications in heat and mass transfer which occurs in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees and damage of crops due to freezing and pollution of the environment, grain storage systems, heat pipes, packed microsphere insulation distillation towers, ion exchange columns, subterranean chemical waste migration, solar power absorbers etc.

Hadhrami, Ellott and Ingham (2003) obtained a new model for viscous dissipation in porous media across a range of permeability values and found unified mathematical theory for the viscous dissipation term in the governing Brinkman equation. Rudraiah and Nagaraj (1977) studied the effect of Darcy and viscous resistances on the fully developed natural convection of a fluid between two heated vertical plates. Effects of viscous dissipation on fully developed forced convection in porous media have been studied by Yew-Mun Hung (2009). Raptis (1983) studied the unsteady free convection flow through a porous medium bounded by an infinite vertical plate. Anjali Devi and Ganga (2009) studied the effects of viscous and joules dissipation on MHD flow with heat and mass transfer past a stretching porous surface embedded in a porous medium. MHD free convection flow through a porous medium in a rotating fluid has been studied by Ram and Jain (1990). Raptis and Perdikis (1983) obtained the velocity and temperature fields when the temperature of the fluid and temperature away from the surface have a difference which varies as some power of time. Maria Neagu (2016) investigated Natural convection process triggered in a fluid saturated thermally stratified porous medium by a vertical impermeable wall of constant heat flux and concentration.

Convective motion in a porous medium has attracted considerable attention from many researches because of its application in geophysics, oil recovery technique, thermal insulation, engineering and heat storage. The study of electrically conducting fluid has many applications in engineering problems such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction and boundary layer in the field of aerodynamics. In view of the applications of free convective and heat transfer flows through porous medium under the influence of magnetic field many researchers have studied magnetohydrodynamic free convective heat transfer flow in a porous medium. The differential solar rotation may be the long-term effect of magnetic drag at the poles of the sun, an magnetohydrodynamic phenomenon due to the Parker spiral shape assumed by the extended magnetic field of the Sun. In this work, steady two dimensional MHD free convection flow of a thermally stratified viscous fluid through a porous medium bounded by a heated vertical plate taking into account both viscous and Darcy dissipations is considered. The main objective is to find the effect of equilibrium temperature gradient of the fluid on the flow.

II. MATHEMATICAL FORMULATION

Consider a steady two dimensional free convection flow of a thermally stratified viscous fluid through a highly porous medium

www.ijraset.com   *Volume 5 Issue II, February 2017*
IC Value: 45.98   *ISSN: 2321-9653*
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

bounded by a vertical plane. The flow is assumed to be in the y direction, which is chosen along the plate in the upward direction and x-axis normal to plate. A uniform magnetic field is applied in the direction perpendicular to the plate. The basic governing equations are

Continuity equation

$$\nabla.\vec{q}' = 0$$

Momentum equation

$$\rho'(\vec{q}'.\nabla)\vec{q}' = -\nabla p - \frac{\mu}{k'}\vec{q}' + \mu\nabla^2\vec{q}' - \rho'\vec{g} - j \times B$$

Energy equation

$$\rho C_p(\vec{q}.\nabla)T' = k\nabla^2 T' + \varphi$$

We take a Cartesian coordinate system with the y'-axis vertically upward along the plate and the x'-axis normal to it. Now the temperature, density and pressure of the fluid can be written as

$$T' = T_e + \theta', \quad \rho' = \rho_e - \alpha\rho_0\theta', \quad P = P_e + \overline{P}, \quad T_e = T_0 + A'_T, \quad \rho_e = \rho_0(1 - \alpha T_e)$$

$$\frac{dp_e}{dy'} = -\rho_e g$$

θ', $\overline{p}$ are the deviation of temperature and pressure, the constant $A'_T(> 0)$ is the equilibrium temperature gradient of the fluid, the constants ρ_0 and T_0 are the reference density and temperature respectively and α is the coefficient of volume expansion. We assume the surface temperature of the plate in the form $T'_w = T_w + T_e$ where $T_w(> 0)$ is constant.

Here the motion takes place only due to the temperature gradient in the plate. The plate being assumed infinite along the y'-axis, the field variables θ' and $\vec{q}'$ are taken to be independent of y'. Then using the equation of continuity and applying Boussinesq approximation the equation of motion (2) becomes

$$\nu\frac{d^2v'}{dx'^2} - \frac{\nu}{k'}v' + \alpha g\theta' - Ha = 0$$

The energy equation becomes

$$k\frac{d^2\theta'}{dx'^2} + \rho_0\nu(\frac{dv'}{dx'})^2 + \frac{\nu\rho_0}{k'}v'^2 - c_p\rho_0 A'_T v' = 0$$

The corresponding boundary conditions for the velocity and temperature fields are

$$v' = 0, \quad \theta' = T_w \quad \text{at} \quad x' = 0$$
$$v' \to 0, \quad \theta' \to 0 \quad \text{at} \quad x' \to \infty$$

Where v' is the velocity component along the y'-axis, k is the thermal conductivity, c_p is the specific heat at constant pressure, $\nu = \mu/\rho_0$ and the second and the third term in the energy equation (6) represent the viscous and Darcy dissipation respectively.

Introducing the non-dimensional variables $v = v'/\beta$, $x = x'\beta/\nu$, $\theta = \theta'/T_w$ where $\beta = (\alpha g\nu T_w)^{1/3}$ has dimension of velocity, the above equations (6) and (7) take the form

$$\frac{d^2v}{dx^2} - \sigma^2 v + \theta - Ha = 0$$

$$\frac{d^2\theta}{dx^2} + N\left(\frac{dv}{dx}\right)^2 + N\sigma^2 v^2 - PA_T v = 0$$

The modified boundary conditions are

$$\theta = 1, \quad v = 0, \quad \text{at} \quad x = 0$$
$$v \to 0, \theta \to 0 \quad \text{as} \quad x \to \infty$$

where

σ	$= \nu/\sqrt{k'}\beta$	(Darcy resistance parameter)
N	$= \rho_0\nu\beta^2/kT_w$	(Buoyancy force parameter)
Pr	$= \rho_0 c_p\nu/k$	(Prandtl number)
Ha	$= \frac{\sigma B_0^2\nu v}{\rho\beta^2}$	(Hartmann number)

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$A_T = vA_T'/T_w\beta$ (equilibrium temperature gradient parameter).

III.   SOLUTION OF THE PROBLEM

The coupled equations (8) and (9) are non-linear due to dissipation terms. However, in many practical problems N is small and therefore, with N as a perturbation parameter the perturbation technique is applied to solve the above equations.

we write

$v(x) = v_0(x) + N v_1(x) + O(N^2)$

$\theta(x) = \theta_0(x) + N\theta_1(x) + O(N^2)$

where

v_0, θ_0 are the solutions corresponding to N = 0 i.e., when the dissipations are neglected.

By substituting equation (11) in equations (8) and (9) and equating the coefficients of like powers of N

The zeroth order equations are

$v_0'' - \sigma^2 v_0 + \theta_0 - Hav_0 = 0$

$\theta_0'' - PA_T v_0 = 0$

The first order equations are

$v_1'' - \sigma^2 v_1 + \theta_1 - Hav_1 = 0$

$\theta_1'' - v_0'^2 + \sigma^2 v_0^2 - PA_T v_1 = 0$

The corresponding boundary condition (10) reduces to

$v_0 = 0, \quad v_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0 \quad$ at $x = 0$

$v_0 \to 0, \quad v_1 \to 0, \quad \theta_0 \to 0, \quad \theta_1 \to 0 \quad$ as $x \to \infty$

Where primes denote differentiation with respect to x.

Solving the equations (12) and (13) with the help of (14) we get $v_0, \theta_0, v_1, \theta_1$ respectively and from (11) we finally get the expressions of velocity and temperature in the form

$$v = M\left(e^{-\beta_1 x} - e^{-\alpha_1 x}\right)$$

$$+ N\left[A_1 e^{-\alpha_1 x} + A_2 e^{-\beta_1 x}\right.$$

$$+ M^2 \left\{ \frac{(\sigma^2 + \alpha_1^2)e^{-2\alpha_1 x}}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4Ha\alpha_1^2} + \frac{(\sigma^2 + \beta_1^2)e^{-2\beta_1 x}}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4Ha\beta_1^2} - \frac{2(\alpha_1\beta_1 + \sigma^2)e^{-(\alpha_1+\beta_1)x}}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + Ha\alpha_1^2 + 2Ha\alpha_1\beta_1} \right\}\left.\right]$$

(15)

$$\theta = (\alpha_1^2 - \sigma^2)[M - A_1 N]e^{-\alpha_1 x} + (\beta_1^2 - \sigma^2)[M - A_2 N]e^{-\beta_1 x}$$

$$+ N\left[\frac{M^2 e^{-2\alpha_1 x}(\sigma^2 + \alpha_1^2)(\sigma^2 - 4\alpha_1^2)}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4Ha\alpha_1^2} + \frac{M^2 e^{-2\beta_1 x}(\sigma^2 + \beta_1^2)(\sigma^2 - 4\beta_1^2)}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4Ha\beta_1^2} - \frac{2M^2 e^{-(\alpha_1+\beta_1)x}(\alpha_1\beta_1 + \sigma^2)(\sigma^2 - (\alpha_1+\beta_1)^2)}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + Ha\alpha_1^2 + 2Ha\alpha_1\beta_1}\right]$$

Where

$$\alpha_1 = \sqrt{\frac{(\sigma^2 + Ha) + \sqrt{\lambda}}{2}}$$

$$\beta_1 = \sqrt{\frac{(\sigma^2 + Ha) - \sqrt{\lambda}}{2}}$$

$$A_1 = \frac{-M^3(\sigma^2 + \alpha_1^2)(4\alpha_1^2 - \beta_1^2)}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4Ha\alpha_1^2} - \frac{M^3(\sigma^2 + \beta_1^2)(3\beta_1^2)}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4Ha\beta_1^2} + \frac{2M^3(\alpha_1\beta_1 + \sigma^2)(\alpha_1^2 + 2\alpha_1\beta_1)}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + Ha\alpha_1^2 + 2Ha\alpha_1\beta_1}$$

$$A_2 = \frac{-M^3(\sigma^2 + \alpha_1^2)(3\alpha_1^2)}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4Ha\alpha_1^2} - \frac{M^3(\sigma^2 + \beta_1^2)(4\beta_1^2 - \alpha_1^2)}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4Ha\beta_1^2} - \frac{2M^3(\alpha_1\beta_1 + \sigma^2)(\beta_1^2 + 2\alpha_1\beta_1)}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + Ha\alpha_1^2 + 2Ha\alpha_1\beta_1}$$

$M = 1/(\alpha_1^2 - \beta_1^2)$ $\quad$ and $\quad$ $\lambda = (\sigma^2 + Ha)^2 - 4PrA_T$.

For $\lambda < 0, \alpha_1$ and β_1 will become complex and the real part of (15) and (16) will be the solutions in this case. We can also obtain a solution in the case $\lambda = 0$ i.e., $A_T = (\sigma^2 + Ha)^2/4Pr$. For very high permeability of the medium, the parameter $K\left(= \frac{1}{\sigma^2}\right)$ will be very large and the solution of the corresponding problem for the free flow of the fluid is obtained by making $\sigma \to 0$ in our solution. The shear stress at the plate, in the non-dimensional form is given by

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$$\tau = \left(\frac{dv}{dx}\right)_{x=0}$$

$$\tau = M\big((-\beta_1) - (-\alpha_1)\big)$$

$$+ N\Bigg[A_1(-\alpha_1) + A_2(-\beta_1)$$

$$+ M^2\Bigg\{\frac{(\sigma^2 + \alpha_1^2)(-2\alpha_1)}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4H\alpha_1^2} + \frac{(\sigma^2 + \beta_1^2)(-2\beta_1)}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4H\beta_1^2} - \frac{2(\alpha_1\beta_1 + \sigma^2)(-(\alpha_1 + \beta_1))}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + H\alpha_1^2 + 2H\alpha_1\beta_1}\Bigg\}\Bigg]$$

The non dimensional co-efficient of heat transfer defined by Nusselt number as

$$Nu = -\left(\frac{d\theta}{dx}\right)_{x=0}$$

$$Nu = -\Bigg((\alpha_1^2 - \sigma^2)[M - A_1N](-\alpha_1) + (\beta_1^2 - \sigma^2)[M - A_2N](-\beta_1)$$

$$+ N\Bigg[\frac{M^2(-2\alpha_1)(\sigma^2 + \alpha_1^2)(\sigma^2 - 4\alpha_1^2)}{3\alpha_1^2(4\alpha_1^2 - \beta_1^2) + 4H\alpha_1^2} + \frac{M^2(-2\beta_1)(\sigma^2 + \beta_1^2)(\sigma^2 - 4\beta_1^2)}{3\beta_1^2(4\beta_1^2 - \alpha_1^2) + 4H\beta_1^2}$$

$$- \frac{2M^2(-(\alpha_1 + \beta_1))(\alpha_1\beta_1 + \sigma^2)(\sigma^2 - (\alpha_1 + \beta_1)^2)}{\alpha_1\beta_1(\beta_1 + 2\alpha_1)(\alpha_1 + 2\beta_1) + H\alpha_1^2 + 2H\alpha_1\beta_1}\Bigg]\Bigg)$$
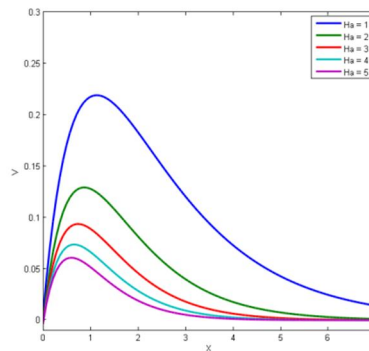

Figure 1: Velocity profile for different Hartmann number (Ha)
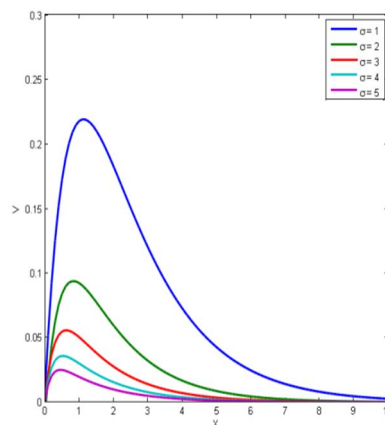
[N = 0.01, Pr = 0.71, $\sigma = 1$, A_T = -0.3]

Figure 2: Velocity profile for different Darcy resistance parameter (σ)

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$[N = 0.01, Pr = 0.71, Ha = 1, A_T = -0.3]$
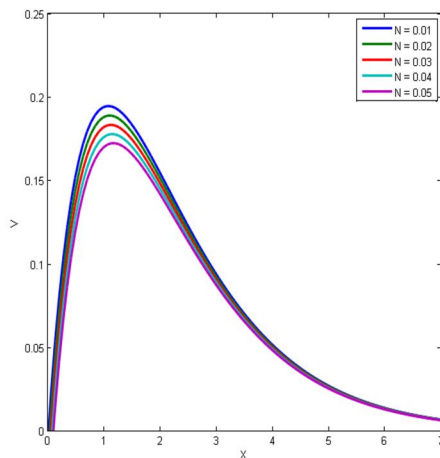

Figure 3: Velocity profile for different Buoyancy force parameter (N)
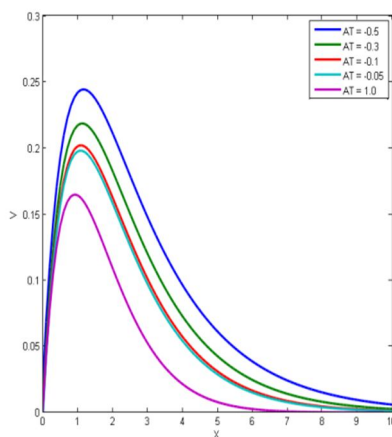
$[\sigma = 1, Pr = 0.71, Ha = 1, A_T = -0.3]$

Figure 4: Velocity profile for different Equilibrium Temperature gradient parameter (A_T)
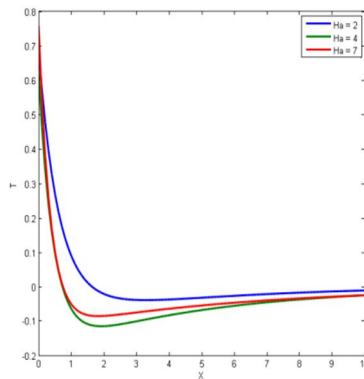
$[\sigma = 1, Pr = 0.71, Ha = 1, N = 0.01]$

Figure 5: Temperature profile for different Hartmann number (Ha)

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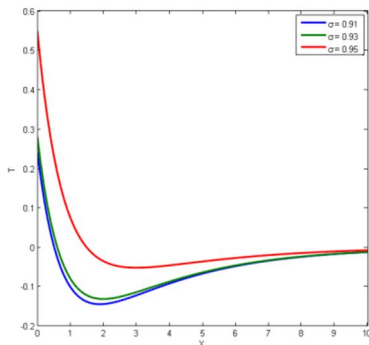
[σ = 1, Pr = 0.71, A_T = 0.3, N = 0.01]

Figure 6: Temperature profile for different Darcy resistance parameter (σ)
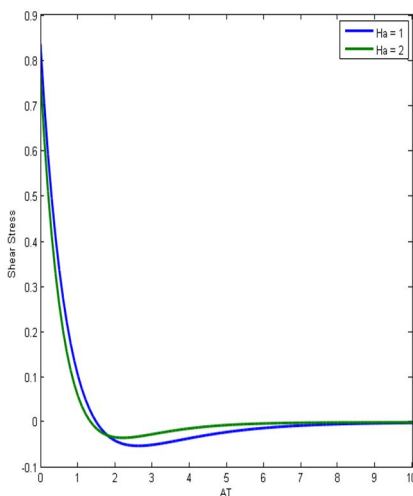
[Pr = 0.71, A_T = 0.3, N = 0.01, Ha = 1]

Figure 7: Shear Stress for different Hartmann number (Ha)

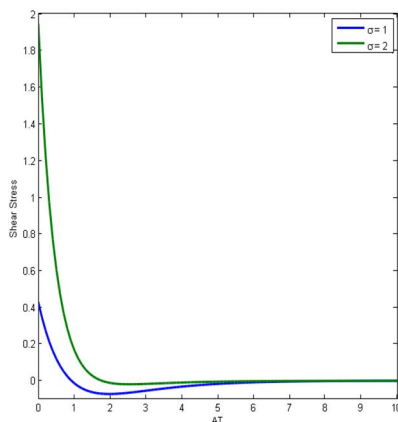[σ = 1, Pr = 0.71, N = 0.01, A_T = -0.3]

Figure 8: Shear Stress for different Darcy resistance parameter (σ)

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$[N = 0.01, Pr = 0.71, Ha = 1, A_T = -0.3]$
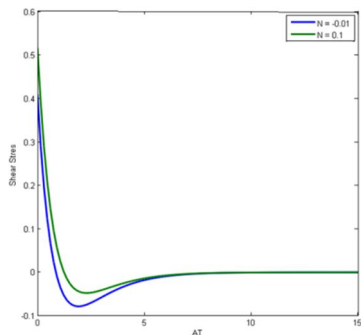

Figure 9: Shear Stress for different Buoyancy force parameter (N)
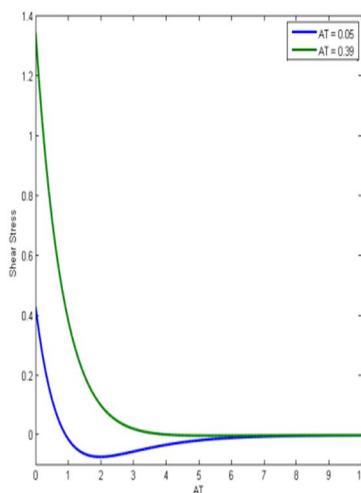
$[\sigma = 1, Pr = 0.71, Ha = 1, A_T = -0.3]$

Figure 10: Shear Stress for different Equilibrium Temperature gradient parameter (A_T)

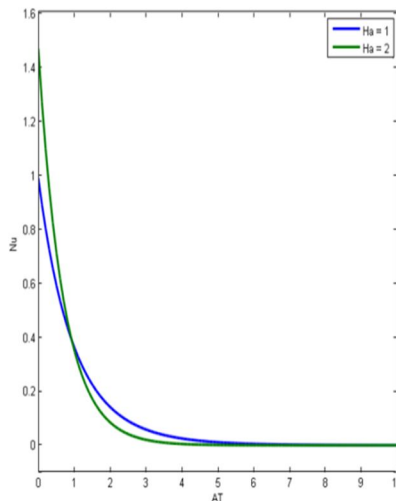$[\sigma = 1, Pr = 0.71, Ha = 1, N = 0.01]$

Figure 11: Nusselt number for different Hartmann number (Ha)

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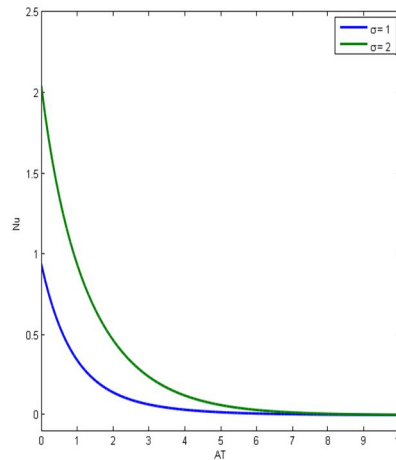
[$\sigma = 1$, Pr = 0.71, A_T = -0.3, N = 0.01]

Figure 12: Nusselt number for different Darcy resistance parameter (σ)

[Ha = 1, Pr = 0.71, A_T = -0.3, N = 0.01]

IV.   RESULTS AND DISCUSSION

The profile for velocity and temperature are shown through Figures 1 - 10. The momentum and energy equations are characterized by the Hartmann number (Ha), Darcy resistance parameter (σ), buoyancy force parameter (Pr), and equilibrium temperature gradient parameter (A_T) . We have calculated the co-efficient of skin friction and the rate of heat transfer in terms of Nusselt number by assigning specific values to the parameter involved in the problem

The velocity profile for various Hartmann number (Ha = 1, 2, 3, 4) is shown in the Figure 1. It is found that with the increasing Hartmann number the velocity profile attains its maximum when x = 1 and converges to zero when x →∞.

The velocity profile for various Darcy resistance parameter (σ = 1, 2, 3, 4, 5) is shown in the Figure 2. The figure disciples that with the increasing Darcy resistance parameter the velocity profile decreases.

The velocity profile for various buoyancy force parameter (N = 0.01, 0.02, 0.03, 0.04, 0.05) is shown in the Figure 3. It is clear from the figure that with the increasing buoyancy forces parameter the velocity profile decreases.

The velocity profile for various equilibrium temperature gradient (A_T = -0.5, -0.3, -0.1, -0.05, 1.0) is shown in the Figure 4. It is known from the figure that with the increasing equilibrium temperature gradient the velocity profile is decreasing.

The temperature profile for various Hartmann number (Ha = 2, 4, 7) is shown in the Figure 5. It is clear from the figure that with the increase in Hartmann number the temperature profile decreases.

The temperature profile for various Darcy resistance parameter (σ = 0.91, 0.93, 0.95) is shown in the Figure 6. It is known that the increasing Darcy resistance parameter the temperature profile decreases.

The shear stress for different Hartmann number (Ha = 1, 2,) is shown in the Figure 7. It is shown that the shear stress increases with increasing Hartmann number.

The shear stress for different Darcy resistance parameter (σ = 1, 2) is shown in the Figure 8. It is understood that the shear stress increases with increasing Darcy resistance parameter.

The shear stress for different buoyancy force parameter (N = -0.01, 0.1) is shown in the Figure 9. It is seen that the shear stress increases with increasing buoyancy force parameter.

The shear stress for different equilibrium temperature gradient parameter (A_T = 0.05, 0.39) is shown in the Figure 10. The shear stress increases with increasing equilibrium temperature gradient parameter

The rate of heat transfer in terms of Nusselt number for different Hartmann number (Ha = 1, 2) is shown in the Figure 11. It shows that the rate of heat transfer increases with decreasing Hartmann number.

The rate of heat transfer in terms of Nusselt number for different Darcy resistance parameter (σ = 1, 2) is shown in the Figure 12. Here the rate of heat transfer increases with increasing Darcy resistance parameter.

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V.   CONCLUSION

In this study exact solution for the velocity field and temperature in the presence of Hartmann number, Darcy resistance parameter, Buoyancy force parameter, Prandtl number, and Equilibrium temperature gradient parameter are constructed. A magnetic field is applied transversely to the flow. The solution so obtained, depending on the initial and boundary conditions are presented as sum of the non dimensional parameters which occurs in the problem under study.

The following conclusions are made Velocity increases with the decrease in Hartmann number, Darcy resistance parameter, buoyancy force parameter, Prandtl number and equilibrium temperature gradient. Temperature increases with decrease in Hartmann number and increases with increase in Darcy resistance parameter. The rate heat transfer increases with the increase in Darcy resistance parameter and increases with the decrease in Hartmann number.

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