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Fixed Point Theorems on Multi Valued Mappings in B-Metric Spaces

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Abstract: In this paper, we prove a fixed point theorem and a common fixed point theorem for multi valued mappings in complete b-metric spaces. Keywords: b-Metric space, Multi-valued mappings, Contraction, Fixed point

INTRODUCTION AND PRELIMINARIES

Fixed point theory plays one of the important roles in nonlinear analysis. It has been applied in physical sciences, computing sciences and Engineering. In 1922, Stefan Banach proved a famous fixed point theorem for contractive mappings in complete metric spaces. Later, Czerwik (1993, 1998) has come up with b-metrics which generalized usual metric spaces. After his contribution, many results were presented in β -generalized weak contractive multifunctions and b-metric spaces (Alikhani et al. 2013; Boriceanu 2009; Mehemet and Kiziltunc 2013). The following definitions will be needed in the sequel:

A. Definition

Nadler (1969) Let X and Y be nonempty sets. T is said to be multi-valued mapping from X to Y if T is a function for X to the power set of Y. we denote a multi-valued map by:

 $T: X \rightarrow 2^{Y}$.

B. Definition

Nadler (1969) A point of $x_0 \in X$ is said to be a fixed point of the multi-valued mapping T if $x_0 \in Tx_0$.

I.

C . Example

Joseph (2013) Every single valued mapping can be viewed as a multi-valued mapping. Let $f:X \to Y$ be a single valued mapping. Define $T:X \to 2^Y$ by $Tx = \{f(x)\}$. Note that *T* is a multi-valued mapping iff for each $x \in X$, $TX \subseteq Y$. Unless otherwise stated we always assume *Tx* is non-empty for each $x, y \in X$.

D. Definition

Banach (1922) Led (X, d) be a metric space. A map $T: X \to X$ is called contraction if there exists $0 \le \lambda < 1$ such that $d(Tx, Ty) \le \lambda d(x, y)$, for all $x, y \in X$.

E. Definition

Nadler (1969) Let (X, d) be a metric space. We define the Hausdorff metric on CB(X) induced by d. That is

$$H(A, B) = max\{\sup x \in A \ d(x, B), \ \sup y \in B \ d(y, A)\}$$

for all $A, B \in CB(X)$, where CB(X) denotes the family of all nonempty closed and bounded subsets of X and $d(x, B) = \inf\{d(x, b): b \in B\}$, for all $x \in X$.

E. Definition

Nadler (1969) Let (X, d) be a metric space. A map $T: X \to CB(X)$ is said to be multi-valued contraction if there exists $0 \le \lambda < 1$ such that $H(Tx, Ty) \le \lambda d(x, y)$, for all $x, y \in X$

F. Lemma

Nadler (1969) If A, $B \in CB(X)$ and $a \in A$, then for each $\epsilon > 0$, there exists $b \in B$ such that $d(a, b) \leq H(A, B) + \epsilon$.

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)

$G. \ Definition$

Aydi et al. (2012) Let X be a nonempty set and let $s \ge 1$ be a given real number. A function $d:X \times X \to \mathbb{R}^+$ is called a b-metric provide that, for all x, y, $z \in X$, d(x, y) = 0 if and only if x = y d(x, y) = d(y, x) $d(x, z) \le s[d(x, y) + d(y, z)].$ A pair(X, d) is called a b-metric space.

H. Example

Boriceanu (2009) The space $l_p(0 , <math>l_p = \{(x_n: \sum_{n=1}^{\infty} |x_n|^p < \infty\}$, together with the function $d: l_p \times l_p \to \mathbb{R}^+$.

I. Example

Boriceanu (2009) The space $L_p(0 for all real function <math>x(t), t \in [0, 1]$ such that $\int_0^1 |x(t)|^p dt < \infty$, is *b*-metric space if we take $d(x, y) = (\int_0^1 |x(t) - y(t)| dt)^{\frac{1}{p}}$.

J.Example

Aydi et al. (2012) Let $X = \{0, 1, 2\}$ and $d(2, 0) = d(0, 2) = m \ge 2$, d(0, 1) = d(1, 2) = d(0, 1) = d(2, 1) = 1 and d(0, 0) = d(1, 1) = d(2, 2) = 0. Then $d(x, y) \le \frac{m}{2} [d(x, z) + d(z, y)]$ for all $x, y, z \in X$. If m > 2, the ordinary triangle inequality does not hold.

K. Definition

Let (X, d) be a *b*-metric space. Then a sequence (x_n) in *X* is called Cauchy sequence if and only if for all $\epsilon > 0$ there exists $n(\epsilon) \in \mathbb{N}$ such that for each $m, n \ge n(\epsilon)$ we have $d(x_n, x_m) < \epsilon$.

L. Definition

Let be a (*X*, *d*) *b*-metric space. Then a sequence (*x_n*) in X is called convergent sequence if and only if there exists $x \in X$ such that for all $\epsilon > 0$ there exists $n(\epsilon) \in \mathbb{N}$ such that for all $n \ge n(\epsilon)$ we have $d(x_n, x) < \epsilon$. In this case we write $\lim n \to \infty xn = x$ Our first result is the following theorem.

II. MAIN RESULTS

A. Definition

Let (X, d) be a *b*-metric space with constant $s \ge 1$. A map $T:X \to CB(X)$ is said to be multi-valued generalized contraction if $H(Tx, Ty) \le a_1 d(x, Tx) + a_2 d(y, Ty) + a_3 d(x, Ty) + a_4 d(y, Tx) + a_5 d(x, y) + a_6 \frac{d(x, Tx) (1 + d(x, Tx))}{1 + d(x, y)}$,(1) for all $x, y \in X$ and $a_i \ge 0$, i = 1, 2, 3, ...6 with $a_1 + a_2 + 2sa_3 + a_4 + a_5 + a_6 < 1$.

B. Theorem

Let (X, d) be a complete b-metric space with constants ≥ 1 . Let $T:X \rightarrow CB(X)$ be a multi-valued generalized contraction mapping. Then T has a unique fixed point.

C. Proof

Fix any $x \in X$. Define $x_0 = x$ and let $x_1 \in Tx_0$. By Lemma 7, we may choose $x_2 \in Tx_1$ such that $d(x_1, x_2) \le H(Tx_0, Tx_1) + (a_1 + sa_3 + a_5 + a_6)$.

Now,

 $\begin{aligned} d(x_1, x_2) &\leq H(Tx0, Tx1) + (a_1 + sa_3 + a_5 + a_6) \\ &\leq a_1 d(x_0, Tx_0) + a_2 d(x_1, Tx_1) + a_3 d(x_0, Tx_1) + a_4 d(x_1, Tx_0) + a_5 d(x_0, x_1) + a_6 \frac{d(x, Tx) (1 + d(x, Tx))}{1 + d(x, y)} + (a_1 + sa_3 + a_5 + a_6) \\ d(x_1, x_2) &\leq a_1 d(x_0, x_1) + a_2 d(x_1, x_2) + a_3 d(x_0, x_2) + a_4 d(x_1, x_1) + a_5 d(x_0, x_1) + a_6 d(x_0, x_1) + (a_1 + sa_3 + a_5 + a_6) \end{aligned}$

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International Journal for Research in Applied Science & Engineering Technology (IJRASET)

 $\leq (a_1+a_5+a_6) d(x_0, x_1)+a_2 d(x_1, x_2)+a_3 s[d(x_0, x_1)+d(x_1, x_2)]+(a_1+sa_3+a_5+a_6)$ $\leq (a_1+sa_3+a_5+a_6) d(x_0, x_1)+a_2 d(x_1, x_2)+sa_3 d(x_1, x_2)+(a_1+sa_3+a_5+a_6)$

 $d(x_1, x_2) \leq \frac{(a_1 + sa_3 + a_5 + a_6)}{1 - (a_2 + Sa_3)} d(x_0, x_1) + \frac{(a_1 + sa_3 + a_5 + a_6)}{1 - (a_2 + Sa_3)}$

By Lemma 7, there exist $x_3 \in Tx_2$ such that $d(x_2, x_3) \le d(Tx_1, x_2) + \frac{(a_1 + sa_3 + a_5 + a_6)^2}{1 - (a_2 + sa_3)^2}$.

Now,

 $d(x_{2}, x_{3}) \leq H(Tx_{1}, x_{2}) + \frac{(a_{1} + sa_{3} + a5 + a6)^{2}}{1 - (a_{2} + Sa_{3})} a_{1}d(x_{1}, Tx_{1}) + a_{2}d(x_{1}, Tx_{2}) + a_{3}d(x_{1}, Tx_{2}) + a_{4}d(x_{2}, Tx_{1}) + a_{5}d(x_{1}, x_{2}) + a_{6}d(x_{1}, x_{2}) + \frac{(a_{1} + sa_{3} + a5 + a6)^{2}}{1 - (a_{2} + Sa_{3})} d(x_{1}, x_{2}) + \frac{(a_{1} + sa_{3} + a5 + a6)^{2}}{(1 - (a_{2} + Sa_{3}))^{2}}$

 $d(x_2, x_3) \le \left(\frac{(a_1 + sa_3 + a_5 + a_6)}{1 - (a_2 + sa_3)}\right)^2 d(x_0, x_1) + 2\left[\frac{(a_1 + sa_3 + a_5 + a_6)}{1 - (a_2 + sa_3)}\right]^2$

Continuing this process, we obtain by induction a sequence $\{x_n\}$ such that $x_n \in Tx_{n-1}, x_{n+1} \in Tx_n$ such that

$$d(xn, xn+1) \leq \frac{(a1+sa3+a5+a6)}{1-(a2+sa3)} d(xn-1, xn) + \left[\frac{(a1+sa3+a5+a6)}{(1-(a2+sa3))}\right]^n$$

for all $n \in \mathbb{N}$ and let $k = \frac{(a1+sa3+a5+a6)}{1-(a2+sa3)}$

$$d(xn, xn+1) \leq kd(x_{n-1}, x_n) + k^n \leq k[kd(x_{n-2}, x_{n-1}) + k^{n-1}] + k^n$$
$$= k^2 d(x_{n-2}, x_{n-1}) + kk^{n-1} + k^n$$

$$d(x_n, x_{n+1}) \le k^n d(x_0, x_1) + nk^n$$

Since k < 1, $\sum k^n$ and $\sum nk^n$ have same radius of convergence. Then, $\{x_n\}$ is a Cauchy sequence. But (X, d) is a complete *b*-metric space, it follows that $\{xn\}\infty n=0$ is convergent.

$$u = \lim n \to \infty x n.$$

Now,

 $d(u, Tu) \le s[d(u, xn+1)+d(xn+1, Tu)]$ $d(u, Tu) \le s[d(u, xn+1)+d(Txn, Tu)]$

Using (1), we obtain,

 $d(u,Tu) \leq s[d(u,xn+1)] + s[a1d(xn,Txn) + a2d(u,Tu) + a3d(xn,Tu) + a4d(u,Txn) + a5d(xn,u) + a6d(xn,u)].$

As $n \rightarrow \infty$,

 $d(u, Tu) \le s[a2d(u, Tu) + a3d(u, Tu)](1 - (a2s + a3s))d(u, Tu) \le 0.$

The above inequality is true unless d(u, Tu) = 0. Thus, Tu = u. Now we show that u is the unique fixed point of T. Assume that v is another fixed point of T. Then we have Tv = v and

 $d(u, v) = d(Tu, Tv) \leq s[d(u, Tv) + d(v, Tu)]$

we obtain, $d(u, v) \le 2sd(u, v)$. This implies that u = v. This completes the proof.

D. Theorem

Let(X, d) be a complete b-metric space with constant $\lambda \ge 1$. Let T, S:X \rightarrow CB(X) be a multi valued mapping satisfies the condition: H(Tx, Sy) $\le a_1d(x, Tx) + a_2d(y, Sy) + a_3d(x, Sy) + a_4d(y, Tx) + a_5d(x, y),$

for all $x, y \in X$ and $a_i \ge 0$, i = 1, 2, ...5, with $(a_1 + a_2)(\lambda + 1) + (a_3 + a_4)(\lambda^2 + \lambda) + 2\lambda a_5 < 2$, $a_1 + a_2 + a_3 + a_4 + a_5 < 1$. Then T and S have a unique common fixed point.

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E. Proof Fix any $x \in X$. Define $x_0 = x$ and let $x_1 \in Tx_0$, $x_2 \in Sx$ such that $x_{2n+1} = Tx_{2n}$, $x_{2n+2} = Sx_{2n+1}$, By Lemma 7, we may choose $x_2 \in Sx_1$ such that $d(x_1, x_2) \leq H(Tx_0, Sx_1) + (a_1 + a_5 + \lambda a_3)$ $d(x_1, x_2) \le a_1 d(x_0, Tx_0) + a_2 d(x_1, Sx_1) + a_3 d(x_0, Sx_1) + a_4 d(x_1, Tx_0) + a_5 d(x_0, x_1) + (a_1 + a_5 + \lambda a_3)$ $= a_1 d(x_0, x_1) + a_2 d(x_1, x_2) + a_3 d(x_0, x_2) + a_4 d(x_0, x_1) + a_5 d(x_0, x_1) + (a_1 + a_5 + \lambda a_3)$ $\leq a_1 d(x_0, x_1) + a_2 d(x_1, x_2) + a_3 \lambda [d(x_0, x_1) + d(x_1, x_2)] + a_5 d(x_0, x_1) + (a_1 + a_5 + \lambda a_3)$ (2) $d(x_1, x_2) \le (a_1 + \lambda a_3 + a_5)d(x_0, x_1) + (a_2 + \lambda a_3)d(x_1, x_2) + (a_1 + a_5 + \lambda a_3)$ $\leq \frac{(a_{1}+a_{5}+\lambda a_{3})}{1-(a_{2}+\lambda a_{3})} d(x_{0},x_{1}) + \frac{(a_{1}+a_{5}+\lambda a_{3})}{1-(a_{2}+\lambda a_{3})}$ On the other hand and by symmetry, we have $d(x_2, x_1) = d(Sx_1, Tx_0)$ $\leq H(Sx_1, Tx_0) + (a_2 + a_5 + \lambda a_4)$ $\leq a_1d(x_1,Sx_1)+a_2d(x_0,Tx_0)+a_3d(x_1,Tx_0)+a_4d(x_0,Sx_1)+a_5d(x_1,x_0)+(a_2+a_5+\lambda a_4)$ $=a_1d(x_1,x_2)+a_2d(x_0,x_1)+a_3d(x_1,x_1)+a_4d(x_0,x_2)+a_5d(x_0,x_1)+(a_2+a_5+\lambda a_4)$ (3) $\leq a_1 d(x_1, x_2) + a_2 d(x_0, x_1) + a_4 [d(x_0, x_1) + d(x_1, x_2)] + a_5 d(x_0, x_1) + (a_2 + a_5 + \lambda a_4) = (a_2 + a_5 + \lambda a_4) d(x_0, x_1) + (a_1 + \lambda a_4) d(x_2, x_1) (a_2 + a_5 + \lambda a_4) = (a_2 + a_5 + \lambda a_4) d(x_2, x_1) (a_2 + a_5 + \lambda a_4) d(x_2, x_1) + (a_2 + a_5 + \lambda a_4) d(x_2, x_1) + (a_3 + a_4) d(x_2, x_1) (a_3 + a_5) d(x_3, x_1) + (a_3 + a_5) d(x_3, x_1) d(x_3, x_1$ $d(x_2, x_1) \leq \frac{(a2+a5+\lambda a4)}{1-(a1+\lambda a4)} d(x_0, x_1) + \frac{(a2+a5+\lambda a4)}{1-(a1+\lambda a4)}$ Adding inequalities (2) and (3), we obtain $d(x1,x2) \le \frac{a1+a2+Sa3+Sa4+2a5}{2-(a1+a2+Sa3+Sa4)} d(x_0,x_1) + \frac{(a1+a2+Sa3+Sa4+2a5)}{2-(a1+a2+Sa3+Sa4)} d(x_0,x_1) + \frac{(a1+a2+Sa3+Sa4+2a5)}{2-(a1+a2+Sa3+Sa4+2a5)} d(x_0,x_1) + \frac{(a1+a2+Sa3+Sa4+2a5)}{2-(a1+a2+Sa3+Sa4+2a5} d(x_0,x_1) + \frac{(a1+a2+Sa3+Sa4+2a5)}{2-(a1+a2+Sa3+Sa4+2a5)} d(x_0,x_1) + \frac{(a1+a2+Sa3+Sa4+2a5)}{$ Similarly, it can be shown that, there exists $x_3 \in Tx_2$ such that $d(x_3, x_2) \le H(Tx_2, Sx_1) + \left(\frac{a_1 + a_2 + \lambda a_3 + \lambda a_4 + 2a_5}{2 - (a_1 + a_2 + \lambda a_3 + \lambda a_4)}\right)^2$ $\leq k^2 d(x_1, x_0) + 2k^2$ Continuing this process, we obtain by induction a sequence $\{x_n\}$ such that $x_{2n+1} \in Tx_{2n}, x_{2n+2} \in Sx_{2n+1}$ such that $d(x_{2n+1}, x_{2n+2}) \le d(Tx_{2n}, Sx_{2n+1}) + (a_1 + a_5 + \lambda a_3)^{2n+1}$ $\leq a_1 d(x_{2n}, Tx_{2n}) + a_2 d(x_{2n+1}, Sx_{2n+1}) + a_3 d(x_{2n}, Sx_{2n+1}) + a_4 d(x_{2n+1}, Tx_{2n}) + a_5 d(x_{2n}, x_{2n+1}) + (a_1 + a_5 + \lambda a_3)^{2n+1}$ (4) $d(x_{2n+1}, x_{2n+2}) \le \frac{(a1+a5+\lambda a3)}{1-(a2+\lambda a3)} d(x_{2n}, x_{2n+2}) + \frac{(a1+a5+\lambda a3)2n+1}{(1-(a2\lambda a3))2n+1}$ Also, $d(x_{2n+2}, x_{2n+1}) \le \frac{(a^2 + a^5 + \lambda a^4)}{1 - (a^2 + \lambda a^4)} d(x_{2n+1}, x_{2n}) + \frac{(a^2 + a^5 + \lambda a^4)^{2n+1}}{(1 - (a^2 \lambda a^3))^{2n+1}}$ (5) From (4) and (5) $d(x_{2n+1}, x_{2n+2}) \le kd(x_{2n+1}, x_{2n}) + k^{2n+1}$ Therefore, $d(x_n, x_{n+1}) \leq \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})n} n \in \mathbb{N} \text{ and let } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})n} n \in \mathbb{N} \text{ and let } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})n} n \in \mathbb{N} \text{ and let } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4})n} n \in \mathbb{N} \text{ and let } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} n \in \mathbb{N} \text{ and let } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} n \in \mathbb{N} \text{ and } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} n \in \mathbb{N} \text{ and } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} n \in \mathbb{N} \text{ and } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} n \in \mathbb{N} \text{ and } \mathbb{k} = \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_3+\lambda a_4+2a_5})n} d(x_{n-1}, x_n) + \frac{a_{1+a_2+\lambda a_3+\lambda a_4+2a_5}}{2-(a_{1+a_2+\lambda a_5$ $d(x_n, x_{n+1}) \le k d(x_{n-1}, x_n) + k^n$ $\leq k(d(x^{n-2}, x_{n-1})+k^{n-1})+k^n$ $=k^2d(x^{n-2}, x^{n-1})+2k^n$

$$\leq \dots \dots$$

$$\leq k^n d(x_0, x_1) + nk^n$$
.

Since 0 < k < 1, $\sum k^n$ and $\sum nk^n$ have same radius of convergence. Then, $\{x_n\}$ is a Cauchy sequence. Since (X, d) is complete, there exists $z \in X$ such that $x_n \rightarrow z$.

We shall prove that z is a common fixed point of T and S.

$$d(z, Sz) \leq \lambda[d(z, x_{2n+1}) + d(x_{2n+1}, Tz)]$$

$$\leq \lambda[d(z, x_{2n+1}) + H(x_{2n+1}, Tz)]$$

$$d(z, Sz) \leq \lambda[d(z, x_{2n+1}) + d(x_{2n+1}, Sz)]$$

$$\leq \lambda[d(z, x_{2n+1}) + H(x_{2n}, Sz)]$$

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(6)

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 $H(x_{2n}, Sz) \le a_1 d(x_{2n}, Tx_{2n}) + a_2 d(z, Sz) + a_3 d(x_{2n}, Sz) + a_4 d(z, Tx_{2n}) + a_5 d(x_{2n}, z)$ (7)

Using (7) in (6) and letting as $n \to \infty$, we obtain,

 $d(z, Sz) \le \lambda d(z, z) + \lambda [a_1 d(z, z) + a_2 d(z, Sz) + a_3 d(z, Sz) + a_4 d(z, z) + a_5 d(z, z)]$

 $=\lambda[a_2d(z,Sz)+a_3d(z,Sz)]$

 $\leq \lambda(a_2+a_3)d(z, Sz)$

 $[1 - \lambda(a_2 + a_3)]d(z, Sz) \le 0.$

1 - $\lambda(a_2 + a_3) \leq 0$ and S(z) is closed. Thus, S(z) = z.

Similarly,
$$T(z) = z$$
.

We show that z is the unique fixed point of S and T. Now,

 $d(z, v) \le H(Tz, Sv)$

 $\leq a_1 d(z, Tz) + a d(v, Sv) + a_3 d(z, Sv) + a_4 d(v, Tz) + a_5 d(z, v)$

$$\leq a_3 d(z, v) + a_4 d(z, v) + a_5 d(z, v).$$

Since $[1 - (a_3 + a_4 + a_5)] > 0$, d(z, v) = 0. Hence, S and T have a unique common fixed point.

III. CONCLUSION

Many authors have contributed some fixed point results for a self mappings in b-metric spaces. In this paper, we have proved the existence and uniqueness of fixed point results for a multivalued mappings in b-metric spaces. Our contraction mappings also generalize various known contractions like Hardy Roger contraction in the current literature.

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