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# $\hat{\delta}\omega$ Closed Sets in Ideal Topological Space

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**Abstract:** In this paper the notion of  $\hat{\delta}\omega$  closed sets is introduced and some of its basic properties are studied. This new class of sets is independent of semi-closed and closed sets. Also the relationship with some of the known closed sets is discussed.

**Keywords:**  $\hat{\delta}\omega$  closed sets, closed sets,  $\omega$  closed sets.

## I. INTRODUCTION

Levine, velicko introduced the notions of generalized closed (briefly gclosed) and  $\delta$  closed sets respectively and studied their basic properties. The notion of  $I_g$  closed sets was first introduced by Dontchev in 1999. Navaneetha Krishnan and Joseph further investigated and characterized  $I_g$  closed sets. Julian Dontchev and maximilian Ganster, Yuksel, Acikgoz and Noiri introduced and studied the notions of  $\delta$  generalized closed (briefly  $\delta g$  closed) and  $\delta$ -I-closed sets respectively. The purpose of this paper is to define a new class of sets called  $\hat{\delta}\omega$  closed sets and also study some basic properties and characterizations.

Throughout this paper  $(X, \tau, I)$  represents a ideal topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset  $A$  of a ideal topological space  $X$ ,  $cl(A)$  and  $int(A)$  denote the closure of  $A$  and the interior of  $A$  respectively.  $X \setminus A$  or  $A^c$  denotes the complement of  $A$  in  $X$ . We recall the following definitions and results.

## II. PRELIMINARIES

A. subset  $A$  of a space  $X$  is called

pre-open set if  $A \subseteq intcl(A)$  and pre-closed set if  $clint(A) \subseteq A$ .

semi-open set if  $A \subseteq clint(A)$  and semi-closed set if  $intcl(A) \subseteq A$ .

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regular open set if  $A = intcl(A)$  and regular closed set if  $A = clint(A)$ .

$\Pi$  -open set if  $A$  is a finite union of regular open sets.

regular semi open if there is a regular open  $U$  such  $U \subseteq A \subseteq cl(U)$

B. A subset  $A$  of  $(X, \tau)$  is called

generalized closed set, if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

regular generalized closed set, if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .

weakly generalized closed set, if  $clint(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

weakly closed set, if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi open in  $X$ .

regular weakly generalized closed set, if  $clint(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$

is regular open in  $X$ .

regular weakly closed if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is regular semi open.

$g$ -closed if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $w$ -open.

Let  $A$  and  $B$  be subsets of an ideal topological space  $(X, \tau, I)$ . Then, the following properties holds.

$A \subseteq \sigma cl(A)$ .

If  $A \subset B$ , then  $\sigma cl(A) \subset \sigma cl(B)$ .

$\sigma cl(A) = \bigcap \{ F \subset X \mid A \subset F \text{ and } F \text{ is } \delta - I - \text{closed} \}$ .

If  $A$  is a  $\delta$  -I-closed set of  $X$  for each  $\alpha \in \Delta$ , then  $\bigcap \{ A\alpha / \alpha \in \Delta \}$  is  $\delta$  -I-closed.

$\sigma cl(A)$  is  $\delta$ -I-closed.

$\delta$ .I closure is  $\{x \in X: int(cl^*(U)) \cap A \neq \emptyset, U \in I\}$ .

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## III. $\hat{\delta}\omega$ - CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

**Definition 3.1 :** A subset  $A$  of an ideal space  $(X, \tau, I)$  is called  $\hat{\delta}\omega$  closed, if  $\sigma cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$  open.  
**Theorem 3.1**

Every g-closed set in  $X$  is  $\hat{\delta}\omega$ -closed set in  $X$ .

**Proof:** Let  $A$  be an arbitrary g-closed set in the space  $X$ . Suppose  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. i.e.,  $A \subseteq U$  and  $U$  is open. Then by the  $\hat{\delta}\omega$  - closed sets in ideal topological space

definition of  $\hat{\delta}\omega$ -closed set, if  $\sigma cl(A) \subseteq U$ , Whenever  $A \subseteq U$  and  $U$  is  $\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of g-closed set belongs to  $U$  and also the arbitrary element  $A$  of  $\hat{\delta}\omega$ -closed set belongs to  $U$ . This implies  $A$  is a  $\hat{\delta}\omega$  - closed set.

The converse of the above theorem is not true, which is verified from the following example.

**Example 3.1:** Let  $X = \{b, c, d\}$  be with topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $X$ . Then g-closed set will be  $\{\phi, \{b\}, \{b, c\}, \{b, d\}, X\}$ . Here,  $A = \{c\}$  is a set.  $\hat{\delta}\omega$ -closed set but not g-closed.

**Theorem 3.2**

Every closed set in  $X$  is  $\hat{\delta}\omega$ -closed set in  $X$ .

**Proof:** Let  $A$  be an arbitrary closed set in the space  $X$ , every closed set is g-closed set and from the theorem 3.1, every g-closed set in  $X$  is  $\hat{\delta}\omega$ -closed. Thus every closed set in  $X$  is  $\hat{\delta}\omega$ -closed.

The converse of the above theorem is not true, which is verified from the following example.

**Example 3.2:** Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$  and  $\tau^c = \{\phi, \{b, d\}, \{b, c\}, \{b\}, X\}$ . Thus, the closed set is  $\{\phi, \{b, d\}, \{b, c\}, \{b\}, X\}$ . Here  $A = \{c, d\}$  is  $\hat{\delta}\omega$ -closed set but not closed set. **Theorem 3.3**

Every regular closed set in  $X$  is  $\hat{\delta}\omega$ -closed

**Proof:** Let  $A$  be an arbitrary regular closed set in the space  $X$ , every regular closed set is closed and from the theorem 3.1, every g-closed set in  $X$  is  $\hat{\delta}\omega$ -closed. This implies every regular closed set in  $X$  is  $\hat{\delta}\omega$ -closed.

The converse of the above theorem is not true, which is verified from the following example.

**Example 3.3:** Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$  and  $\tau^c = \{\phi, \{b, d\}, \{b, c\}, \{b\}, X\}$ , the regular closed set, here is  $\{\phi, \{b, d\}, \{b, c\}, X\}$ . Then  $A = \{c, d\}$  is  $\hat{\delta}\omega$ -closed set but not regular closed.

**Theorem 3.4**

Every regular generalized closed set in  $X$  is  $\hat{\delta}\omega$ -closed.

**Proof:** Let  $A$  be an arbitrary regular generalized closed set in the space  $X$ . Suppose  $cl(A) \subseteq U$ . Whenever  $A \subseteq U$  and  $U$  is regular open. i.e.,  $A \subseteq U$  and  $U$  is regular open, every regular open set in  $X$  is open. Then by the definition of  $\hat{\delta}\omega$ -closed set, if  $\sigma cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of regular generalized closed set belongs to  $U$  and the arbitrary element  $A$  of  $\hat{\delta}\omega$ -closed set belongs to  $U$ . This implies that  $A$  is a  $\hat{\delta}\omega$ -closed set.

The converse of above theorem need not be true, which is verified from the following example.

**Example 3.4:** Let  $X = \{b, c, d\}$  be with topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$  and  $\tau^c = \{\phi, \{b, d\}, \{b, c\}, \{b\}, X\}$ . Then if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ . Then regular generalized closed set will be  $\{\phi, \{b, d\}, \{b, c\}, \{c, d\}, \{b\}, X\}$ . Here  $A = \{c\}$  is a  $\hat{\delta}\omega$ -closed set but not regular generalized closed set.

**Theorem 3.5**

Every weakly generalized closed set in  $X$  is  $\hat{\delta}\omega$ -closed.

**Proof:** Let  $A$  be an arbitrary weakly generalized closed set in the space  $X$ . Then by definition of weakly generalized closed set and  $\hat{\delta}\omega$ -closed set the arbitrary element  $A$  of weakly generalized closed set belongs to  $U$  and the arbitrary element  $A$  of  $\hat{\delta}\omega$ -closed set belongs to  $U$ . This implies that  $A$  is a  $\hat{\delta}\omega$ -closed set.

The converse of the above theorem need not to be true, which is verified from the following example.

**Example 3.5:** Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Now, if  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ . Then wg-closed set will be  $\{\phi, \{b, c\}, \{b, d\}, \{b\}, X\}$ . Here  $A = \{c\}$  is a  $\hat{\delta}\omega$ -closed set but not weakly generalized closed.

**Theorem 3.6**

Every semi closed set in  $X$ , is  $\hat{\delta}\omega$ -closed.

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Proof: Let  $A$  be an arbitrary semi closed set in the space  $X$ , every semi closed set is closed and from the theorem 3.1 every closed set in  $X$  is  $\delta\omega$  -closed set. This implies, every semi closed set in  $X$  is  $\delta\omega$  closedset.

The converse of the above theorem is not true, which is verified from the following example.

Example 3.6: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Then from the definition of semi closed set  $\{\phi, \{b, d\}, \{b, c\}, X\}$ . Here  $A = \{c\}$  is  $\delta\omega$  -closedset, but not semi-closed.

Theorem 3.7

Every weakly closed set in  $X$  is  $\delta\omega$  -closed set.

Proof: Let  $A$  be a weakly closed set in the space  $X$ . Suppose  $\text{cl}(A) \subseteq U$  When ever  $A \subseteq U$  and  $U$  is semi open in  $X$ . i.e., when ever  $A \subseteq U$  and  $U$  is semi Open, every semi open set is open in  $X$ . Then by the definition of  $\delta\omega$  -closed set. if  $\text{sc}l(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$ open in  $X$ . Hence, the arbitrary element  $A$  of weakly closed set belongs to  $U$  and the arbitrary element  $A$  of  $\delta\omega$  -closed set belongs to  $U$ . This implies that  $A$  is a  $\delta\omega$  -closed set in  $X$ .

The converse of the above theorem need not to be true, which is verified from the following example.

Example 3.7: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is semiopen in  $X$ . Then  $U = \{\phi, \{c\}, \{d\}, X\}$ . Then weakly closed set will be  $\{\phi, \{b\}, \{b, d\}, \{b, c\}, \{c, d\}, X\}$ . Here  $A = \{c\}$  is a closed set in  $\delta\omega$  - closed set, but not Weakly  $X$ .

Theorem 3.8

Every regular weakly generalized closed set in  $X$  is  $\delta\omega$  - closed.

Proof: Let  $A$  be a regular weakly generalized closed set in the space  $X$ . Suppose  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ . i.e.,  $A \subseteq U$  and  $U$  is open, every regular open set in  $X$  is open. Then by the definition of  $\delta\omega$  closed set, if  $\text{sc}l(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\delta\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of regular weakly generalized closed set belongs to  $U$  and the arbitrary element  $A$  of  $\delta\omega$  -closed set belongs to  $U$ . This implies that  $A$  is a  $\delta\omega$  -closed set in  $X$ .

example 3.8: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $\text{cl}(\text{int}(A)) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ . Then  $U = \{\phi, \{c\}, \{d\}, X\}$ . Then regular weakly generalized closed set will be  $\{\phi, \{b\}, \{b, d\}, \{b, c\}, \{c, d\}, X\}$ . Here  $A = \{c\}$  is a set,  $\delta\omega$  closed set but not regular weakly generalized.

Theorem 3.9

Every regular semi closed set in  $X$  is  $\delta\omega$  closed set.

Proof: Let  $A$  be an arbitrary regular semi closed set in the space in  $X$ . Suppose  $U \subseteq A \subseteq \text{cl}(U)$  whenever  $U$  is regular open set in  $X$  is open. i.e.,  $U$  is open. Then by the definition of  $\delta\omega$  -closed set, if  $\text{sc}l(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\delta\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of regular semi closed set belongs to  $U$  and the arbitrary element  $A$  of  $\delta\omega$  -closed set belongs to  $U$ . Thus we can say that This implies that  $A$  is a  $\delta\omega$  -closed set.

The converse of the above theorem need not to be true, which is verified from the following example.

Example 3.9: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$  and  $\tau^c = \{\phi, \{b, d\}, \{b, c\}, \{b\}, X\}$ . Then, from the definition of regular semi closed set is  $\{\phi, \{c\}, \{d\}, \{b, d\}, \{b, c\}, \{c, d\}, X\}$ . Here let  $A = \{b\}$  is  $\delta\omega$  -closed set but not regular semi closed.

Theorem 3.10

Every regular weakly closed set in  $X$  is  $\delta\omega$  - closed.

Proof: Let  $A$  be an arbitrary regular weakly closed set in the space  $X$ , every semi open set is open and from the theorem 3.2, every closed set in  $X$  is  $\delta\omega$  -closed set This implies that every regular weakly closed set in  $X$  is  $\delta\omega$  -closed set

The converse of the above theorem is not true, which is verified using following example

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Example 3.10: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\phi, \{c\}, \{d\}, \{c, d\}, X\}$ . Then by the definition of regular weakly closed set  $\{\phi, \{c\}, \{d\}, \{b\}, \{b, c\}, \{b, d\}, X\}$ . Here  $A = \{c, d\}$  is  $\delta\omega$  - closed set but not regular weakly closed.

Theorem 3.11

Every  $*g$  -closed set in  $X$  is  $\delta\omega$  - closed.

Proof: Let  $A$  be an arbitrary  $*g$  -closed set in the space  $X$ . Suppose  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is semi open  $X$ . i.e., whenever  $A \subseteq U$  and  $U$  is semi open, every weakly open set is open in  $X$ . Then by the definition of  $\delta\omega$  -closed set, if  $\text{sc}l(A) \subseteq U$



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$U$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of  $\ast g$ -closed set belongs to  $U$  and the arbitrary element  $A$  of  $\delta\omega$  closed set belongs to  $U$ . This implies that  $A$  of  $\delta\omega$  - closed set in  $X$ .

the converse of the above theorem need not to be true, which is verified using following example.

Example 3.11: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is weakly open in  $X$ . Then  $U = \{\emptyset, \{c, d\}, \{c\}, \{b\}, \{d\}, X\}$ . Then  $\ast g$  closed set will be  $\{\emptyset, \{b\}, \{b, d\}, \{b, c\}, X\}$ . Here  $A = \{c\}$  is a  $\delta\omega$  closed set, but not  $\ast g$ -closed set in  $X$ .

**Theorem 3.12**

Every  $\theta$ -closed set in  $X$  is  $\delta\omega$ -closed set.

Proof: Let  $A$  be an arbitrary  $\theta$ -closed set in space  $X$ . Suppose  $\text{cl}_\theta(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open. i.e.,  $A \subseteq U$  and  $U$  is open. Then by the definition of  $\delta\omega$ -closed set, if  $\sigma\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\delta\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of  $\theta$ -closed set belongs to  $U$  and also the arbitrary element  $A$  of  $\delta\omega$ -closed set belongs to  $U$ . This implies that,  $A$  is  $\delta\omega$  closed.

The converse of the above theorem is not true, which is verified from the following example.

Example 3.12: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $\text{cl}_\theta(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $X$ . Then  $\theta$ -closed set will be  $\{\emptyset, \{b, c\}, \{b, d\}, \{b\}\}$ . Here  $A = \{c\}$  is a  $\delta\omega$ -closed set, but not  $\theta$ -closed set.

**Theorem 3.13**

Every  $\delta$ -closed set in  $X$  is  $\delta\omega$ -closed set in  $X$ .

Proof: Let  $A$  be an arbitrary  $\delta$ -closed set in space  $X$ . Suppose  $\text{cl}_\delta(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open. i.e.,  $A \subseteq U$  and  $U$  is open. Then by the definition  $\delta\omega$ -closed set, if  $\sigma\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\omega$  open in  $X$ . Hence, the arbitrary element  $A$  of  $\delta$ -closed set belongs to  $U$  and also the arbitrary element  $A$  of  $\delta$ -closed set belongs to  $U$ . This implies that,  $A$  is a  $\delta\omega$  closed.

The converse of the above theorem is not true, which is verified from the following example

Example 3.13: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$ . Now if  $\text{cl}_\delta(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is open in  $X$ . Then  $\delta$ -closed set will be  $\{\emptyset, \{b, c\}, \{b, d\}, \{b\}\}$ . Here  $A = \{c\}$  is a  $\delta\omega$  closed set, but not  $\delta$ -closed.

### IV. SOME OPERATIONS ON $\delta\omega$ - CLOSED SETS

**Theorem 4.1**

The union of two  $\delta\omega$  closed sets of  $X$  is also an  $\delta\omega$  -closed sets of  $X$ .

Proof: Assume that  $A$  and  $B$  are  $\delta\omega$  - closed set in  $X$ . Let  $U$  be open in  $X$ , such that  $A \cup B \subseteq U$ . Thus  $A \subseteq U$  and  $B \subseteq U$ . Since  $A$  and  $B$  are  $\delta\omega$  - closed set so  $\sigma\text{cl}(A) \subseteq U$  and  $\sigma\text{cl}(B) \subseteq U$ . Hence  $\sigma\text{cl}(A \cup B) = \sigma\text{cl}(A) \cup \sigma\text{cl}(B) \subseteq U$ . i.e.,  $\sigma\text{cl}(A \cup B) \subseteq U$ . Hence  $A \cup B$  is an  $\delta\omega$  - closed set in  $X$ .

**Theorem 4.2**

If a subset  $A$  of  $X$  is  $\delta\omega$  - closed in  $X$ , then  $\sigma\text{cl}(A) \setminus A$  does not contain any non-empty open set in  $X$ .

Proof: Suppose that  $A$  is  $\delta\omega$  -closed set in  $X$ . Let  $U$  be open set such that  $\sigma\text{cl}(A) \setminus A \subseteq U$  and  $U \neq \emptyset$ . Now  $U \subseteq \sigma\text{cl}(A) \setminus A$ , i.e.,  $U \subseteq X \setminus A$  which implies that  $A \subseteq X \setminus U$ . As  $U$  is open,  $X \setminus U$  is also open in  $X$ . Since  $A$  is an  $\delta\omega$   $\delta\omega$ -Closed sets in ideal topological  $\omega$ closed set in  $X$ , by definition of  $\delta\omega$ -closed set, we have  $\sigma\text{cl}(A) \subseteq X \setminus U$ . So  $U \subseteq X \setminus \sigma\text{cl}(A)$ . Therefore  $U \subseteq \sigma\text{cl}(A) \cap (X \setminus \sigma\text{cl}(A)) = \emptyset$ . This show that  $U = \emptyset$ , which is contradiction. Hence  $\sigma\text{cl}(A) \setminus A$  does not contain any nonempty open set in  $X$ .

**Theorem 4.3**

For an element  $x \in X$ , the set  $X \setminus \{x\}$  is  $\delta\omega$  - closed or  $\omega$  open.

Proof: Let  $x \in X$ . Suppose  $X \setminus \{x\}$  is not  $\omega$  open. Then  $X$  is the only  $\omega$  open set containing  $X \setminus \{x\}$ , which means that the only choice of  $\omega$  open set containing  $X \setminus \{x\}$  is  $X$ . i.e.,  $X \setminus \{x\} \subset X$ . Also, we know  $X \setminus \{x\}$  is not  $\delta\omega$ -closed. To prove  $X \setminus \{x\}$  is open. Suppose  $X \setminus \{x\}$  is not open. As  $X \setminus \{x\}$  is a subset of  $x$  and  $X \setminus \{x\}$  only but  $X \setminus \{x\}$  is not open. Thus the only open set in  $X$ . Also  $\sigma\text{cl}(X \setminus \{x\}) \subset X$ . Therefore by the definition of  $\delta\omega$  - closed sets  $X \setminus \{x\}$  is  $\delta\omega$  - closed, which is a contradiction. Hence  $X \setminus \{x\}$  is  $\omega$  open.

**Theorem 4.4**

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If  $A$  is an  $\delta\omega$ -closed subset of  $X$  such that  $A \subset B \subset \sigma\text{cl}(A)$ , then  $B$  is an  $\delta\omega$ -closed set in  $X$ .

Proof: Let  $A$  be an  $\delta\omega$ -closed in  $X$ , such that  $A \subset B \subset \sigma\text{cl}(A)$ . Let  $U$  be open set such that  $B \subset U$ , then  $A \subset U$ . Since  $A$  is  $\delta\omega$ -closed, we have  $\sigma\text{cl}(A) \subset U$ . Now as  $B \subset \sigma\text{cl}(A)$ . So  $\sigma\text{cl}(B) \subset \sigma\text{cl}(\sigma\text{cl}(A)) \subset \sigma\text{cl}(A) \subset U$ . Thus  $\sigma\text{cl}(B) \subset U$ , whenever  $B \subset U$  and  $U$  is  $\omega$  open. Therefore  $B$  is a  $\delta\omega$ -closed in  $X$ .

Converse of the theorem is not true, which is verified from the following example.

Example 4.1: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$   $\delta\omega$ -closed set is  $\{\emptyset, \{c\}, \{d\}, \{b\}, \{b, c\}, \{c, d\}, \{b, d\}, X\}$ . Now  $\sigma\text{cl}(A) = \{d\}$  which is contained in each  $\omega$  open set.  $\sigma\text{cl}(B) = \{c\}$ , which is also contained in each open set. Thus by the definition,  $A$  and  $B$  both are  $\delta\omega$ -closed.

Theorem 4.5

If a subset  $A$  of a topological space  $X$  is both  $\omega$  open and  $\delta\omega$ -closed then it is  $\omega$  closed.

Proof: Suppose a subset  $A$  of a topological space in  $X$  is both  $\omega$  open and  $\delta\omega$ -closed. Now  $A \subset A$  then by definition of  $\delta\omega$ -closed we have  $\sigma\text{cl}(A) \subset A$ . So  $A \subset \sigma\text{cl}(A)$ . Thus we have  $\sigma\text{cl}(A) = A$ . Finally  $A$  is open.

Theorem 4.6

If a subset  $A$  of a topological space  $X$  is both open and  $\delta\omega$ -closed then it is closed.

Proof: Suppose a subset  $A$  of a topological space in  $X$  is both open and  $\delta\omega$ -closed. Now  $A \subset A$  then by definition of  $\delta\omega$ -closed we have  $\sigma\text{cl}(A) \subset A$ . So  $A \subset \sigma\text{cl}(A)$ . Thus we have  $\sigma\text{cl}(A) = A$ . Finally  $A$  is open.

Theorem 4.7

In a topological space  $X$  if open of  $X$  are  $\{X, \emptyset\}$ , then every subset of  $X$  is an  $\delta\omega$ -closed set.

Proof: Let  $X$  be topological space and open. i.e.,  $\{X, \emptyset\}$ . Suppose  $A$  be any arbitrary subset of  $X$ , if  $A = \emptyset$  then  $X$  is an  $\delta\omega$ -closed set in  $X$ . If  $A \neq \emptyset$  then  $X$  is the only open set containing  $A$  and so  $\sigma\text{cl}(A) \subset X$ . Hence by the definition  $A$  is  $\delta\omega$ -closed in  $X$ .

The converse is not true, which is verified by following example.

Example 4.2: Let  $X = \{b, c, d\}$  be with the topology  $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$ , every subset of  $X$  is an  $\delta\omega$ -closed set in  $X$ . Thus for every subset of  $X$  is an  $\delta\omega$ -closed set, we need not be open set only if  $\{X, \emptyset\}$ .

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