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Solution of Third Order Korteweg -De Vries Equation by Homotopy Perturbation Method Using Elzaki Transform

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Abstract: This Paper is discussing the theoretical approach of Elzaki transform [1] coupled with Homotopy Perturbation Method [3] that can be applied to higher order partial differential equations for finding exact as well as approximate solutions of the equations. Here Homotopy Perturbation Method using Elzaki transform[1],[2] has been applied to Korteweg-de vries equation which is of third order homogenous partial differential equation.

Keywords: Homotopy perturbation, Korteweg-de vries equation, Elzaki transforms.

1. INTRODUCTION

The korteweg-de vries equation [4], [5] $\frac{\partial u}{\partial t} + a\frac{\partial^3 u}{\partial x^3} + bu\frac{\partial u}{\partial x} = 0$ (1.1)

Was first derived by korteweg and vries(1895) to the water waves in shallow canal, when the study of water waves was of vital interest for applications in naval architecture and for the knowledge of tides and floods.

The canonical Korteweg-de Vries (KdV) equation $u_t + 6uu_x + u_{xxx} = 0$ is widely recognized as a paradigm for the description of weakly nonlinear long waves in many branches of physics and engineering. Here u(x, t) is an appropriate field variable, t is the time coordinates, and x is the space coordinate in the relevant direction. It describes how waves evolve under the competing but comparable effects of weak nonlinearity and weak dispersion.

2. ELZAKI TRANSFORM ON KDV EQUATION: A THEORETICAL APPROACH

Consider the operator form of KDV equation (1.1) as

$$Du + Ru + Nu = 0 \tag{2.1}$$

$$U(x, 0) = f(x) \tag{2.2}$$

Where D is a linear differential operator with respect to t, R is a linear differential operator with respect to x and N is a non linear differential operator.

On Taking Elzaki transform on both sides of equation (2.2) it gives,

$$E[D\,u(x,t)] + E[R\,u(x,t)] + E[N\,u(x,t)] = 0 \tag{2.3}$$

By using the differentiation property of Elzaki transforms and above initial conditions it yields,

$$E[u(x,t)] = v^2 f(x) - v\,E\left[a\frac{\partial^3 u}{\partial x^3} + bu\frac{\partial u}{\partial x}\right] \tag{2.4}$$

By using Elzaki inverse on both sides of equation (2.4), it yields,

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$$u(x,t) = f(x) - E^{-1}\left[vE\left[a\frac{\partial^3 u}{\partial x^3} + bu\frac{\partial u}{\partial x}\right]\right]$$
(2.5)

Now by applying Homotopy Perturbation Method to (2.5),it gives

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = f(x) - p\{E^{-1}[vE[\sum_{n=0}^{\infty} aR\, u(x,t) + N\, bu(x,t)]]\} \qquad (2.6)$$

The Nonlinear term in equation (2.6) can be decomposed as:

$$N\, u(x,t) = \sum_{n=0}^{\infty} p^n H_n(u)$$
(2.7)

Where $H_n(u)$ are given by

$$H_n(u_0, u_1, \dots \dots \dots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}(p^i u_i)$$

Equation (2.6) becomes

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = f(x) - p\left\{E^{-1}\left[vE\left[\sum_{n=0}^{\infty} p^n b\, H_n(u) + \sum_{n=0}^{\infty} p^n a\frac{\partial^3 u_n}{\partial x^3}\right]\right]\right\} \qquad (2.8)$$

Where $H_n(u)$ are He's polynomials. The first few components of He's polynomial are given by

$$H_0(u) = u_0\frac{\partial u_0}{\partial x}$$

$$H_1(u) = u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x}$$

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On comparing the coefficient of like powers of p, it gives

$$p^0 : u_0(x,t) = f(x)$$

$$p^1 : u_1(x,t) = -E^{-1}\{v\, E[Ra + bH_0(u)]\}$$

$$p^2 : u_2(x,t) = -E^{-1}\{v\, E[Ra + bH_1(u)]\}$$

$$p^3 : u_3(x,t) = -E^{-1}\{v\, E[Ra + bH_2(u)]\}$$

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Hence the approximate solution of equation (3.1.1) is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) \dots \dots \dots \dots \dots \dots \dots \dots$$
(2.9)

Equation (2.9) is the approximate solution of KDV equation which converges to the exact solution.

SPECIAL CASES OF KORTEWEG-DE VRIES EQUATION

CASE1: For a=1 b=1

Equation (2.1) becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$
(2.10)

We consider this problem with initial condition

$$u(x,0) = (1-x)$$
(2.11)

Using the differentiation property of Elzaki transforms and initial condition (2.11), it gives

$$E[u(x,t)] = v^2(1-x) - v\, E\left[u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3}\right]$$
(2.12)

By taking inverse of Elzaki transform on both sides of (2.12), it gives

$$u(x,t) = 1 - x - E^{-1}\left[vE\left[u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3}\right]\right]$$
(2.13)

By applying Homotopy Perturbation Method in equation (2.13) yields

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = 1 - x - p\left\{E^{-1}\left[vE\left[\sum_{n=0}^{\infty} p^n H_n(u) + \sum_{n=0}^{\infty} p^n\frac{\partial^3 u}{\partial x^3}\right]\right]\right\} \qquad (2.14)$$

Where $H_n(u)$ are He's Polynomial to be determined.

Now the He's Polynomial are

$$H_0(u) = u_0\frac{\partial u_0}{\partial x}$$

$$H_1(u) = u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x}$$

$$H_2(u) = u_0\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_0}{\partial x}$$

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On comparing the coefficient of like power of p on both sides,

$p^0 : u_0(x,t) = 1-x$

$H_0(u) = (1-x)(-1) = -(1-x)$

$p^1 : u_1(x,t) = -E^{-1}\left[vE\left[H_0(u) + \frac{\partial^3 u_0}{\partial x^3}\right]\right]$

$= -E^{-1}[vE[-(1-x) + 0]]$

$= (1-x)t$

$H_1(u) = u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x}$

$= (1-x)(-t) + (1-x)t(-1)$

$= -2(1-x)t$

$p^2 : u_2(x,t) = -E^{-1}\left[vE\left[H_1(u) + \frac{\partial^3 u_1}{\partial x^3}\right]\right]$

$= -E^{-1}[vE[-2(1-x)t + 0]]$

$= 2(1-x)\frac{t^2}{2!} = (1-x)t^2$

$H_2(u) = u_0 \frac{\partial u_2}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_0}{\partial x}$

$= (1-x)(-t^2) + (1-x)t(-t) + (1-x)t^2(-1)$

$= -3(1-x)t^2$

$p^3 : u_3(x,t) = -E^{-1}\left[vE\left[H_2(u) + \frac{\partial^3 u_2}{\partial x^3}\right]\right]$

$= -E^{-1}[vE[-3(1-x)t^2 + 0]]$

$= 3(1-x)E^{-1}[vE(t^2)]$

$= 6(1-x)\frac{t^3}{3!}$

Similarly the higher terms p^4, p^5….. Can be determined.

Therefore $u(x,t) = u_0 + u_1 + u_2 + u_3$……………………………….

$= (1-x) + (1-x)t + (1-x)t^2 + (1-x)t^3 +$……………………

$= (1-x)[1 + t + t^2 + t^3 + \dots\dots\dots\dots\dots]$ (2.15)

Equation (2.15) is the approximate solution of (2.10) which converges to the exact solution $u(x,t) = \frac{1-x}{1-t}$.

CASE2: For $a = -1, b = -6$

Equation (2.1) becomes

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^3} - 6u\frac{\partial u}{\partial x} = 0 \qquad (2.16)$$

$$u(x,0) = 1-x \qquad (2.17)$$

Taking Elzaki transform on equation (2.16) subject to initial condition (2.17) gives, $E[u(x,t)] = v^2(1-x) + v\,E\left[6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3}\right]$ (2.18)

By taking inverse of Elzaki transform of equation (2.18) it yields,

$$u(x,t) = 1 - x - E^{-1}\left[vE\left[6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3}\right]\right] \qquad (2.19)$$

Now we apply Homotopy Perturbation Method to equation (3.2.19) it gives,

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = 1 - x - p\left\{E^{-1}\left[vE\left[\sum_{n=0}^{\infty} 6p^n H_n(u) + \sum_{n=0}^{\infty} p^n \frac{\partial^3 u}{\partial x^3}\right]\right]\right\} \qquad (2.20)$$

Where $H_n(u)$ are He's Polynomial.

$H_0(u) = u_0 \frac{\partial u_0}{\partial x}$

$H_1(u) = u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x}$

$H_2(u) = u_0 \frac{\partial u_2}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_0}{\partial x}$

Comparing the coefficient of like power of p,

$p^0 : u_0(x,t) = 1-x$

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$H_0(u) = (1-x)(-1) = -(1-x)$

$p^1:\ u_1(x,t) = E^{-1}\left[vE\left[6H_0(u) + \frac{\partial^3 u_0}{\partial x^3}\right]\right] = E^{-1}[vE[-6(1-x) + 0]] = -6(1-x)t$

$H_1(u) = u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x} = (1-x)(6t) + -6(1-x)t(-1) = 12(1-x)t$

$p^2:\ u_2(x,t) = E^{-1}\left[vE\left[6H_1(u) + \frac{\partial^3 u_1}{\partial x^3}\right]\right] = E^{-1}[vE[72(1-x)t + 0]] = 72(1-x)\frac{t^2}{2!} = 36(1-x)t^2$

$H_2(u) = u_0\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_0}{\partial x}$

$= (1-x)(-36t^2) - 6(1-x)t(6t) + 36(1-x)t^2(-1)$

$= -108(1-x)t^2$

$p^3:\ u_3(x,t) = E^{-1}\left[vE\left[6H_2(u) + \frac{\partial^3 u_2}{\partial x^3}\right]\right]$

$= E^{-1}[vE[-648(1-x)t^2 + 0]]$

$= -648(1-x)E^{-1}[vE(t^2)]$

$= -648(1-x)\frac{t^3}{3!} = -108(1-x)t^3$

$H_3(u) = u_0\frac{\partial u_3}{\partial x} + u_2\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + u_3\frac{\partial u_0}{\partial x} = 432(1-x)t^3$

$p^4:\ u_4(x,t) = E^{-1}\left[vE\left[6H_3(u) + \frac{\partial^3 w_2}{\partial x^3}\right]\right]$

$= E^{-1}[vE[2592(1-x)t^2 + 0]]$

$= 2592(1-x)E^{-1}[vE(t^2)]$

$= 2592(1-x)\frac{t^4}{4!} = 108(1-x)t^4$

Similarly we will find $p^4, p^5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$u(x,t) = u_0 + u_1 + u_2 + u_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= (1-x) - 6(1-x)t + 36(1-x)t^2 - 108(1-x)t^3 + 108(1-x)t^4 + \ldots..$

$= (1-x)[1 - 6t + 36t^2 - 108t^3 + 108t^4 - \ldots\ldots\ldots\ldots\ldots]$

$= (1-x)\{1 - 6t + 36t^2 - 108t^3[1 - t + t^2 - \cdots.]\}$ (2.21)

$= (1-x)\left\{1 - 6t + 36t^2 - \frac{108t^3}{(1+t)}\right\}$ (2.22)

Therefore (2.21) is the approximate solution of equation (2.16) which finally converges to the exact solution.

CONCLUSION

In this paper, korteweg-de vries equation is solved by Homotopy Perturbation Method using Elzaki transform to get its solution in exact form for the given initial conditions. Solutions of Korteweg de Vries (KDV) with initial are calculated to present an exact result of the KDV Equation.

REFERENCES

[1]Elzaki, Tarig M (2011), The New Integral Transform "Elzaki Transform" Global Journal of Pure and Applied Mathematics, ISSN 0973-1768.

[2]Elzaki, Tarig M & Eman M. A.(2012), Homotopy Perturbation and Elzaki Transform for Solving Nonlinear Partial Differential Equations , Mathematical Theory and Modeling ,ISSN 2224-5804 Vol.2.

[3]Hemeda A. A.(2012).Homotopy Perturbation Method for Solving Systems of Nonlinear Coupled Equations. Applied Mathematical Sciences, Vol. 6

[4] The Korteweg-de Vries Equation:History, exact Solutions, and graphical Representation by Klaus Brauer, University of Osnabrück/Germany ,May 2000

[5] The Korteweg–de Vries equation: Its place in the development of nonlinear physics Peter S. Riseborough,Department of Physics, Temple University, Philadelphia, PA 19122, USA.for Solving Nonlinear Partial Differential Equations , Mathematical Theory and Modeling ,ISSN 2224-5804 Vol.2.