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# Chromatic Prime Number for Circular Embedded Graph of the Tensor Product Graph 

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#### Abstract

The Chromatic number of tensor product $(G \otimes H)$ of $G$ and $H$ has vertex set $V(G)$ and $V(H)$. In this paper, we introduced chromatic prime number on circular embedded graph of tensor product Keywords: Circular embedded Graph, Chromatic number and Chromatic prime number. $P^{+}$and $P^{-}$are positive and negative edge prime numbers.


## I. INTRODUCTION

In this paper, we provide few results on chromatic prime number on circular embedded of tensor product graphs for connected, butterfly and pentagon graphs. A graph consists of a set of vertices $V(G)$ and a set of edges $E(G)$. For every vertices $u_{1}, u_{2} V(G)$, the edge connecting $u_{1}$ and $u_{2}$ is denoted by $e_{1}$. Here, we introduced the chromatic prime number for the circular embedded graph of tensor product. We first prove that $\chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H}) \leq \min \left\{\chi_{\mathrm{p}}(\mathrm{G}), \chi_{\mathrm{p}}(\mathrm{H})\right\}$. An assignment of colors to the vertices of a graph, so that no two adjacent vertices get the same color is called a coloring of the graph. The chromatic number of a graph $G$ is the minimum number of colors needs to color the graph G.

## II. PRELIMINARIES

In this section, we introduced the chromatic prime number on tensor product graph with respect to the positive and negative prime number
Definition 1: The Tensor product, $G \otimes H$, of graph $G$ and $H$ is the graph with vertex set $v(G) \times v(H)$ and $(a, x)(b, y) \in E(G \otimes H)$ whenever $a b \in E(G)$ and $x y \in E(H)$.
Definition 2: The chromatic number, $\chi(\mathrm{G})$, of G is the smallest number n for which G has an n -coloring.
Definition 2: The chromatic prime number, $\chi_{p}(G)$, of $G$ is the smallest number $n_{p}$ for which $G$ has an $n_{p}$-coloring.

## III. MAIN RESULTS AND DISCUSSION

## A. Theorem 1

Let G and H be the 2-prime colorable connected graph and Butterfly graph respectively, then their product satisfies the circular embedding through the tensor product, that is, $\chi_{p}(G \otimes H) \leq \min \left\{\chi_{p}(G), \chi_{p}(H)\right\}$

## B. Proof

Let $G$ be a 2-prime colorable connected graph and butterfly graph with vertices $u_{1}, u_{2}, u_{3}$, and $v_{1}, v_{2}, \ldots, v_{7}$ respectively.
Let the value of the vertices of graph $G, u_{1}, u_{3}=2$ and $u_{2}=3$. $\left(u_{1}+u_{2}\right)$ and $\left(u_{2}+u_{3}\right)$ are positive prime numbers

$$
\begin{equation*}
\chi(\mathrm{G}) 2=\chi_{\mathrm{p}}(\mathrm{G}) \tag{1}
\end{equation*}
$$

Let the value of the vertices of graph $H, v_{1}, v_{6}, v_{5}=2 ; v_{2}, v_{4}, v_{7}=3$ and $v_{3}=5 .\left(v_{1}+v_{2}\right),\left(v_{1}+v_{3}\right),\left(v_{3}+v_{6}\right),\left(v_{3}+v_{5}\right),\left(v_{4}+v_{5}\right)$ and $($ $\left.\mathrm{v}_{3}-\mathrm{v}_{2}\right),\left(\mathrm{v}_{3}-\mathrm{v}_{4}\right),\left(\mathrm{v}_{3}-\mathrm{v}_{7}\right)$ are positive and negative prime numbers respectively.

$$
\begin{equation*}
\chi(\mathrm{H})=3=\chi_{\mathrm{p}}(\mathrm{H}) \tag{2}
\end{equation*}
$$

Let the tensor product of $G$ and $H, \chi_{p}(G \otimes H)$ which has satisfies the circular embedding. Let us denote, $u_{1} v_{i}, u_{2} v_{i}, u v_{i}$; where $i=$ $1,2, \ldots, 7$.
The tensor product $\chi_{p}(G \otimes H)$, which has prime numbers of vertices on addition and subtraction. We get positive and negative prime numbers.

$$
\begin{equation*}
\chi(\mathrm{H} \otimes \mathbf{G})=2=\chi_{\mathrm{p}}(\mathrm{H} \otimes \mathbf{G}) \tag{3}
\end{equation*}
$$

From (1),(2) and (3), we get

$$
\begin{gathered}
\chi(\mathrm{G} \otimes \mathrm{H}) \leq \min \{\chi(\mathrm{G}), \chi(\mathrm{H})\} \\
\chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H}) \leq \min \left\{\chi_{\mathrm{p}}(\mathrm{G}), \chi_{\mathrm{p}}(\mathrm{H})\right\}
\end{gathered}
$$

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Fig. 1

## C. Example

1) Theorem 2: Let G and H be the 3-prime colourable complete graph and star graph respectively, then their product satisfies the circular embedding through the tensor product, ie, $\chi_{p}(\mathrm{G} \otimes \mathrm{H}) \leq \min \left\{\chi_{p}(\mathrm{G}), \chi_{p}(\mathrm{H})\right\}$
2) Proof: Let G be a 3-prime colourable complete graph with vertices $\mathrm{u}_{1}, \mathrm{u}_{2}$ and $\mathrm{u}_{3}$. Let H be a star graph with vertices $\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}$, $\mathrm{v}_{4}$ and $\mathrm{v}_{5}$.
Let the value of the vertices of graph G, $u_{1}=2, u_{2}=3$, and $u_{3}=5 .\left(u_{1}+u_{2}\right),\left(u_{1}+u_{3}\right)$ and $\quad\left(u_{3}-u_{2}\right)$ are positive and negative prime numbers respectively.

$$
\begin{equation*}
\chi(\mathrm{G})=3=\chi_{\mathrm{p}}(\mathrm{G}) \tag{1}
\end{equation*}
$$

Let the value of the vertices of graph $H, v_{1}, v_{2}=2 ; v_{3}, v_{4}=3$ and $v_{5}=5 ;\left(v_{1}+v_{3}\right),\left(v_{1}+v_{4}\right),\left(v_{2}+v_{4}\right),\left(v_{2}+v_{5}\right),\left(v_{3}+v_{1}\right)$ and $\left(v_{5}-\right.$ $v_{3}$ ) are positive and negative prime numbers respectively.

$$
\begin{equation*}
\chi(\mathrm{H})=3=\chi_{\mathrm{p}}(\mathrm{H}) \tag{2}
\end{equation*}
$$

The tensor product $\chi_{p}(G \otimes H)$, which satisfies the circular embedding; $u_{1} v_{i}, u_{2} v_{i}, u_{3} v_{i}$, where $i=1,2, \ldots, 5$. The tensor product $\chi_{p}(G$ $\otimes \mathrm{H}$ ), which has prime numbers of vertices on addition and subtraction, we get positive and negative prime numbers.

$$
\begin{equation*}
\chi(\mathrm{G} \otimes \mathrm{H})=3 \text { therefore } \chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H})=3 \tag{3}
\end{equation*}
$$

From (1),(2) and (3), we get

$$
\begin{gathered}
\chi(\mathrm{G} \otimes \mathrm{H}) \leq \min \{\chi(\mathrm{G}), \chi(\mathrm{H})\} \\
\chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H}) \leq \min \left\{\chi_{\mathrm{p}}(\mathrm{G}), \chi_{\mathrm{p}}(\mathrm{H})\right\}
\end{gathered}
$$

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## D. Example



Fig. 2

1) Theorem 3: Let G and H be the pentagon graph and 3-prime colourable complete graph respectively, then their product satisfies the circular embedding by the tensor product,
2) Proof: similarly to the above theorem.
3) Theorem 4: Let G and H be Hexagon graph and merge graph respectively, this product satisfies the circular embedding by the tensor product, then does not exit.

Proof: Let G and H be a 2-prime colorable hexagon graph and 3-prime colorable merge graph with vertices $\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{u}_{4}, \mathrm{u}_{5}$. and $\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{6}$ respectively. Let the value of the vertices of graph $\mathrm{G}, \mathrm{u}_{1}, \mathrm{u}_{3}, \mathrm{u}_{5}=2$ and $\mathrm{u}_{2}, \mathrm{u}_{4}, \mathrm{u}_{6}=3 .\left(\mathrm{u}_{1}+\mathrm{u}_{2}\right),\left(\mathrm{u}_{2}+\mathrm{u}_{3}\right),\left(\mathrm{u}_{3}+\mathrm{u}_{4}\right),\left(\mathrm{u}_{4}+\mathrm{u}_{5}\right)$, $\left(u_{5}+u_{6}\right)$ and $\left(u_{6}+u_{1}\right)$ are positive prime numbers and their

$$
\begin{equation*}
\chi(\mathrm{G})=2 \text { and } \chi_{\mathrm{p}}(\mathrm{G})=2 . \tag{1}
\end{equation*}
$$

Let the value of the vertices of graph $H, v_{1}, v_{3}, v_{6}=2 ; v_{2}, v_{5}=3$ and $v_{4}=5 .\left(v_{1}+v_{4}\right),\left(v_{3}+v_{4}\right),\left(v_{4}+v_{6}\right),\left(v_{5}+v_{6}\right)$ and $\left(v_{4}-v_{2}\right),\left(v_{4}-\right.$ $\mathrm{v}_{6}$ ) are positive and negative prime numbers respectively.

$$
\begin{equation*}
\chi(\mathrm{H})=3 \text { and their } \chi_{p}(\mathrm{H})=3 \tag{2}
\end{equation*}
$$

Let the tensor product of G and $\mathrm{H}, \chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H})$ which has satisfies the circular embedding.
Let us assign the values, the tensor product of the vertices uivj, where $\mathrm{i}, \mathrm{j}=1,2, \ldots, 6$
The tensor product $\chi_{p}(G \otimes H)$, which has prime number of vertices on addition and subtraction, we get positive and negative prime numbers.

$$
\begin{equation*}
\chi_{p}(\mathrm{G} \otimes \mathrm{H})=3 \text { and } \chi_{p}(\mathrm{G} \otimes \mathrm{H})=3 \tag{3}
\end{equation*}
$$

But $\chi_{\mathrm{p}}(\mathrm{GH})=3$ is not possible by the definition of chromatic prime number.
From (1), (2) and (3) we get

$$
\begin{gathered}
\chi(\mathrm{G} \otimes \mathrm{H}) \leq \min \{\chi(\mathrm{G}), \chi(\mathrm{H})\} \\
\chi_{\mathrm{p}}(\mathrm{G} \otimes \mathrm{H}) \leq \min \left\{\chi_{\mathrm{p}}(\mathrm{G}), \chi_{\mathrm{p}}(\mathrm{H})\right\}
\end{gathered}
$$

## International Journal for Research in Applied Science \& Engineering Technology (IJRASET) <br> IV. CONCLUSION

In this paper, we observed that the circular embedding by the tensor product graph does not possess prime chromatic number for all size and order.

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