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Operations on Hesitant Fuzzy Hypergraph

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Abstract : The concept of hypergraphs was extended to fuzzy hypergraph. In this paper, we extend the concepts of fuzzy hypergraphs into that of hesitant fuzzy hypergraphs. Based on the definition of hesitant fuzzy graph, order of the hfhg, size of the hfhg, operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined for hesitant fuzzy graphs. The authors further proposed to apply these operations in clustering techniques.

I. INTRODUCTION

Hypergraph theory, originally developed by C.Berge in 1960, is a generalization of graph theory. The concept of hypergraphs can model more general types of relations than binary relations. The notion of hypergraphs has been extended in fuzzy theory and the concept of fuzzy hypergraphs was proposed by Lee-Kwang and S.M.Chen. The authors have already introduced the concept of Hesitant fuzzy hypergraph. Operations on HFHGs have also been analyzed to a real-life problem with a numerical example.

II. PRELIMINARIES

A. Definition 1.1

Let X be a fixed set, a Hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1]. The HFS is defined by a mathematical symbol:

$$A = \{< x, h_A(x) > / x \in X\}$$

where $h_A(x)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element $x \in X$ to the set A. Xu and Xia (2011b) called $h = h_A(x)$ *a* hesitant fuzzy element (HFE) and Θ the set of all HFEs.

B. Definition 1.2

A Hesitancy Fuzzy Graph is of the form G = (V, E), where

$V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1: V \to [0,1], \gamma_1: V \to [0,1]$ and $\beta_1: V \to [0,1]$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ respectively and $\mu_1(v_i)+\gamma_1(v_i)+\beta_1(v_i) = 1$ for every $v_i \in V$, where $\beta_1(v_i) = 1-[\mu_1(v_i) + \gamma_1(v_i)]$ and $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$, $\gamma_2: V \times V \to [0,1]$ and $\beta_2: V \times V \to [0,1]$ are such that,

$$\mu_2(v_i, vj) \le \min[\mu_1(v_i), \mu_1(v_j)]$$
$$\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$$
$$\beta_2(v_i, v_j) \le \min[\beta_1(v_i), \beta_1(v_j)]$$

and $0 \le \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) + \beta_2(v_i,v_j) \le 1$ for every $(v_i,v_j) \in E$.

C. Definition 1.3

1) Hypergraph H is an ordered pair H = (V, E) where
a) $V = \{v_1, v_2, v_3 \ldots v_n\}$, a finite set of vertices.
b) $E = \{E_1, E_2, E_3 \ldots E_m\}$, a family of subsets of V .
c) $E_j \neq \emptyset, j = 1,2,\ldots m$ and $\cup E_j = V$.

The set V is called the set of vertices and E is the set of edges (or hyper edges).

D. Definition 1.4

In a hypergraph, two or more vertices $x_1, x_2, \ldots, x_n$ are said to be adjacent if there exist an edge Ej which contains those vertices. In a hypergraph two edges Ei & Ej, $i \neq j$ is said to be adjacent if their intersection is not empty.

E. Definition 1.5

A regular hyper graph is one in which every vertex is contained in k edges for some constant k.

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F. Definition 1.6

1) Hesitant Fuzzy HyperGraph (HFHG) is an ordered pair H = <V, E> where

a) $V = \{v_1, v_2, v_3 \ldots v_n\}$, a finite set of vertices

b) $E = \{E_1, E_2, E_3 \ldots E_m\}$, a family of Hesitant fuzzy subsets of V.

c) $E_j = \{<v_i, \mu_j(v_i), \gamma_j(v_i), \beta_j(v_i)> / \mu_j(v_i), \gamma_j(v_i), \beta_j(v_i) \geq 0$ and $0 \leq \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) + \beta_2(v_i,v_j) \leq 1, j = 1,2,\ldots m\}$

d) $E_j \neq \emptyset, j = 1,2,\ldots m$

Here the edges E_j are HFHGs. $\mu_j(v_i)$, $\gamma_j(v_i)$, $\beta_j(v_i)$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ respectively of the vertex v_i to edge E_j.

G. Definition 1.7

Let H = (X, E) be a Regular Hesitant Fuzzy Hypergraph. The **order** of a Regular Hesitant Fuzzy Hypergraph, H in

$$O(H) = \left(\sum_{x \in X} \mu_{E_i}(x), \quad \sum_{x \in X} \gamma_{E_i}(x), \sum_{x \in X} \beta_{E_i}(x) \right) \text{ for every } x \in X.$$

The **size** of Regular Hesitant Fuzzy Hypergraph is

$$S(H) = \sum_{i=1}^{n} S(E_i) \text{ where } S(E_i) = \left(\sum_{x \in E_i} \mu_{E_i}(x), \quad \sum_{x \in E_i} \gamma_{E_i}(x), \sum_{x \in E_i} \beta_{E_i}(x) \right)$$

1) Example 1: Consider the Hesitant Fuzzy Hypergraph H = (X, E), Define X = {a, b, c, d} and $E = \{E_1, E_2, E_3, E_4\}$, where

E_1 = {(a, 0.1, 0.7, 0.2) (b, 0.1, 0.7, 0.2)}
E_2 = {(b, 0.1, 0.7, 0.2) (c, 0.1, 0.7, 0.2)}
E_3 = {(c, 0.1, 0.7, 0.2) (d, 0.1, 0.7, 0.2)}
E_4 = {(d, 0.1, 0.7, 0.2) (a, 0.1, 0.7, 0.2)}

Here, Order of the HFHG, O (H) = (0.4, 2.8, 0.8),
Size of the HFHG, S (H) = (0.8, 5.6, 1.6).

III. OPERATIONS ON HESITANT FUZZY HYPERGRAPHS

A. Definition 2.1

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two HFHGs. Then the **Union** of G_1 and G_2 is an HFHG $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$ defined by

$$(\mu_1 \cup \mu_1')(v) = \begin{cases} \mu_1(v) & if \ v \in V_1 - V_2 \\ \mu_1'(v) & if \ v \in V_2 - V_1 \end{cases}$$

$$(\gamma_1 \cup \gamma_1')(v) = \begin{cases} \gamma_1(v) & if \ v \in V_1 - V_2 \\ \gamma_1'(v) & if \ v \in V_2 - V_1 \end{cases}$$

$$(\beta_1 \cup \beta_1')(v) = \begin{cases} \beta_1(v) & if \ v \in V_1 - V_2 \\ \beta_1'(v) & if \ v \in V_2 - V_1 \end{cases}$$

And $$(\mu_2 \cup \mu_2')(v_i v_j) = \begin{cases} \mu_{2ij} & if \ e_{ij} \in E_1 - E_2 \\ \mu_{2ij}' & if \ e_{ij} \in E_2 - E_1 \end{cases}$$

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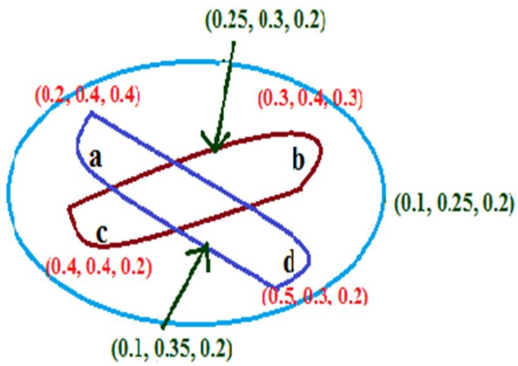
$$(\gamma_2 \cup \gamma_2^{'})(v_i v_j) = \begin{cases} \gamma_{2ij} & if\ e_{ij} \in E_1 - E_2 \\ \gamma_{2ij}^{'} & if\ e_{ij} \in E_2 - E_1 \end{cases}$$

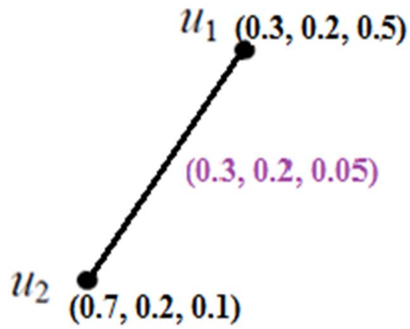
$$(\beta_2 \cup \beta_2^{'})(v_i v_j) = \begin{cases} \beta_{2ij} & if\ e_{ij} \in E_1 - E_2 \\ \beta_{2ij}^{'} & if\ e_{ij} \in E_2 - E_1 \end{cases}$$

where $(\mu_1, \gamma_1, \beta_1)$ and $(\mu_1', \gamma_1', \beta_1')$ refer the vertex membership, non-membership and hesitant element of G_1 and G_2 respectively; $(\mu_2, \gamma_2, \beta_2)$ and $(\mu_2', \gamma_2', \beta_2')$ refer the edge membership, non-membership and hesitant element of G_1 and G_2 respectively.

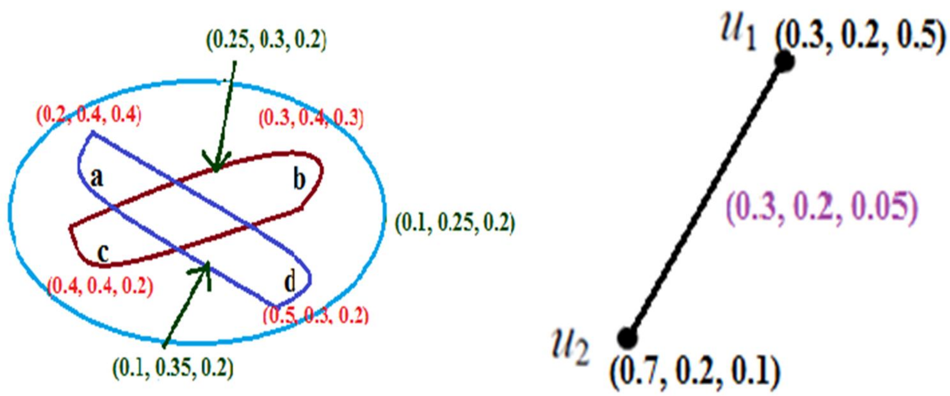
1) *Example 2:* Consider the Hesitant Fuzzy Hypergraphs $G_1 = (V_1, E_1)$ where $V_1 = \{a, b, c, d\}$ and $G_2 = (V_2, E_2)$ where $V_2 = \{u_1, u_2\}$.



G_1



G_2



$G_1 \cup G_2$

B. Definition 2.2

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two HFHGs. Then the **Join** of G_1 and G_2 is an HFHG $G = G_1 + G_2 = (V_1 \cup V_2, E_1 \cup E_2 E')$ defined by

$$(\mu_1 + \mu_1^{'})(v) = (\mu_1 \cup \mu_1^{'})(v) \qquad \text{if } v \in V_1 \cup V_2$$

$$(\gamma_1 + \gamma_1^{'})(v) = (\gamma_1 \cup \gamma_1^{'})(v) \qquad \text{if } v \in V_1 \cup V_2$$

$$(\beta_1 + \beta_1^{'})(v) = (\beta_1 \cup \beta_1^{'})(v) \qquad \text{if } v \in V_1 \cup V_2$$

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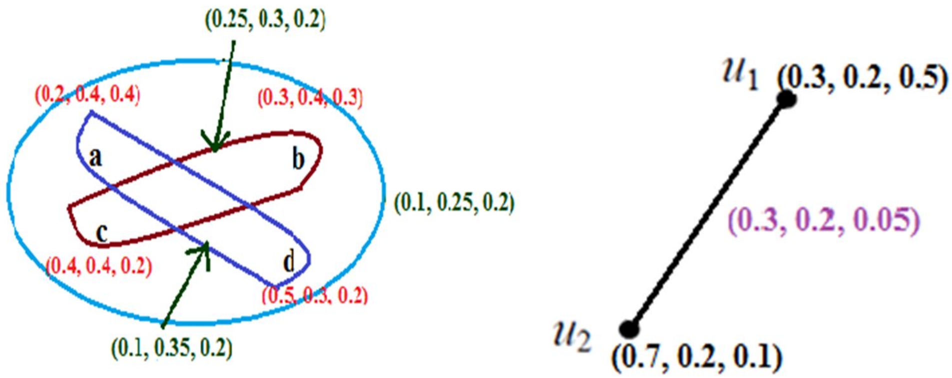
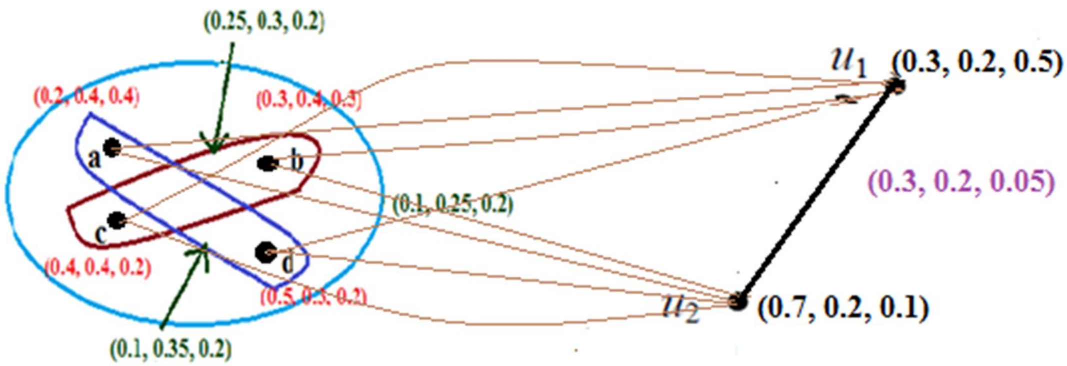
$$(\mu_1 + \mu_1^{'})(v_i v_j) = \begin{cases} (\mu_1 + \mu_1^{'})(v_i v_j) & if \ v_i v_j \in E_1 \cup E_2 \\ \mu_1(v_i)\mu_1^{'}(v_j) & if \ v_i v_j \in E^{'} \end{cases}$$

$$(\gamma_1 + \gamma_1^{'})(v_i v_j) = \begin{cases} (\gamma_1 + \gamma_1^{'})(v_i v_j) & if \ v_i v_j \in E_1 \cup E_2 \\ \gamma_1(v_i)\gamma_1^{'}(v_j) & if \ v_i v_j \in E^{'} \end{cases} \quad (\beta_1 + \beta_1^{'})(v_i v_j) = \begin{cases} (\beta_1 + \beta_1^{'})(v_i v_j) & if \ v_i v_j \in E_1 \cup E_2 \\ \beta_1(v_i)\beta_1^{'}(v_j) & if \ v_i v_j \in E^{'} \end{cases}$$

1) *Example 3:* Consider the Hesitant Fuzzy Hypergraphs $G_1 = (V_1, E_1)$ where $V_1 = \{a, b, c, d\}$ and $G_2 = (V_2, E_2)$ where $V_2 = \{u_1, u_2\}$.



G_1 **G_2**



G_1+G_2

C. Definition 2.3

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two HFHGs. Then the **Intersection** of HFHGs G_1 and G_2, denoted by G_1 I G_2, is an HFHG defined by

$$(\mu_1 \ I \ \mu_1^{'})(v) = \begin{cases} \mu_1(v) & if \ v \in V_1 and \ V_2 \\ \mu^{'}{}_1(v) & if \ v \in V_2 and \ V_1 \end{cases}$$

$$(\gamma_1 \ I \ \gamma_1^{'})(v) = \begin{cases} \gamma_1(v) & if \ v \in V_1 and \ V_2 \\ \gamma^{'}{}_1(v) & if \ v \in V_2 and \ V_1 \end{cases}$$

$$(\beta_1 \ I \ \beta_1^{'})(v) = \begin{cases} \beta_1(v) & if \ v \in V_1 and \ V_2 \\ \beta^{'}{}_1(v) & if \ v \in V_2 and \ V_1 \end{cases}$$

$$(\mu_2 \ I \ \mu_2^{'})(v_i v_j) = \begin{cases} \mu_{2ij} & if \ e_{ij} \in E_1 and \ E_2 \\ \mu^{'}{}_{2ij} & if \ e_{ij} \in E_2 and \ E_1 \end{cases}$$

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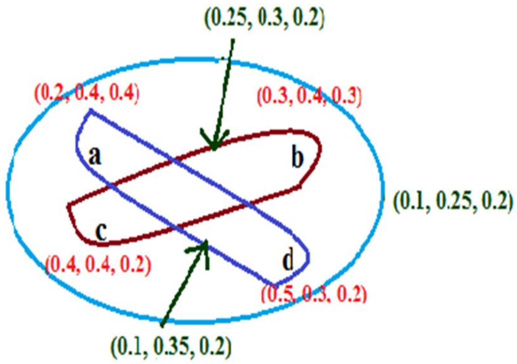
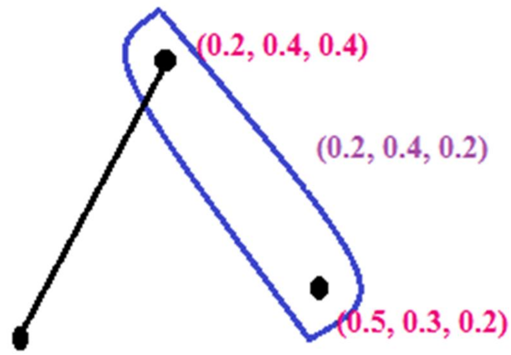
$$(\gamma_2\ I\ \gamma_2^{'})(v_i v_j) = \begin{cases} \gamma_{2ij} & if\ e_{ij} \in E_1 and\ E_2 \\ \gamma^{'}_{2ij} & if\ e_{ij} \in E_2 and\ E_1 \end{cases}$$

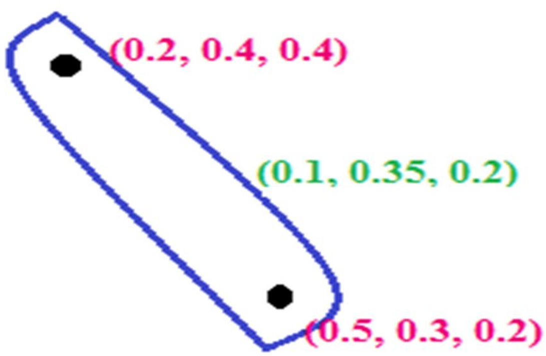
$$(\beta_2\ I\ \beta_2^{'})(v_i v_j) = \begin{cases} \beta_{2ij} & if\ e_{ij} \in E_1 and\ E_2 \\ \beta^{'}_{2ij} & if\ e_{ij} \in E_2 and\ E_1 \end{cases}$$

1) Example 4:





G_1 **G_2**

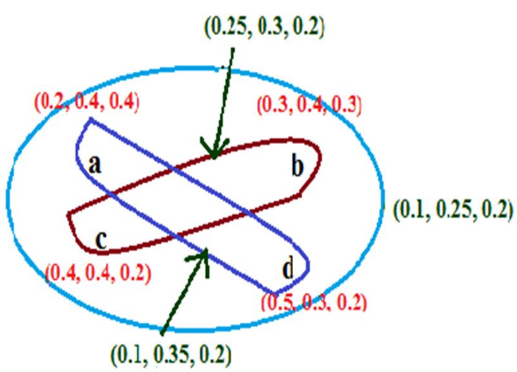


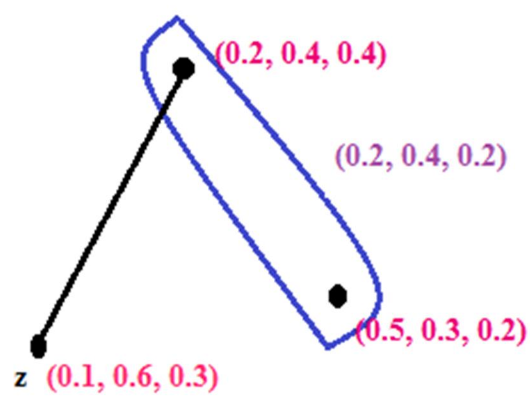
G_1 I G_2

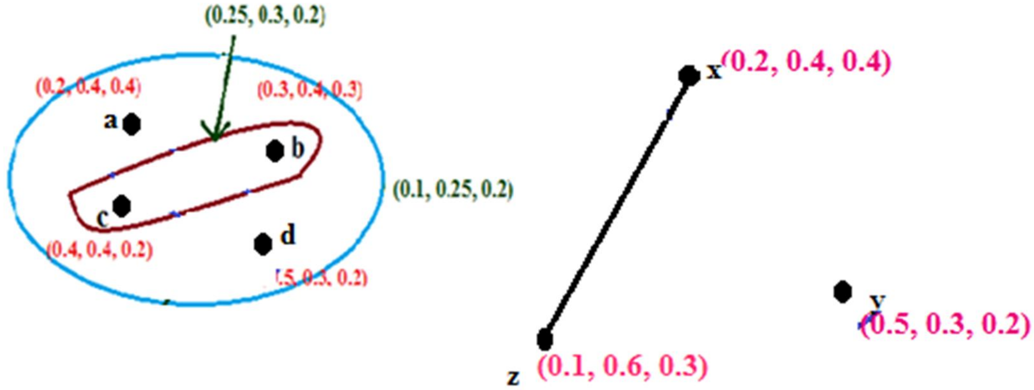
D. Definition 2.4

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two HFHGs. Then the ringsum of HFHGs G_1 and G_2 is an HFHG defined by

$$G_1 \oplus G_2 = (G_1\ U\ G_2) - (G_1\ I\ G_2).$$







$G_1 \oplus G_2$

E. Definition 2.5

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two HFHGs. Then the Cartesian product of HFHGs G_1 and G_2 is an HFHG, denoted by $G = G_1 x G_2$, defined by $G = (V, E'')$ $V = V_1 x V_2$ and

$E'' = \{(u, u_2)(u, v_2) : u\epsilon V_1, u_2v_2 \epsilon E_2\} \cup \{(u_1,w)(v_1,w) : w\epsilon V_2, u_1v_1 \epsilon E_1\}$

where

(i)

$(\mu_1 \times \mu_1')(u_1, u_2) = \mu_1(u_1).\mu_1'(u_2)$ for every $(u_1,u_2) \epsilon V$

$(\gamma_1 \times \gamma_1')(u_1, u_2) = \gamma_1(u_1).\gamma_1'(u_2)$ for every $(u_1,u_2) \epsilon V$ and

$(\beta_1 \times \beta_1')(u_1, u_2) = \beta_1(u_1).\beta_1'(u_2)$ for every $(u_1,u_2) \epsilon V$

(ii)

$(\mu_2 \times \mu_2')(u,u_2)(u,v_2) = \mu_1(u).\mu_2(u_2v_2)$ for every $u \epsilon V_1$, $u_2v_2 \epsilon E_2$

$(\gamma_2 \times \gamma_2')(u,u_2)(u,v_2) = \gamma_1(u).\gamma_2(u_2v_2)$ for every $u \epsilon V_1$, $u_2v_2 \epsilon E_2$

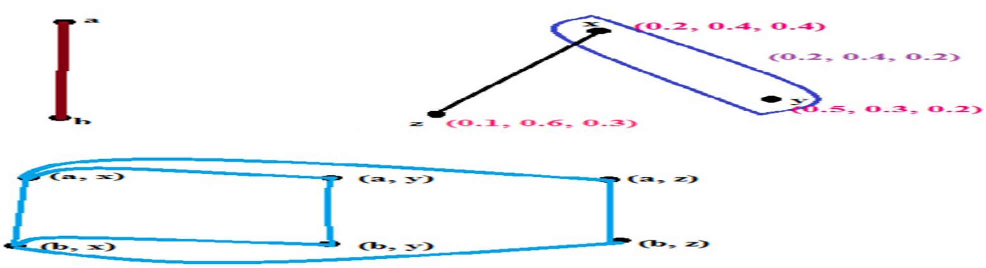
$(\beta_2 \times \beta_2')(u,u_2)(u,v_2) = \beta_1(u).\beta_2(u_2v_2)$ for every $u \epsilon V_1$, $u_2v_2 \epsilon E_2$ and

$(\mu_2 \times \mu_2')(u_1, w)(v_1, w) = \mu_1(w).\mu_2(u_1v_1)$ for every $w \epsilon V_2$, $u_1v_1 \epsilon E_1$

$(\gamma_2 \times \gamma_2')(u_1, w)(v_1, w) = \gamma_1(w).\gamma_2(u_1v_1)$ for every $w \epsilon V_2$, $u_1v_1 \epsilon E_1$

$(\beta_2 \times \beta_2')(u_1, w)(v_1, w) = \beta_1(w).\beta_2(u_1v_1)$ for every $w \epsilon V_2$, $u_1v_1 \epsilon E_1$

1) Example 5:



F. Definition 2.6

Let $G = G_1 \circ G_2 = (V_1xV_2, E)$ be the Composition of two graphs G_1 and G_2 where $V = V_1 \times V_2$ and

$E = \{(u, u_2)(u, v_2) : u\epsilon V_1, u_2v_2 \epsilon E_2\} \cup \{(u_1, w)(v_1, w) : w \epsilon V_2, u_1v_1\epsilon E_1\}$
$\cup \{(u_1, u_2)(v_1, v_2) : u_1v_1\epsilon E_1, u_2 \neq v_2\}$

Then, the "Composition of HFHGs G_1 and G_2", denoted by $G = G_1 o G_2$, is an HFHG defined by

(i)

$(\mu_1 \circ \mu_1')(u_1, u_2) = \mu_1(u_1).\mu_1'(u_2)$ for every $(u_1,u_2) \epsilon V_1xV_2$

$(\gamma_1 \circ \gamma_1')(u_1, u_2) = \gamma_1(u_1).\gamma_1'(u_2)$ for every $(u_1,u_2) \epsilon V_1xV_2$ and

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$(\beta_1 \circ \beta_1')(u_1, u_2) = \beta_1(u_1).\beta_1'(u_2)$ for every $(u_1,u_2) \in V_1 x V_2$

(ii)

$(\mu_2 \circ \mu_2')(u,u_2)(u,v_2) = \mu_1(u).\mu_2(u_2v_2)$ for every $u \in V_1$, $u_2v_2 \in E_2$

$(\gamma_2 \circ \gamma_2')(u,u_2)(u,v_2) = \gamma_1(u).\gamma_2(u_2v_2)$ for every $u \in V_1$, $u_2v_2 \in E_2$

$(\beta_2 \circ \beta_2')(u,u_2)(u,v_2) = \beta_1(u).\beta_2(u_2v_2)$ for every $u \in V_1$, $u_2v_2 \in E_2$

$(\mu_2 \circ \mu_2')(u_1, w)(v_1, w) = \mu_1(w).\mu_2(u_1v_1)$ for every $w \in V_2$, $u_1v_1 \in E_1$

$(\gamma_2 \circ \gamma_2')(u_1, w)(v_1, w) = \gamma_1(w).\gamma_2(u_1v_1)$ for every $w \in V_2$, $u_1v_1 \in E_1$

$(\beta_2 \circ \beta_2')(u_1, w)(v_1, w) = \beta_1(w).\beta_2(u_1v_1)$ for every $w \in V_2$, $u_1v_1 \in E_1$ and

$(\mu_2 \circ \mu_2')(u_1, u_2)(v_1, v_2) = \mu_1'(u_2).\ \mu_1'(v_2).\mu_2(u_1v_1)$ for every $(u_1,u_2), (v_1,v_2) \in E - E''$

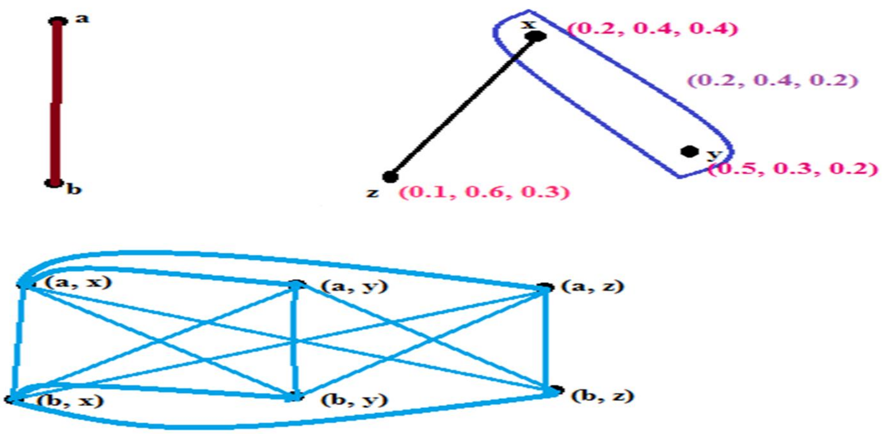
$(\gamma_2 \circ \gamma_2')(u_1, u_2)(v_1, v_2) = \gamma_1'(u_2).\ \gamma_1'(v_2).\ \gamma_2(u_1v_1)$ for every $(u_1,u_2), (v_1,v_2) \in E - E''$

$(\beta_2 \circ \beta_2')(u_1, u_2)(v_1, v_2) = \beta_1'(u_2).\beta_1'(v_2).\beta_2(u_1v_1)$ for every $(u_1,u_2), (v_1,v_2) \in E - E''$

where

$E'' = \{(u, u_2)(u, v_2) : u \in V_1, u_2v_2 \in E_2\} \cup \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}$

1) Example 6:



IV. CONCLUSION

In this paper, order of the HFHG, size of the HFHG, Operations like complement, join, union, intersection, ringsum, Cartesian product, composition on HFHGs are defined. Currently, the authors are working on existing clustering techniques. Further, it is proposed to apply the properties of HFHGs to develop a new clustering algorithm and the same may be checked with a numerical dataset.

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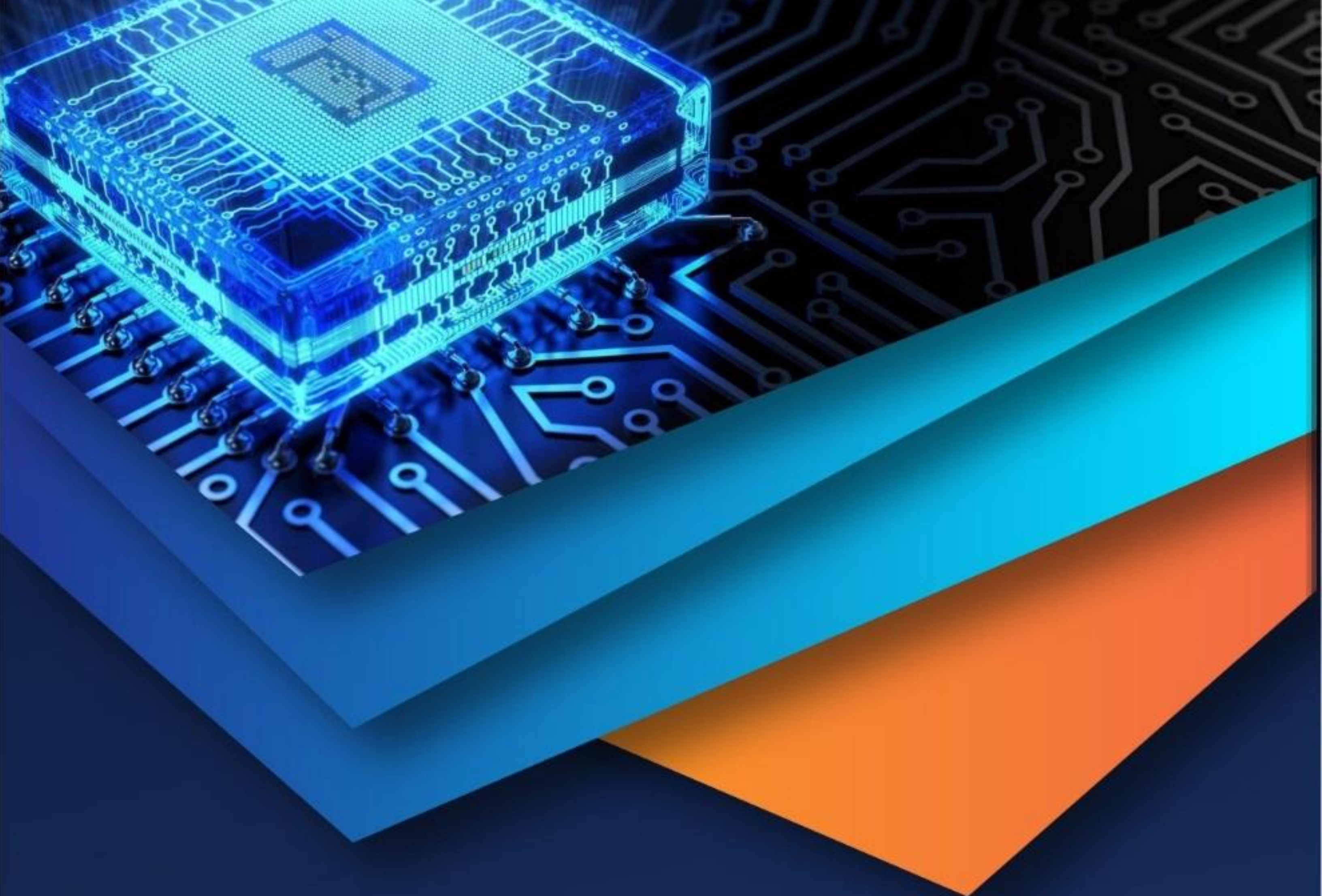

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