# Unsteady Convective Heat and Mass Transfer Flow In A Vertical Channel with Dissipative and Radiation Effects 

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#### Abstract

We analyze the thermo diffusion effect on unsteady convective heat and mass transfer flow of a viscous fluid through a porous media in a vertical channel on whose walls a traveling thermal wavy is imposed. The governed equation non-linear coupled equation governed the flow phenomena are solved by employing perturbation techniques with $\Delta$ the slope of the boundary temperature as perturbation parameter. The effect of dissipation thermo diffusion on the unsteady double different heat transfer flow of viscous dissipative fluid is discussed graphically nusslet number shear wood number on the walls $\pm 1$ on discussed for a different values of the parameters.


Keywords: heat and mass transfer, vertical channel, radiation effects, thermo diffusion, convection.

## I. INTRODUCTION

The time dependent thermal convection flows have applications in chemical Engineering, space technology etc. These flows can also be achieved by either time dependent movement of the boundary or unsteady temperature of the boundary. The unsteady may also be attributed due to the free stream oscillations or oscillatory flux or temperature oscillations. . Nelson and wood (5,6). Lee et al(3), have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. For along channel (low Rayleigh numbers) the numerical solutions approach the fully developed flow analytical solutions. At intermediate Rayleigh numbers it is observed that the parallel plate heat and mass transfer is higher than that for a single plate. Vajravelu and Debnath (12) have made an interesting and a detailed study of non-linear convection heat transfer and fluid flows, induced by traveling thermal waves. Ravindra (9) has analysed the mixed convection flow of a viscous fluid through a porous medium in a vertical channel. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundaries. Purushothama Reddy (8) has analysed the unsteady mixed convective effects on the flow induced by imposing traveling thermal waves on the boundaries. Nagaraja (4) has investigated the combined heat and mass transfer effects on the flow of a viscous fluid through a porous medium in a vertical channel, with the traveling thermal waves imposed on the boundaries while the concentration is maintained uniform on the boundaries Sivanjaneya $\operatorname{Prasad}(10)$ has analysed heat and mass transfer effects on the flow of an incompressible viscous fluid through a porous medium in vertical channel. Recently, Sulochana et al (11) have considered the unsteady convective heat and mass transfer through a porous medium due to the imposed traveling thermal wave boundary through a horizontal channel bounded by non -uniform walls. Tanmay Basak et al(2) have analysed the natural convection flows in a square cavity filled with a porous matrix for uniformly and non-uniformly heated bottom wall and adiabatic top wall maintaining crust temperature of cold vertical walls Darcy - Forchheimer model is used to simulate the momentum transfer in the porous medium. Yan and Lin (13) have examined the effects of the latent heat transfer associated with the liquid film vaporization on the heat transfer in the laminar forced convection channel flows. Results are presented for an air-water system under various conditions. The effects of system temperature on heat and mass transfer are investigated. Recently Atul Kumar Singh et al (1) investigated the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Convection fluid flows generated by traveling thermal waves have also received attention due to applications in physical problems. The linearised analysis of these flows has shown that a traveling thermal wave can generate a mean shear flow within a layer of fluid, and the induced mean flow is proportional to the square of the amplitude of the wave. From a physical point of view, the motion induced by traveling thermal waves is quite interesting as a purely fluid-dynamical problem and can be used as a possible explanation for the observed four - day retrograde zonal motion of the upper atmosphere of venus. Also, the heat transfer results will have a definite bearing on the design of oil or gas

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-fired boilers.

## II. FORMULATION OF THE PROBLEM:

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at $\mathrm{y}=\mathrm{L}$ while the boundary at $y=-L$ is maintained at constant temperature $T_{1}$. The walls are maintained at constant concentrations. A uniform magnetic field of strength H0 is applied transverse to the walls. Assuming the magnetic Reynolds to be small we neglect the induced magnetic field in comparison to the applied magnetic field. Assuming that the flow takes place at low concentration we neglect the Duffor effect .The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. Also the kinematic viscosity $v$, the thermal conducting $k$ are treated as constants. We choose a rectangular Cartesian system 0 ( x , y ) with $x$-axis in the vertical direction and $y$-axis normal to the walls. The walls of the channel are at $y= \pm L$.
The equations governing the unsteady flow and heat transfer are

## A. Equation of linear momentum

$$
\begin{align*}
& \rho_{e}\left(\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}\right)-\rho g-\left(\frac{\mu}{k}\right) u  \tag{2.1}\\
& \rho_{e}\left(\frac{\partial v}{\partial t}+u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{\partial y^{2}}\right)-\left(\frac{\mu}{k}\right) v \tag{2.2}
\end{align*}
$$

B. Equation of continuity

$$
\begin{equation*}
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.3}
\end{equation*}
$$

C. Equation of energy

$$
\begin{align*}
& \rho_{e} C_{p}\left(\frac{\partial T}{\partial t}+u \frac{\partial T}{\partial x}+v \frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}\right)+Q+  \tag{2.4}\\
& \mu\left(\left(\frac{\partial u}{\partial y}\right)^{2}+\left(\frac{\partial v}{\partial x}\right)^{2}\right)+\left(\frac{\mu}{\lambda k}+\sigma \mu_{e}^{2} H_{o}^{2}\right)\left(u^{2}+v^{2}\right)-\frac{\partial\left(q_{R}\right)}{\partial y}
\end{align*}
$$

D. Equation of Diffusion

$$
\begin{equation*}
\left(\frac{\partial C}{\partial t}+u \frac{\partial C}{\partial x}+v \frac{\partial C}{\partial y}\right)=D_{1}\left(\frac{\partial^{2} C}{\partial x^{2}}+\frac{\partial^{2} C}{\partial y^{2}}\right)+k_{11}\left(\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}\right) \tag{2.5}
\end{equation*}
$$

## E. Equation of state

$$
\begin{equation*}
\rho-\rho_{e}=-\beta \rho_{e}\left(T-T_{e}\right)-\beta^{*} \rho_{e}\left(C-C_{e}\right) \tag{2.6}
\end{equation*}
$$

where $\rho_{e}$ is the density of the fluid in the equilibrium state, $\mathrm{Te}, \mathrm{Ce}$ are the temperature and Concentration in the equilibrium state, $(\mathrm{u}, \mathrm{v})$ are the velocity components along $\mathrm{O}(\mathrm{x}, \mathrm{y})$ directions, p is the pressure, $\mathrm{T}, \mathrm{C}$ are the temperature and Concentration in the flow region, $\rho$ is the density of the fluid, $\mu$ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, $\lambda$ is the coefficient of thermal conductivity, k is the permeability of the porous medium , $\mathrm{D}_{1}$ is the molecular diffusivity , $\mathrm{k}_{11}$ is the , $\beta$ is the coefficient of thermal expansion, $\beta^{*}$ is the and Q is the strength of the constant internal heat source and qr is the radiative heat flux. Invoking Rosseland approximation for radiation

$$
\begin{equation*}
q_{r}=-\frac{4 \sigma^{\cdot}}{3 \beta_{R}} \frac{\partial\left(T^{\prime 4}\right)}{\partial y} \tag{2.7}
\end{equation*}
$$

Expanding $T^{\prime 4}$ in Taylor's series about Te neglecting higher order terms

$$
T^{\prime 4} \cong 4 T_{e}^{3} T^{\prime}-3 T_{e}^{4}
$$

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where $\sigma^{\bullet}$ is the Stefan-Boltzmann constant $\beta_{R}$ is the Extinction coefficient.
In the equilibrium state

$$
0=-\frac{\partial p_{e}}{\partial x}-\rho_{e} g
$$

where $p=p_{e}+p_{D}, p_{D}$ being the hydrodynamic pressure.
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$
\begin{equation*}
Q=\frac{1}{2 L} \int_{-L}^{L} u d y \tag{2.8}
\end{equation*}
$$

The boundary conditions for the velocity and temperature fields are $\mathrm{u}=0, \mathrm{v}=0, \mathrm{~T}=\mathrm{Tl} \quad \mathrm{C}=\mathrm{Cl} \quad$ on $\mathrm{y}=-\mathrm{L}$ $u=0, v=0, T=T_{2}+\Delta T_{e} \operatorname{Sin}(m x+n t), C=C_{2} \quad$ on $\mathrm{y}=\mathrm{L}$
where $\Delta T_{e}=T_{2}-T_{1}$ and $\operatorname{Sin}(m x+n t)$ is the imposed traveling thermal wave
In view of the continuity equation we define the stream function $\psi$ as

$$
\begin{equation*}
\mathrm{u}=-\psi_{\mathrm{y}}, \mathrm{v}=\psi_{\mathrm{x}} \tag{2.10}
\end{equation*}
$$

Eliminating pressure p from equations (2.2)\&(2.3)and using the equations governing the flow in terms of $\psi$ are

$$
\begin{gather*}
{\left[\left(\nabla^{2} \psi\right)_{t}+\psi_{x}\left(\nabla^{2} \psi\right)_{y}-\psi_{y}\left(\nabla^{2} \psi\right)_{x}\right]=v \nabla^{4} \psi-\beta g\left(T-T_{0}\right)_{y}-}  \tag{2.11}\\
\left(\frac{v}{k}\right) \nabla^{2} \psi-\left(\frac{\sigma \mu{ }_{e}^{2} H_{o}^{2}}{\rho_{e}}\right) \frac{\partial^{2} \psi}{\partial y^{2}} \\
\rho_{e} C_{p}\left(\frac{\partial \theta}{\partial t}+\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right)=\lambda \nabla^{2} \theta+Q+\mu\left(\left(\frac{\partial^{2} \psi}{\partial y^{2}}\right)^{2}+\left(\frac{\partial^{2} \psi}{\partial x^{2}}\right)^{2}\right)+  \tag{2.12}\\
\left.\left(\frac{\mu}{k}+\left(\frac{\sigma \mu_{e}^{2} H_{o}^{2}}{\rho_{e}}\right)\right)\left(\left(\frac{\partial \psi}{\partial x}\right)^{2}+\left(\frac{\partial \psi}{\partial y}\right)^{2}\right)\right)+\frac{16 \sigma \sigma^{\cdot} T_{e}^{3}}{\beta_{R} \lambda} \frac{\partial^{2} \theta}{\partial y^{2}} \\
\left(\frac{\partial C}{\partial t}+\frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x}-\frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y}\right)=D_{1} \nabla^{2} C+\frac{S c S}{N}{ }^{2} \nabla^{2} \theta \tag{2.13}
\end{gather*}
$$

Introducing the non-dimensional variables in (2 .11)-(2.13) as

$$
\begin{equation*}
x^{\prime}=m x, y^{\prime}=y / L, t^{\prime}=t v m^{2}, \Psi^{\prime}=\Psi / v, \theta=\frac{T-T_{e}}{\Delta T_{e}}, C^{\prime}=\frac{C-C_{1}}{C_{2}-C_{1}} \tag{2.14}
\end{equation*}
$$

(under the equilibrium state ${ }_{\Delta} T_{e}=T_{e}(L)-T_{e}(-L)=\frac{Q L^{2}}{\lambda}$ )
the governing equations in the non-dimensional form (after dropping the dashes ) are

$$
\begin{equation*}
\delta R\left(\delta\left(\nabla_{1}^{2} \psi\right)_{t}+\frac{\partial\left(\psi, \nabla_{1}^{2} \psi\right)}{\partial(x, y)}\right)=\nabla_{1}^{4} \psi+\left(\frac{G}{R}\right) \theta_{y}-D^{-1} \nabla_{1}^{2} \psi-M^{2} \frac{\partial^{2} \psi}{\partial y^{2}} \tag{2.15}
\end{equation*}
$$

The energy equation in the non-dimensional form is

$$
\begin{gather*}
\delta P\left(\delta \frac{\partial \psi}{\partial t}+\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x}-\frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right)=\nabla_{1}^{2} \theta+\alpha+\left(\frac{P R^{2} E_{c}}{G}\right)\left(\left(\frac{\partial^{2} \psi}{\partial y^{2}}\right)^{2}+\delta^{2}\left(\frac{\partial^{2} \psi}{\partial x^{2}}\right)^{2}\right)+  \tag{2.16}\\
\left.+\left(D^{-1}\right)\left(\delta^{2}\left(\frac{\partial \psi}{\partial x}\right)^{2}+\left(\frac{\partial \psi}{\partial y}\right)^{2}\right)\right)+\frac{4}{3 N_{1}} \frac{\partial^{2} \theta}{\partial y^{2}}
\end{gather*}
$$

The Diffusion equation is

$$
\begin{equation*}
\delta S c\left(\delta \frac{\partial C}{\partial t}+\frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x}-\frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y}\right)=\nabla_{1}^{2} C \tag{2.17}
\end{equation*}
$$

where
$R=\frac{U L}{v} \quad$ (Reynolds number)
$G=\frac{\beta g \Delta T_{e} L^{3}}{v^{2}} \quad$ (Grashof number)
$\mathrm{P}=\frac{\mu c_{p}}{k_{1}} \quad$ ( Prandtl number), $D^{-1}=\frac{L^{2}}{k} \quad$ (Darcy parameter),

# International Journal for Research in Applied Science \& Engineering Technology (IJRASET) <br> $E_{c}=\frac{\beta g L^{3}}{C_{p}} \quad$ (Eckert number) <br> $\delta=m L \quad$ ( Aspect ratio) 

$\gamma=\frac{n}{v m^{2}} \quad$ (non-dimensional thermal wave velocity) $S c=\frac{v}{D_{1}}$
(Schimdt Number )
$N=\frac{\beta^{*} \Delta C}{\beta \Delta T} \quad$ (Buoyancy ratio ) $\quad$ So $=\frac{k_{11}}{v} \quad$ (Soret Parameter )
$N_{1}=\frac{\beta_{R} \lambda}{4 \sigma \cdot T_{e}{ }^{3}}$ (Radiation parameter) $\quad N_{2}=\frac{3 N_{1}}{3 N_{1}+4} P_{1}=P N_{2} \quad \alpha_{1}=\alpha N_{2}$
$\nabla_{1}^{2}=\delta^{2} \frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}$
The corresponding boundary conditions are

$$
\begin{align*}
& \psi(+!)-\psi(-!)=1 \\
& \frac{\partial \psi}{\partial x}=0, \quad \frac{\partial \psi}{\partial y}=0 \quad \text { at } \quad y= \pm 1  \tag{2.18}\\
& \theta(x, y)=1 \quad, \quad C(x, y)=0 \\
& \theta(x, y)=\operatorname{Sin}(x+\gamma t), \mathrm{C}(\mathrm{x}, \mathrm{y})=1 \quad \text { on } \mathrm{y}=1 \\
& \frac{\partial \theta}{\partial y}=0, \frac{\partial C}{\partial y}=0 \quad \text { at } \mathrm{y}=-1 \\
& y=0
\end{align*}
$$

The value of $\psi$ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (2.8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function $t$.

## III. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio $\delta$ to be small.
We adopt the perturbation scheme and write

$$
\begin{align*}
& \psi(x, y)=\psi_{0}(x, y)+\delta \psi_{1}(x, y)+\delta^{2} \psi{ }_{2}(x, y)+ \\
& \theta(x, y)=\theta_{0}(x, y)+\delta \theta_{1}(x, y)+\delta^{2} \theta_{2}(x, y)+ \\
& C(x, y)=C_{0}(x, y)+\delta C_{1}(x, y)+\delta^{2} C_{2}(x, y)+ \tag{3.1}
\end{align*}
$$

On substituting (3.1) in (2.16) - (2.17) and separating the like powers of $\delta$ the equations and respective conditions to the zeroth order are

$$
\begin{align*}
& \psi_{0, y y y y y}-M_{1}^{2} \psi_{0, y y}=-G\left(\theta_{0, y}+N C_{o, y}\right)  \tag{3.2}\\
& \theta_{o, y y}+\alpha_{1}+\frac{P_{1} E_{c} R^{2}}{G}\left(\psi_{o, y}\right)^{2}+\frac{P_{1} E_{c} M_{1}^{2}}{G}\left(\psi_{o, y}^{2}\right)=0  \tag{3.3}\\
& C_{o, y Y}=0 \tag{3.4}
\end{align*}
$$

with $\quad \psi_{0}(+1)-\Psi(-1)=1$,

$$
\begin{align*}
& \psi_{0, \mathrm{y}}=0, \psi_{0, \mathrm{x}}=0 \quad \text { at } \mathrm{y}= \pm 1  \tag{3.5}\\
& \theta_{o}=1, C_{0}=0 \quad \text { on } \quad y=-1 \\
& \theta_{o}=\operatorname{Sin}(x+\gamma t), C_{0}=1 \quad \text { on } \quad y=1 \tag{3.6}
\end{align*}
$$

and to the first order are

$$
\begin{gather*}
\psi_{1, y y y y}-M_{1}^{2} \psi_{1, y y}=-G\left(\theta_{1, y y}+N C_{1, y}\right)+\left(\psi_{0, y} \psi_{0, x y y}-\psi_{0, x} \psi_{0, y y y}\right)  \tag{3.7}\\
\theta_{1, y y}=\left(\psi_{0, x} \theta_{o, y}-\psi_{0, y} \theta_{o x}\right)+\frac{2 P_{1} E_{c} R^{2}}{G}\left(\psi_{0, y y} \cdot \psi_{1, y y}\right)+\frac{2 P_{1} E_{c} M_{1}^{2}}{G}\left(\psi_{0, y} \cdot \psi_{1, y}\right) \tag{3.8}
\end{gather*}
$$

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\begin{equation*} C_{1, y y}=\left(\psi_{0, x} C_{o, y}-\psi_{0, y} C_{o x}\right) \tag{3.9} \end{equation*}
$$

with

$$
\begin{align*}
& \psi_{1(+1)-} \psi_{1(-1)}=0 \\
& \psi_{1, y}=0, \psi_{1, x}=0 \text { at } \mathrm{y}= \pm 1  \tag{3.10}\\
& \theta_{1}( \pm 1)=0, \mathrm{C}_{1}( \pm 1)=0 \quad \text { at } \mathrm{y}= \pm 1 \tag{3.11}
\end{align*}
$$

Assuming $\mathrm{Ec} \ll 1$ to be small we take the asymptotic expansions as

$$
\begin{align*}
& \psi_{0}(x, y)=\psi_{00}(x, y)+E c \psi_{01}(x, y)+\ldots \ldots \ldots . \\
& \psi_{1}(x, y)=\psi_{10}(x, y)+E c \psi_{11}(x, y)+\ldots \ldots \ldots . \\
& \theta_{0}(x, y)=\theta_{00}(x, y)+\theta_{01}(x, y)+\ldots \ldots \ldots \ldots \ldots . \\
& \theta_{1}(x, y)=\theta_{10}(x, y)+\theta_{11}(x, y)+\ldots \ldots \ldots . . \\
& C_{0}(x, y)=C_{00}(x, y)+C_{01}(x, y)+\ldots \ldots \ldots . \\
& C_{1}(x, y)=C_{10}(x, y)+C_{11}(x, y)+\ldots . \tag{3.12}
\end{align*}
$$

Substituting the expansions(3.12) in equations (3.2)-(3.4) and separating the like powers-of Ec we get the following

$$
\begin{align*}
& \theta_{00, y y}=-\alpha_{1} \quad, \quad \theta_{00}(-1)=1, \theta_{00}(+1)=\operatorname{Sin} D_{1} \\
& C_{00, y y}=0 \quad, \quad C_{00}(-1)=0, C_{00}(+1)=1  \tag{3.14}\\
& \psi_{00, v y y}-M_{1}^{2} \psi_{00, y y}=-G\left(\theta_{00, y}+N C_{00, y}\right) \text {, }  \tag{3.15}\\
& \psi_{00}(+1)-\psi_{00}(-1)=1, \psi_{00, y}=0, \psi_{00, x}=0 \text { at } y= \pm 1 \\
& \theta_{01, y y}=-\frac{P_{1} R}{G} \psi^{2}{ }_{00, y y}-\frac{P_{1} M_{1}^{2}}{G} \psi^{2}{ }_{00, y}^{2} \quad, \quad \theta_{01}( \pm 1)=0  \tag{3.16}\\
& C_{01, y y}=0 \quad, \quad C_{01}(-1)=0, C_{01}(+1)=0  \tag{3.17}\\
& \psi_{01, y y y}-M_{1}^{2} \psi_{01, y y}=-G\left(\theta_{01, y}+N C_{01, y}\right)  \tag{3.18}\\
& \psi_{01}(+1)-\psi_{01}(-1)=0, \psi_{01, y}=0, \psi_{01, x}=0 \text { at } y= \pm 1 \\
& \theta_{10, y y}=G P_{1}\left(\psi_{00, y} \theta_{00, x}-\psi_{00, x} \theta_{00, y}\right) \quad \theta_{10}( \pm 1)=0  \tag{3.19}\\
& C_{10, y y}=S c\left(\psi_{00, y} C_{00, x}-\psi_{00, x} C_{00, y}\right) \quad C_{10}( \pm 1)=0  \tag{3.20}\\
& \psi_{10, y y y}-M_{1}^{2} \psi_{10, y y}=-G\left(\theta_{10, y}+N C_{10, Y}\right)+ \\
& \left(\psi_{00, y} \psi_{00, x y y}-\psi_{00, x} \psi_{00, x y y}\right)  \tag{3.21}\\
& \left.-\theta_{01, y} \psi_{0, x}\right)-\frac{2 P_{1} R^{2}}{G} \psi_{00, y y} \psi_{10, y y}-\frac{2 P_{1} M_{1}^{2}}{G} \psi_{00, y} \psi_{10, y}, \theta_{1}( \pm 1)=0  \tag{3.22}\\
& C_{11, y y}=S c\left(\psi_{00, y} C_{01, x}-\psi_{01, x} C_{00, y}+C_{00, x} \psi_{01, y}-C_{01, y} \psi_{0, x}\right) \tag{3.23}
\end{align*}
$$

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$$
\begin{align*}
\psi_{11, y y y}-M_{1}^{2} \psi_{1, y y}= & -G\left(\theta_{11, y}+N C_{11, y}\right)+\left(\psi_{00, y} \psi_{11, x y y}-\right. \\
& \left.\psi_{00, x} \psi_{01, y y y}+\psi_{01, y} \psi_{00, x y y}-\psi_{01 . x} \psi_{00, y y y}\right) \tag{3.24}
\end{align*}
$$

$$
\psi_{11}(+1)-\psi_{11}(-1)=0, \psi_{11, y}=0, \psi_{11, x}=0 \quad \text { at } \quad y= \pm 1
$$

## IV. SOLUTION OF THE PROBLEM

Solving the equations (3.13)- (3.24) subject to the relevant boundary conditions we obtain

$$
\begin{aligned}
& \theta_{o o}(y, t)=\left(\frac{\alpha_{1}}{2}\right)\left(1-y^{2}\right)+\left(\frac{\operatorname{Sin}\left(D_{1}\right)-1}{2}\right) y+\left(\frac{\operatorname{Sin}\left(D_{1}\right)+1}{2}\right) \\
& C_{00}=0.5\left(y^{2}-1\right)
\end{aligned}
$$

$$
\psi_{o o}(y, t)=a_{3}\left(y^{3}-y\right)+\frac{\left(1+2 a_{3}\right)}{\left(M_{1} C h\left(M_{1}\right)-S h\left(M_{1}\right)\right)}\left(\operatorname{Sh}_{1} y\right)
$$

$$
\left.-y \operatorname{Sh}\left(M_{1}\right)\right)+1+a_{1}+a_{3} \operatorname{Ch}\left(M_{1} y\right)+a_{4} y^{2}
$$

$$
\theta_{01}(y, t)=a_{15}\left(y^{2}-1\right)+a_{16}\left(y^{4}-1\right)+a_{17}\left(y^{6}-1\right)
$$

$$
+a_{18}\left(y \operatorname{Sh}\left(M_{1} y\right)-\operatorname{Sh}\left(M_{1}\right)\right)-a_{19}\left(\operatorname{Ch}\left(M_{1} y\right)-\right.
$$

$$
\left.C h\left(M_{1}\right)\right)+a_{20}\left(C h\left(2 M_{1} y\right)-C h\left(2 M_{1}\right)\right)
$$

$$
C_{01}(y, t)=a_{23}\left(y^{2}-1\right)+a_{24}\left(y^{4}-1\right)+a_{25}\left(y^{6}-1\right)+
$$

$$
a_{26}\left(y \operatorname{Sh}\left(M_{1} y\right)-\operatorname{Sh}\left(M_{1}\right)\right)+a_{27}\left(\operatorname{Ch}\left(M_{1} y\right)-\right.
$$

$$
\left.\operatorname{Ch}\left(M_{1}\right)\right)+a_{28}\left(\operatorname{Ch}\left(2 M_{1} y\right)-\operatorname{Ch}\left(2 M_{1}\right)\right)
$$

$$
\psi_{01}(y, t)=a_{34}+a_{35} y+a_{36} C h\left(M_{1} y\right)+a_{37} \operatorname{Sh}\left(M_{1} y\right)+a_{38} y^{2}+a_{39} y^{4}+
$$

$$
\left.a_{40} y^{6}+a_{41} y C h\left(M_{1} y\right)+a_{42} \operatorname{Ch}\left(2 M_{1} y\right)\right)
$$

$$
\theta_{10}(y, t)=b_{1}\left(y^{2}-1\right)+b_{2}\left(y^{3}-y\right)+b_{3}\left(y^{4}-1\right)+b_{4}\left(y^{5}-y\right)+b_{5}\left(y^{6}-1\right)+
$$

$$
b_{6}\left(C h\left(\left(M_{1} y\right)-C h\left(M_{1}\right)\right)+b_{7}\left(\operatorname{Sh}\left(M_{1} y\right)-\right.\right.
$$

$$
\left.y \operatorname{Sh}\left(M_{1}\right)\right)+b_{8}\left(y \operatorname{Sh}\left(M_{1} y\right)-\operatorname{Sh}\left(M_{1}\right)\right)
$$

$$
C_{10}(y, t)=d_{59}\left(y^{2}-1\right)+d_{60}\left(y^{3}-y\right)+d_{61}\left(y^{4}-1\right)+d_{62}\left(y^{5}-y\right)+d_{63}\left(y^{6}-1\right)+
$$

$$
\left(d_{64}+y d_{67}\right)\left(\operatorname{Ch}\left(\left(M_{1} y\right)-\operatorname{Ch}\left(M_{1}\right)\right)+d_{65}\left(\operatorname{Sh}\left(M_{1} y\right)-y \operatorname{Sh}\left(M_{1}\right)\right)+\right.
$$

$$
d_{66}\left(y \operatorname{Sh}\left(M_{1} y\right)-\operatorname{Sh}\left(M_{1}\right)\right)
$$

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$\psi_{10}=d_{80} y^{2}+d_{81} y^{3}+d_{82} y^{4}+d_{83} y^{5}+d_{84} y^{6}+d_{85} y^{7}+d_{86} y \operatorname{Sh}\left(M_{1} y\right)+$

$$
\begin{aligned}
& d_{87} y C h\left(M_{1} y\right)+d_{88} y^{2} \operatorname{Sh}\left(M_{1} y\right)+d_{89} y^{2} C h\left(M_{1} y\right)+d_{90} C h\left(M_{1} y\right)+ \\
& d_{91} \operatorname{Sh}\left(M_{1} y\right)+d_{92} y+d_{93}
\end{aligned}
$$

## V. GRAPHS



Fig. 1 Variation of $u$ with $G$
$\mathrm{D}^{-1}=1 \times 10^{2}, \mathrm{a}=2, \mathrm{~N}=1, \mathrm{~S}_{\mathrm{C}}=1.30, \mathrm{~S}_{0}=0.50, \mathrm{E}_{\mathrm{C}}=0.05$, $\mathrm{M}=2 \mathrm{x}+\gamma \mathrm{t}=\pi / 4$

|  |  | II | III | IV |
| :--- | :--- | :--- | :--- | :--- |
| G | $1 \times 10^{3}$ | $3 \times 10^{3}$ | $-1 \times 10^{3}$ | $-3 \times 10^{3}$ |



Fig. 2 Variation of $u$ with $D^{-1}$
$\mathrm{G}=1 \times 10^{3}, \alpha=2, \mathrm{~N}=1, \mathrm{~S}_{\mathrm{C}}=1.30, \mathrm{~S}_{0}=0.50, \mathrm{E}_{\mathrm{C}}=0.05$, $\mathrm{M}=2 \mathrm{x}+\gamma \mathrm{t}=\pi / 4$
$\mathrm{D}^{-1} \quad 1 \times 10^{2} \quad \stackrel{\text { I }}{ } 2 \times 10^{2} \quad{ }^{\text {II }} 3 \times 10^{2}{ }^{\text {III }}$


Fig. 3 Variation of $v$ with $\alpha$


Fig. 4 Variation of $\mathrm{v} \times 10^{2}$ with N

$$
\begin{aligned}
& D^{-1}=1 \times 10^{2}, G=1 \times 10^{3}, N=1, S_{C}=1.30, S_{0}=0.50, E_{C}=0.05 \\
& M=2 x+\gamma t=\pi / 4
\end{aligned}
$$

$\mathrm{D}^{-1}=1 \times 10^{2}, \mathrm{G}=1 \times 10^{3}, \alpha=0.5, \mathrm{~S}_{\mathrm{C}}=1.30, \mathrm{~S}_{0}=0.50, \mathrm{E}_{\mathrm{C}}=0.05$, $\mathrm{M}=2 \mathrm{x}+\gamma \mathrm{t}=\pi / 4$

Fig. 5 Variation Of $\theta$ with $\mathrm{S}_{\mathrm{C}}$
$\mathrm{D}^{-1}=1 \times 10^{2}, \mathrm{G}=1 \times 10^{3}, \alpha=0.5, \mathrm{~N}=1, \mathrm{~S}_{0}=.50, \mathrm{E}_{\mathrm{C}}=0.05$, $\mathrm{M}=2 \mathrm{x}+\gamma \mathrm{t}=\pi / 4$

|  |  | I | II | III |
| :--- | ---: | ---: | ---: | ---: |
| $\mathrm{S}_{\mathrm{C}}$ | 1.30 | 0.60 | 0.24 |  |



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Fig. 7 Variation of c with $\mathrm{E}_{\mathrm{C}}$
$\mathrm{D}^{-1}=1 \times 10^{2}, \mathrm{G}=2 \times 10^{3}, \mathrm{a}=0.5, \mathrm{~N}=1, \mathrm{~S}_{\mathrm{C}}=1.30, \mathrm{~S}_{0}=0.50$, $\mathrm{M}=2 \mathrm{x}+\gamma \mathrm{t}=\pi / 4$
$\begin{array}{cccll} & \text { I } & \text { II } & \text { III } & \text { IV } \\ \mathrm{E}_{\mathrm{C}} & 0.07 & 0.05 & 0.03 & 0.01\end{array}$


Fig. 8 Variation of $c$ with $x+\gamma t$

$$
\begin{aligned}
& \mathrm{D}^{-1}=1 \times 10^{2}, \mathrm{G}=2 * 10^{3}, \alpha=0.5, \mathrm{~S}_{\mathrm{C}}=1.30, \mathrm{~S}_{0}=0.50, \mathrm{E}_{\mathrm{C}}=0.05 \text {, } \\
& \mathrm{N}=1
\end{aligned}
$$

## VI. SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

The shear stress on the channel walls is given by

$$
\tau=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \quad y= \pm L
$$

which in the non- dimensional form reduces to

$$
\begin{aligned}
\tau=\left(\frac{\frac{\tau}{\mu U}}{a}\right) & =\left(\psi_{y y}-\delta^{2} \psi_{x x}\right) \\
& =\left[\psi_{00, y y}+E c \psi_{01, y y}+\delta\left(\psi_{10, y y}+E c \psi_{11, y y}+O\left(\delta^{2}\right)\right]_{y= \pm 1}\right.
\end{aligned}
$$

and the corresponding expressions are

$$
\begin{aligned}
& (\tau)_{y=+1}=b_{90}+\delta b_{91}+O\left(\begin{array}{ll}
2
\end{array}\right) \\
& (\tau)_{y=-1}=b_{92}+\delta b_{93}+O\left(\begin{array}{ll}
2
\end{array}\right)
\end{aligned}
$$

The local rate of heat transfer coefficient (Nusselt number Nu ) on the walls has been calculated using the formula

$$
N u=\frac{1}{\theta_{m}-\theta_{w}}\left(\frac{\partial \theta}{\partial y}\right)_{y= \pm 1}
$$

and the corresponding expressions are

$$
\begin{aligned}
&\left(\begin{array}{ll}
N & u
\end{array}\right)_{y=+1}=\frac{\left(b_{51}+\delta b_{52}\right)}{\left(b_{44}-\operatorname{Sin}\left(D_{1}\right)+\delta b_{45}\right)} \\
&\left(\begin{array}{ll}
N & u
\end{array}\right)_{y=-1}\left.=\frac{\left(b_{53}+\right.}{}+\delta b_{54}\right) \\
&\left(\begin{array}{llllll}
b_{44} & -1 & + & \delta & b & 45
\end{array}\right)
\end{aligned}
$$

The local rate of mass transfer coefficient (Sherwood number) ( Sh ) on the walls has been calculated using the formula

$$
S h=\frac{1}{C_{m}-C_{w}}\left(\frac{\partial C}{\partial y}\right)_{y= \pm 1}
$$

and the corresponding expressions are

$$
\begin{aligned}
(S h)_{y=+1} & =\frac{\left(b_{65}+\delta b_{63}\right)}{\left(b_{58}-1+\delta b_{57}\right)} \\
(S h)_{y=-1} & =\frac{\left(-b_{65}+\delta b_{63}\right)}{\left(b_{58}+\delta b_{57}\right)}
\end{aligned}
$$

A. Discussion of The Numerical Results
we discuss the effect of dissipation and thermo- diffusion on convective heat and moss transfer flow of a viscous fluid through a porous medium in vertical channel on whose walls a traveling thermal wave is imposed. The velocity, temperature and

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concentration are discussed for different values of $\mathrm{G}, \mathrm{D}^{-1}, \alpha, \mathrm{~N}, \mathrm{Sc}, \mathrm{So}, \mathrm{E}_{\mathrm{C}}$ and $x+\gamma t$. Fig. 1 represents variation of u with Grashof number ' G '. The actual axial flow is in the vertically upward direction. That is $\mathrm{u}>0$ is the actual flow and $\mathrm{u}<0$ is the reversal flow. It is found that $u$ exhibits a reversal flow in the vicinity of left boundary $y=-1$ for $G>0$ and for $G<0$ we notice a reversal flow in the region $(0.4,0.8)$ and the region of reversal flow shrinks with increase $|\mathrm{G}|$ with maximum $|\mathrm{u}|$ occurring at $\mathrm{y}=0.6$. The variation of u with Darcy parameter ' $\mathrm{D}^{-1}$ ' is shown in fig.2.It is found that a reversal flow which occurs at $\mathrm{y}=-0.8$ spreads to the region $(-0.8,0.2)$ and region reversal flow enlargers with increase in $\mathrm{D}^{-1}$ Also lesser the permeability of the porous medium larger $|\mathrm{u}|$ in the flow region. The secondary velocity (v) which is due to the traveling thermal wave imposed on the wall is shown in $\mathrm{Fig}(3-4)$. An increase in the strength of heat source results in a depreciation $|v|$ everywhere in the region(fig3). When the molecular buoyancy force dominates over the thermal buoyancy force the secondary velocity depreciates in the first half and enhances in the second half when the buoyancy forces act in the same direction and for the forces in opposite direction a reversed effect is observed in the behavior of $|\mathrm{v}|$ (fig.4).The non-dimensional temperature distribution $(\theta)$ is shown in figs(.5-6) for different parametric values. From fig. 4 we observe that lesser the molecular diffusivity larger the temperature in the entire flow region. Fig. 5 represents the variation of $\theta$ with $S_{0}$.It shows that an increases in $S_{0}>0$ enhances the actual temperature while it depreciates with $\left|\mathrm{S}_{0}\right|$ The non-dimensional concentration distribution (C) is shown in figs7-8 for different parametric values. We follow the convention that the nondimensional concentration is positive or negative according as the actual concentration is greater/ lesser than $\mathrm{C}_{2}$. The variation of C with Ec shows that higher the dissipative effects smaller the concentration (fig.31). An increase in the phase $\mathrm{x}+\gamma t$ of traveling thermal wave shows that the actual concentration depreciates at $\mathrm{x}+\gamma t=\pi / 2$, enhances at $\mathrm{x}+\gamma t=\pi$ and again depreciates at $\mathrm{x}+\gamma t=2 \pi$.

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