Volume 5 Issue VI, June 2017
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Estimation Of Power Harmonics Using Kalman Filter

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Abstract: This paper presents an optimal method for tracking the harmonics in power system voltage or current waveforms. A Kalman filter is used to estimates the harmonics of a distorted measurement signal. The Kalman filter performance depends the noise covariance matrices Q and R. In this paper tuning of these matrices are discussed. The Kalman filter's response time is investigated for a sudden variation in the magnitude and phase of one of the harmonics present in the signal.
Keywords—Kalman filter, harmonic signal estimation, adaptive filtering, noise covariance matrices

I. INTRODUCTION

The electrical power system has been increasing in complexity at a rapid rate in the last few decades. Many measures have been introduced to improve its reliability and security. However, the system is more and more polluted owing to the increasing use of power-electronic converters and controllers for industrial processes and drives, among other types of disturbing loads. The effect is the contamination of the 50 Hz supply by a wide range of frequencies up to the radio frequencies, and thus power system monitoring becomes a necessity to check the state of health of the power network. Negative effects of harmonic currents and voltages, such as increased I^2R losses and the reduction of the lifespan of sensitive equipment, has prompted the establishment of a number of standards and guidelines regarding acceptable harmonic levels. Accurate analysis of power system measurements is essential to determine harmonic levels and effectively design of filters. Harmonic levels and fundamental frequency are load dependant. The problem thus becomes one of determining the frequency content of a non-stationary signal. The effectiveness of standard Fourier transform techniques such as the FFT and the STFT is limited by averaging, spectral leakage and the picket fence effect, prompting the search for alternative methods. A wide variety of techniques have been investigated. Modified versions of the standard DFT analysis have been proposed by Zuhua Ren et al[1], whilst wavelet analysis has been successfully applied by Pham and Wong et al in [2]. The conventional weight least square method and a new singular decomposition methods were also verified Moghadasian, M et al[3]. Neural networks have also been extensively studied as a means of harmonic extraction [4][5].

One method in particular which has attracted much interest is the recursive filtering algorithm, Kalman filter. The Kalman filter offers the possibility of dynamic tracking of the harmonic content, increasing the accuracy of the analysis, and providing a possible input to a feedback control scheme, such as an activc filter. One of the first papers to apply the Kalman filter to the harmonic estimation problem was Girgis et al [6]. Since then, a variety of work has been published in the area investigating a number of different models and approaches [7] [8]. Whilst the potential of the Kalman filter as a tool for estimation harmonics has been demonstrated, practical application has been limited by implementation difficulties. The response of the filter is governed by the error covariance matrices Q and R, which act as "tuning" parameters for the estimation, balancing between accurate speed of tracking and filter divergence. In practice, choosing appropriate parameters for desired filter operation can be an arduous task; limiting the success of the application.

This paper explores the application of the Kalman filter to the estimation of power system harmonics. The tuning methods of noise covariance matrices are presented. Finally, the effectiveness of the Kalman filter analysis is assessed during typical power system operation and during disturbances such as a sudden change in one of the harmonic frequency component. In this paper the state variable representation of the signal is presented section II and Kalman filter description is given in section III. Section IV discuses the tuning methods of the Kalman filter. Finally simulation results and conclusions are presented sections V and VI respectively.

II. STATE VARIABLE REPRESENTATION OF THE SIGNAL

Considering a reference rotating frequency at w, the noise-free signal may be expressed[9] as:

$$\begin{aligned} Z_k &= A_k \cos(wk\Delta t + \phi_k) \\ &= A_k \cos(\phi_k)\cos(wk\Delta t) - A_k \sin(\phi_k)\cos(wk\Delta t) \end{aligned} \tag{1}$$

Let X_{1k} be $A_k \cos(\phi_k)$ and X_{2k} be $A_k \sin(\phi_k)$; therefore both X_{1k} and X_{2k} include two components, which are unknown. The variables X_{1k} and X_{2k} represent the in-phase and quadrature-phase components and referred to as state variables. This leads to the following state equation:

$$X_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X_k + W_k \tag{2}$$

Where W_k allows the state variables for random walk and it is a white noise sequence and $X_k = [X_{1k} \quad X_{2k}]^T$. The measurement equation would include the signal and noise and it can be represented as:

$$Z_k = [\cos(wk\Delta t) \quad -\sin(wk\Delta t)] X_k + V_k \tag{3}$$

Where V_k is measurement noise covariance vector.

Now a noise free voltage or current measurement signal that includes n harmonics may be represented by:

$$Z_k = \sum_{i=1}^{n} A_{ik} \cos(iwk\Delta t + \phi_{ik}) \tag{4}$$

Where :

A_{ik} is the magnitude of the phasor quantity representing the i^{th} harmonics at instant k.

ϕ_{ik} is the phase angle of the i^{th} harmonics at instant k.

n is the harmonic order and Δt is the sampling interval.

Each frequency component requires two state variable, thus the total number of state variables is $2n$. These state variables are defined as follows:

$$\begin{array}{ll} X_{1k} = A_{1k} \cos(\phi_{1k}), & X_{2k} = A_{1k} \sin(\phi_{1k}) \\ X_{3k} = A_{2k} \cos(\phi_{2k}), & X_{4k} = A_{2k} \sin(\phi_{2k}) \\ \vdots & \vdots \\ X_{2n-1k} = A_{nk} \cos(\phi_{nk}), & X_{2nk} = A_{nk} \sin(\phi_{nk}) \end{array} \tag{5}$$

These equations (5) represent the in-phase and quadrature phase components of the harmonics with respect to a rotating reference, respectively. Thus the state variable equations may be represented as:

$$X_{k+1} = [I] X_k + W_k \tag{6}$$

Where I is the identity matrix of order 2n X 2n, and the measurement equation can be then expressed as:

$$\begin{aligned} Z_k = [&\cos(w_1 k\Delta t) \quad -\sin(w_1 k\Delta t) \\ &\cdots \cos(w_n k\Delta t) \quad -\sin(w_n k\Delta t)] X_k + V_k \end{aligned} \tag{7}$$

Where $X_k = [X_{1k} \quad X_{2k} \quad \cdots \quad X_{2nk}]^T$.

III. DESCRIPTION OF KALMAN FILTER

The Kalman filter is a technique for estimating the unknown state of a dynamical system with additive noise. The Kalamn filter has long been regarded as the optimal solution to many tracking and state prediction tasks [10]. The strength of Kalamn filter algorithm is that it computes on-line. This implies that we don't have to consider all the previous data again to compute the current estimates; we only need to consider the estimates from the previous time step and the current measurement. Popular applications include, state estimation [11], navigation, guidance, radar tracking [12], sonar ranging, satellite orbit computation, etc. These applications can be summarized into various classes such as denoising, tracking and control problems. The basic Kalamn filter is optimal in the mean

square error sense (given certain assumptions), and is the best possible of all filters, if state and measurement inputs are Gaussian vectors and the additive noise is white and has zero mean [10].
We assume that the system can be modelled by the state transition equation,

$$X_{k+1} = \Phi X_k + W_k \tag{8}$$

where X_k is the state at time *k*, W_k is additive noise from either the system or the process and Φ is the state transition matrix. The measurement system can be represented by a linear equation of the form,

$$Z_k = HX_k + V_k \tag{9}$$

where Z_k is the measurement prediction made at time *k*, V_k is additive measurement noise and H is the observation matrix. With the noise error vector W_k and V_k we can define:

$$E[w, w_j^T] = Q\delta_{ij} \tag{10}$$

$$\text{and } E[v, v_j^T] = R\delta_{ij} \tag{11}$$

The Kalman Filter uses a feed-back control for process estimation. The KF algorithm consists of two steps: (i) a prediction step and (ii) an update step as described below.

A. Prediction (time-update)
This predicts the state and process covariance at time *k+1* dependent on information at time *k*.

B. Update (measurement update)
This updates the state, process covariance and Kalman gain at time *k+1*.

Summary of Kalman Filter Equations are given below:

C. Step 1: Predicted Sate

$$X_{k+1/k} = \Phi_k X_{k/k} \tag{12}$$

D. Step2: Predicted Measurement

$$Z_{k+1/k} = H_k X_{k+1/k} \tag{13}$$

E. Step3: Predicted Sate Covariance

$$P_{k+1/k} = \Phi_k P_{k/k} \Phi_k^T + Q_k \tag{14}$$

F. Step4: Predicted Kalman Gain

$$K_{k+1} = P_{k+1/k} H_k^T \left[H_k P_{k+1/k} H_k^T + R_k \right]^{-1} \tag{15}$$

G. Step5: Actual Measurement

$$Z_{k+1} \tag{16}$$

H. Step6: Updated (Estimated or Corrected) Sate

$$X_{k+1/k+1} = X_{k+1/k} + K_{k+1}(Z_{k+1} - Z_{k+1/k}) \tag{17}$$

I. Step7: Updated (Estimated or Corrected) Sate Covariance

$$P_{k+1/k+1} = (1 - K_{k+1} H_k) P_{k+1/k} \tag{18}$$

The above seven equations constitute Kalman Filter Algorithm [10]. By knowing the initial conditions (State X and its Covariance P) and the noise covariance matrices (Process noise Q and Measurement noise R) the steps 1 to 4 can be executed. As soon as measurement is available steps 6 & 7 can be executed and cycle can be repeated for next measurement.

IV. TUNING OF KALMAN FILTER

The practical application of the Kalman filter has been limited by the difficulties involved in accurately tuning the filter for desired

Volume 5 Issue VI, June 2017
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performance. The Kalman filter optimally combines model and measurement information to estimate the state vector X_k. The error covariances Q and R determine the relative weighting of the model and measurement information, in essence, acting as tuning parameters balancing the dynamic response of the filter against sensitivity to noise. The effect of Q and R on the Kalman filter performance is a topic which has been investigated in detail [12][13].

Whilst there are a number of excellent texts on the subject, the calculation of the error covariance matrix Q and measurement error R is often a non-trivial task [14][15]. Inaccuracies due to modelling error and linearisation error can often be difficult to quantify. Effects such as cross-correlation between model and measurement error and the coloration of the noise processes W_k and V_k must be taken into consideration. Also, research has indicated that in many cases, particularly for nonlinear systems, theoretical values for Q and R do not necessarily yield the most accurate results [13]. These factors combined ensure that the tuning of the Kalman filter can often he a challenging task. However, the process can be facilitated by adopting a methodical approach as follows:

A. Calculate Theoretical Values If Possble

While sometimes difficult, if sufficient information is available regarding the propagation of states and measurement, then Q and R can be calculated using equations (10) and (11). These theoretical values might not always necessarily yield desired results. However they will often offer a good initial starting point from which a desired response can he obtained with minimum adjustment of actual values.

B. Examine The Effect Of Q/R Ratio

Assuming no cross-correlation between states Q is typically a diagonal matrix of form:

$$diag\ (Q) = [q^1 q^2 \cdots q^n] \tag{19}$$

while for a single measurement, R reduces to a scalar value. In many cases it is not the actual value of Q and R that determines the filter response, but rather the ratio, Q/R [13]. If this is the case, R can be set to a nominal value e.g. $R = 1$ and Q can be tuned accordingly in order to give desired response.

C. Investigate Variants Of The Kalman Filter Algorithm

In certain systems, the use of the Q/R ratio as a tuning parameter may not be valid, and it is often difficult to tune the filter effectively. There are also a number of algorithms proposing adaptive updating of the Q and R parameters [16]. The adaptive algorithm adopted in this paper was first proposed by Jazwinski in [17]. Since then it has been studied and employed by a number of authors [18]. It can be derived by assuming Q matrix to be of the form $Q = q_k I$ with q_k updated each iteration by:

$$\hat{q}_k = f\left(\frac{e_k^2 - (H_k P_{k+1/k} H_k^T + R_k)}{H_k H_k^T}\right)$$

$$\text{Where}\quad f(x) = \begin{cases} x \ if \ \ x \geq 0 \\ 0 \quad otherwise \end{cases} \tag{20}$$

Here e_k^2 is the predictor error, and it is defined as:

$$e_k = Z_k - H_k X_{k+1/k} \tag{21}$$

In order to increase statistical significance the estimate is smoothed by:

$$q_k = \alpha q_{k-1} + (1-\alpha)\hat{q}_k \tag{22}$$

where α is positive scalar. Similarly the measurement error variance r_k van be recursively calculated by:

$$\begin{aligned} \hat{r}_k &= f(e_k^2 - H_k P_{k+1/k} H_k^T); \\ r_k &= \alpha r_{k-1} + (1-\alpha)\hat{r}_k \end{aligned} \tag{23}$$

For practical application a lower bound is placed on r_k to prevent the values dropping below minimum measurement noise.

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V. SIMULATION RESULTS

Simulation studies were performed on a 12 state Kalman filter to estimate the harmonics of the distorted measurement voltage or current signal. The following signal was taken as measurement signal which includes 3rd,5th,7th,9th and 11th harmonics of 50 Hz and is given by

$$Z(t) = 1.0\cos(wt + 20) + 0.1\cos(3wt + 210) + 0.06\cos(5wt + 180) + 0.02\cos(7wt - 200) + 0.009\cos(9wt - 145) + 0.005\cos(11wt + 30) \tag{24}$$

The wave form of this signal is depicted in fig.1. This signal is applied to a 12 state Kalman filter. The harmonic components along with the fundamental(i.e., 50 Hz, 3rd,5th,7th,9th and 11th) estimates are shown in fig.2 & fig.3. It can be seen from the figures that with zero initial state vector the filter is capable of undertaking real-time assessment of the magnitudes calculated is nearly half a cycle of the fundamental frequency(10 ms).

To evaluate the Kalman filter's response time to a sudden variation in the magnitude and phase of one of the harmonics present in the signal, the signal Z(t) has been modified with a variation in the magnitude and phase of the 5th harmonic after third cycle. The magnitude and phase of this harmonic becomes 0.12 and +90˚. The magnitude of the 50 Hz, 3rd, 5th, 7th,9th and 11th components and THD is obtained by applying the modified signal samples to a 12 state Kalman filter. It can be seen from the figures that the time it takes the filter to respond to a sudden change in the magnitude and phase of the 5th harmonic is less than half a cycle of the fundamental frequency.

VI. CONCLUSIONS

In this paper, the application of the Kalman filter to power system harmonic signal analysis was investigated. A 12 state Kalman filter was used to estimate harmonic components along with the fundamental component (i.e.,50Hz,3rd,5th,7th,9th,11th), of the distorted voltage or current measurement signal. The response time of the filter was examined by applying a variation in the magnitude and phase of 5th harmonic component. In this paper the tuning methods of the Q and R matrices were also discussed.

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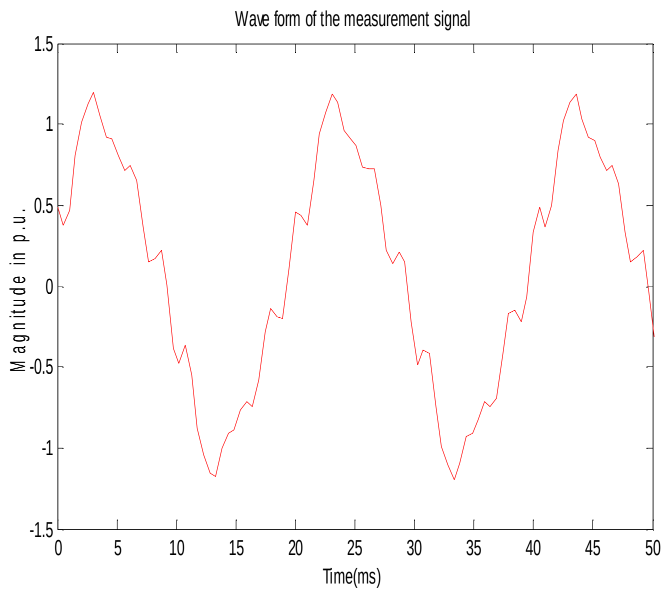

Fig. 1 Waveform of the Measurement Signal

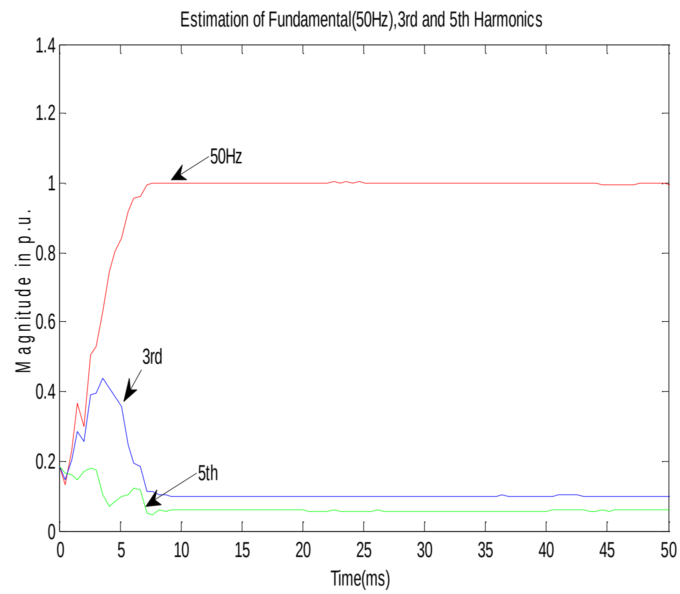

Fig. 2 Waveform of the Measurement Signal Estimation of Fundamental (50z), 3rd & 5th Harmonics

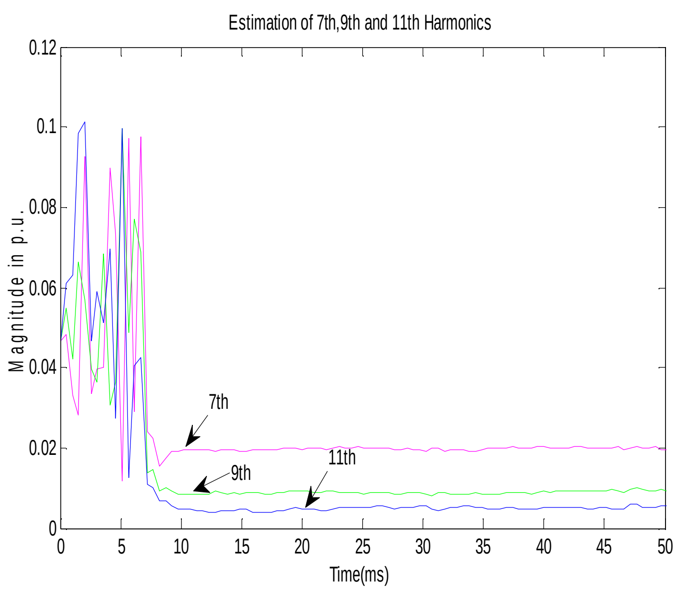

Fig. 3 Estimation of 7th, 9th and 11th Harmonics

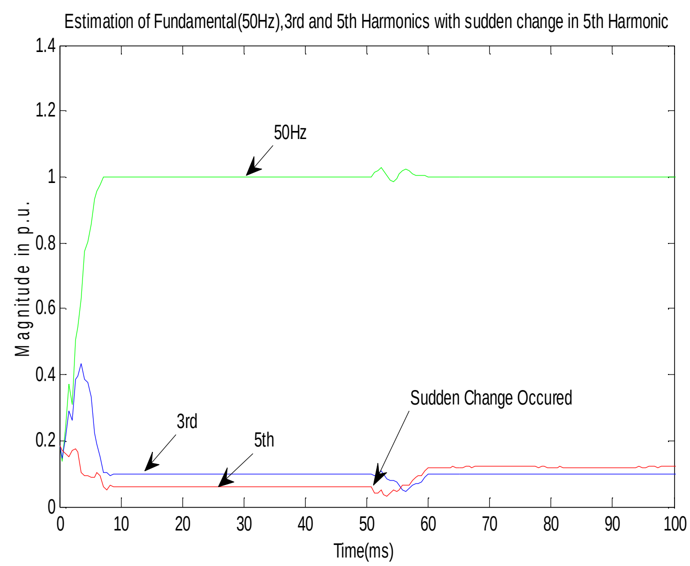

Fig. 4 Estimation of Fundamental (50z), 3rd & 5th Harmonics with Sudden Change in 5th Harmonic

Volume 5 Issue VI, June 2017
ISSN: 2321-9653

International Journal for Research in Applied Science & Engineering Technology (IJRASET)

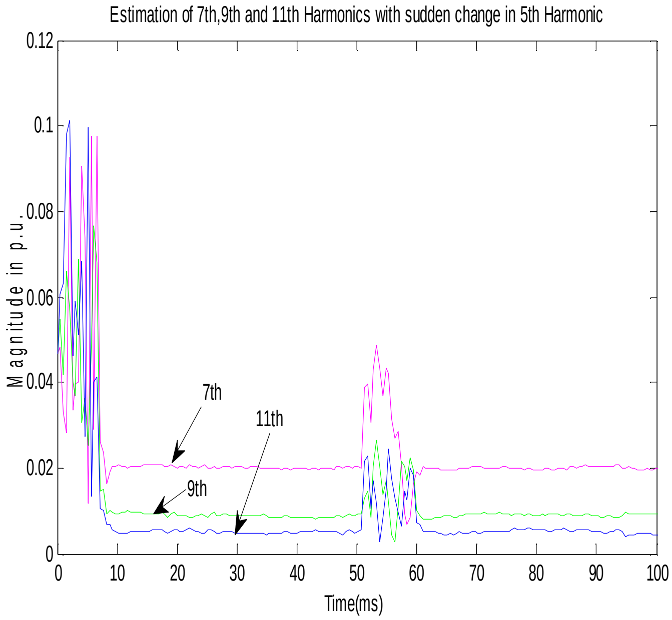

Fig. 5 Estimation of 7^{th}, 9^{th} & 11^{th} Harmonics with Sudden Change in 5^{th} Harmonic

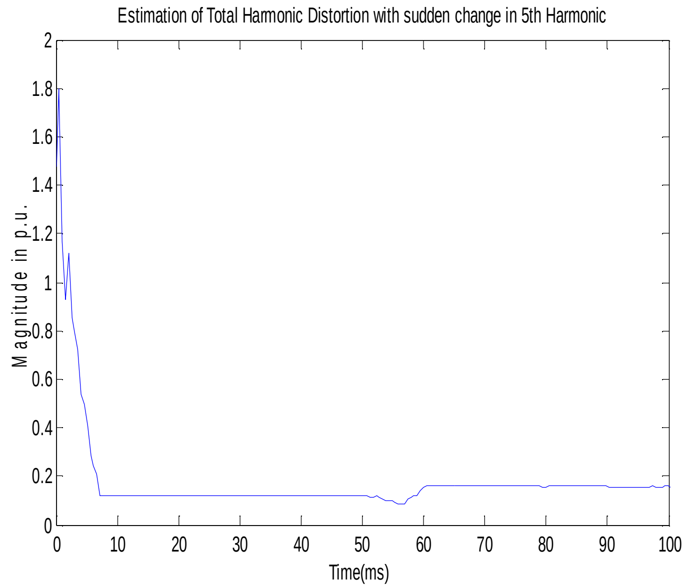

Fig. 6 Estimation of Total Harmonic Distortion(THD) with Sudden Change in 5^{th} Harmonic