

Limitation of Classical Bending Theory in Isotropic and FRP Beams

Bapi Raju V¹, Balakrishna Murthy V², Prasanna K. G. L.³,

¹Assistant Professor, Department of Mechanical Engineering, V. R. Siddhartha Engineering College, Vijayawada-520 007, India

²Professor, Department of Mechanical Engineering, V. R. Siddhartha Engineering College, Vijayawada-520 007, India

³ Student, Department of Mechanical Engineering, V. R. Siddhartha Engineering College, Vijayawada-520 007, India

Abstract: *In this paper, the deflections and the bending stresses of beam due to a uniform load, for various dimensions of the beam, are calculated by using classical beam bending theory and the same is once again calculated by modeling the problem as a 3- dimensional and 2-dimensional plane stress problem using ANSYS. The results are then compared to find the limitations of using 1 dimensional and 2 dimensional approaches for calculations of deflections and stresses*

Keywords: *FEM; Non-linear analysis; 1D and 3D comparison of beams; Limitations of using 1D and 2D, Modelling of beams*

I. INTRODUCTION

When a beam deforms, under a load the deflections and bending stresses can be calculated using classical beam bending theory, provided one of the dimensions is relatively very large compared to other two, so that the problem can be treated as one dimensional (1-D). However when one of smaller dimensions starts getting bigger relatively then we need to go for 2 dimensional (2-D) approach using finite element method (FEM) to calculate deflections and bending stresses and if none of the dimensions are relatively small then we need to go for 3 dimensional approach using FEM to achieve the same

II. PREVIOUS STUDY

uiz A. et al [1] discussed an eight-node hexahedral element with uniform reduced integration, which is free of volumetric and shear locking and has no spurious singular modes, is implemented for geometrically non-linear static structural analysis. Numerical examples verify the computational efficiency and the potential of the three-dimensional element in the analysis of shells, plates and beams undergoing large displacements and rotations. Results are compared to those employing classical plate and shell elements.

Chandrashekhara et al [2] presents the flexural analysis of fiber-reinforced composite beams based on a higher-order shear deformation theory. The influence of boundary conditions, beam geometries, and ply orientations on the deflections and stresses of laminated beams is shown both in tabular and graphical forms.

Damian [3] developed a three-dimensional finite element model to examine the structural behavior of the Horsetail Creek Bridge in Oregon both before and after applying FRP laminates. The comparisons between ANSYS predictions and field data are made in terms of concrete strains. The analysis shows that the FE bridge model does not crack under the applied service truckloads

III. PURPOSE OF PRESENT STUDY

In order to identify the limitations of 1-D and 2-D approaches comparisons of deflections and bending stresses of a beam under the action of a uniformly distributed load (UDL) are done by using beams of various dimensions

IV. GEOMETRICAL MODELING OF THE PROBLEM

A. Comparison of 1D and 3D modelling for isotropic beam

For calculating the deflections and stresses in 1-D modelling formulas from classical mechanics are used

A 3-D model of beam is modeled using ANSYS software (Fig 1). The length of the beam is taken as 1m and cross sectional dimensions of the beam are height h which varies according to different span to height ratios (s) such as s=10, 20, 40, 60, 80, 100. The width of the beam is taken according to width to height ratio, a=1. The finite element mesh is generated with solid-95 element type that provides accurate results for hexagonal mapped mesh. The number of element divisions made are 100 along the span of the beam, 10 divisions along the width of the beam and 10 divisions along the height of the beam. Simply supported boundary conditions are applied to the beam. A uniform transverse pressure of 1MPa is applied on the top surface of the beam.

The following material properties are considered for the present analysis.

Young's Modulus, E=200GPa

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Poisson's Ratio, $\nu = 0.3$

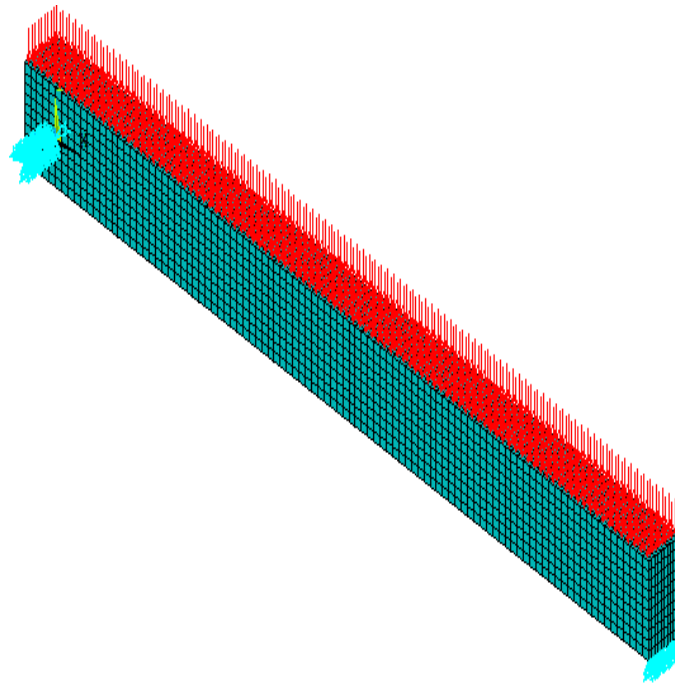


Fig. 1 FE model showing meshing, boundary conditions, and loading ($s=10$, $a=0.3$)

B. Comparison of 2D and 3D modeling Isotropic beam

A 2-D model of beam is modeled using ANSYS software by modeling longitudinal section of the beam (Fig. 2). The length of the beam is taken as 1m. The analysis is done for different aspect ratios 'a' (width to height ratio). The finite element mesh is generated with 8-node quadratic Plane-82 elements. For the beam simply supported boundary conditions are applied. A uniform transverse pressure of 1MPa is applied on the top surface of the beam.

3-D model is modeled as explained in the previous section.

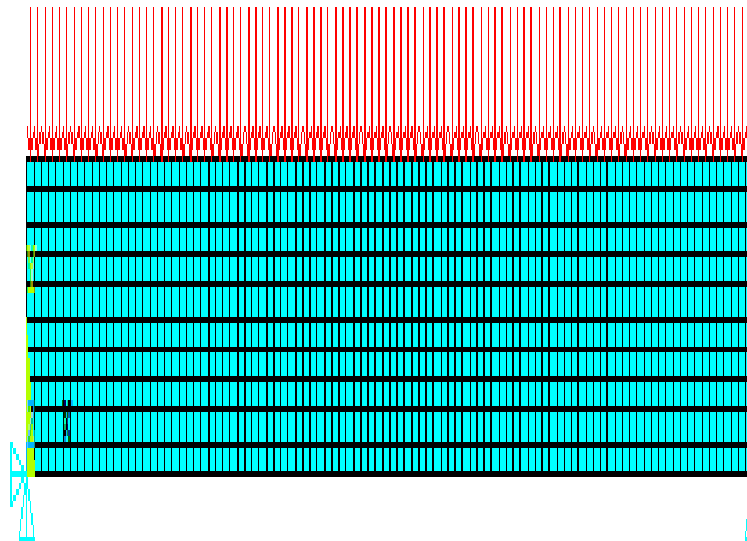


Fig. 2 2-D FE model showing meshing, boundary conditions, and loading ($s=10$, $a=0.2$)

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C. Comparison of 2D and 3D modeling orthotropic beam

The modelling of the problem is done as explained in the previous section. However instead of isotropic beam an orthotropic beam is used.

The details of orthotropic materials used for the analysis of composite beams (0/90/90/0) are listed in Table 1

TABLE .1 MATERIAL PROPERTIES OF GRAPHITE-EPOXY AT VF =0.6

	$\theta=0^\circ$	$\theta=90^\circ$
Ex	141.6764GPa	12.3857GPa
Ey	12.3857GPa	12.3857GPa
Ez	12.387GPa	141.6764GPa
vxy	0.257	0.4205
vyz	0.4206	0.256
vzx	0.257	0.256
Gxy	4.0301GPa	4.3592GPa
Gyz	4.3592GPa	4.0301GPa
Gxz	4.0301GPa	4.0301GPa

V. RESULTS AND DISCUSSION

A. Comparison of 1D and 3D modelling for isotropic beam

Comparison of 1-D and 3-D results is listed in Tables 2 and 3. Error graphs are drawn with respect to 's'. These graphs are useful to state the limitations of 1-D approach.

TABLE 2 COMPARISON OF 1-D AND 3-D DEFLECTIONS

S	1-D	3-D	% Error
10	0.000781	0.000818	4.492665
20	0.00625	0.006303	0.840869
40	0.05	0.050089	0.177684
60	0.16875	0.169087	0.19907
80	0.4	0.40049	0.12235
100	0.78125	0.781386	0.017405

TABLE 3 COMPARISON OF 1-D AND 3-D STRESSES

S	1-D	3-D	% Error
10	75000000	7.73E+07	2.97542
20	3E+08	3.00E+08	3.97E-14
40	1.2E+09	1.20E+09	0.033322
60	2.7E+09	2.70E+09	0.022227
80	4.8E+09	4.80E+09	0.00625
100	7.5E+09	7.51E+09	0.133156

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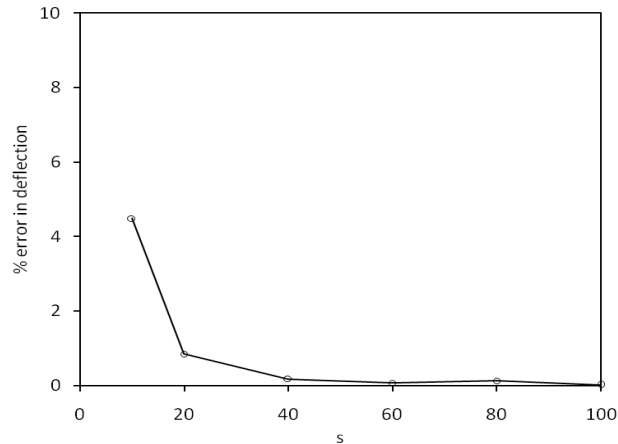


Fig. 3 Variation of %error in deflection w.r.t S

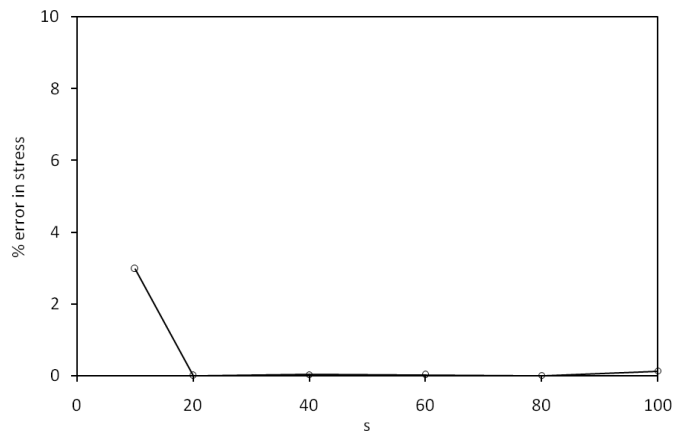


Fig. 4. Variation of %error in stress w.r.t. S

From these graphs it can be observed that the percentage error decreases with increase in 's'. For the values of 's' below 20, 3-D analysis may be suggested as there is considerable variation in 1-D and 3-D results.

B. Comparison of 2D and 3D modeling Isotropic beam

Comparison of 2-D and 3-D results is listed in Tables 4 and 5. Graphs are drawn with respect to 'a'. These graphs are useful to state the limitations of 2-D approach.

TABLE 4 COMPARISON OF 2-D AND 3-D DEFLECTIONS

a	2D	3D	%Error
0.2	8.18E-04	8.17E-04	0.241909
0.3	8.18E-04	8.16E-04	0.248018
0.4	8.18E-04	8.16E-04	0.245574
0.5	8.18E-04	8.17E-04	0.237022
1	8.19E-04	8.18E-04	0.129417

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TABLE 5 COMPARISON OF 2-D AND 3-D STRESSES

a	2D	3D	%Error
0.2	7.52E+07	7.52E+07	1.99E-02
0.3	7.52E+07	7.52E+07	3.99E-03
0.4	7.52E+07	7.52E+07	3.99E-03
0.5	7.52E+07	7.52E+07	3.99E-03
1	7.52E+07	7.53E+07	3.59E-02

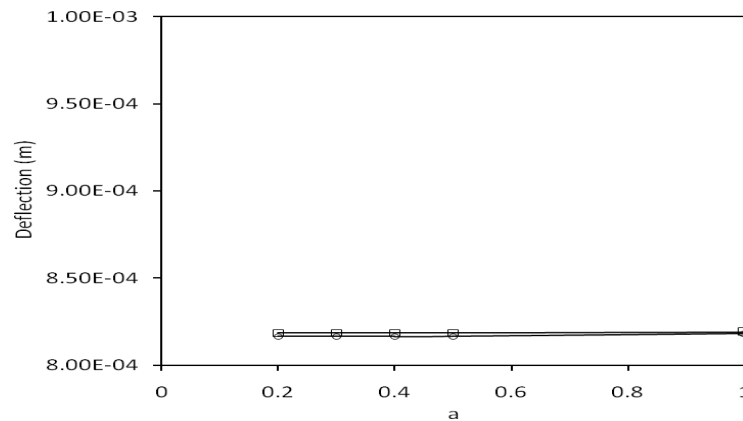


Fig.5 variation of deflection w.r.t 'a'.

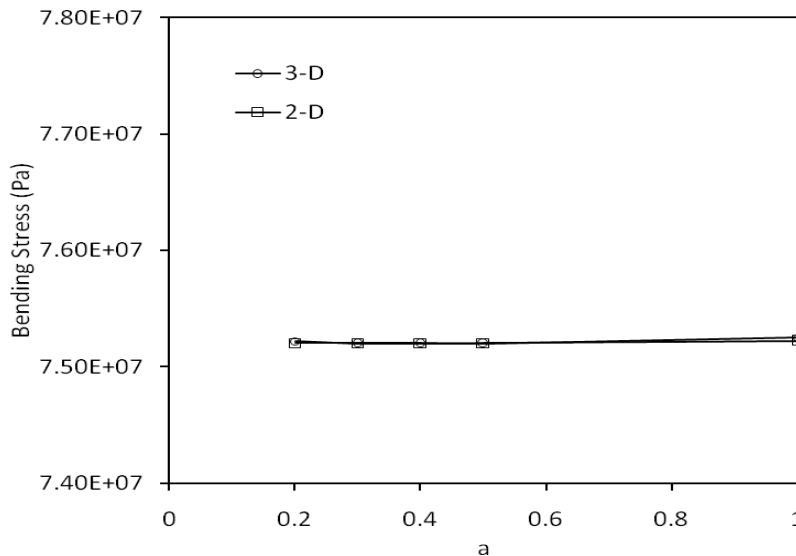


Fig.6 variation of bending stress w.r.t 'a'.

From the above graphs it is observed that the deflection and stress of both the beams is almost same. There is no significant variation in the results of 2-D and 3-D approaches. Hence 2-D approach is sufficient.

C. Comparison of 2D and 3D modeling orthotropic beam

Comparison of 2-D and 3-D results is listed in Tables 6 and 7. Graphs are drawn with respect to 'a'. These graphs are useful to state the limitations of 2-D approach for composite beams.

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TABLE 6 COMPARISON OF 2-D AND 3-D DEFLECTIONS

a	2-D	3-D	%error
0.1	1.84E-03	1.80E-03	1.78E+00
0.2	0.001836	1.80E-03	1.91E+00
0.3	0.001839	0.001801	2.07E+00
0.4	1.84E-03	1.80E-03	2.15E+00
1	0.001836	1.79E-03	2.36E+00

TABLE 7 COMPARISON OF 2-D AND 3-D STRESSES

a	2-D	3-D	%error
0.1	8.67E+07	8.73E+07	0.606193
0.2	8.67E+07	8.73E+07	0.687285
0.3	8.67E+07	8.73E+07	0.687285
0.4	8.67E+07	8.73E+07	0.62669
1	8.67E+07	8.73E+07	0.63695

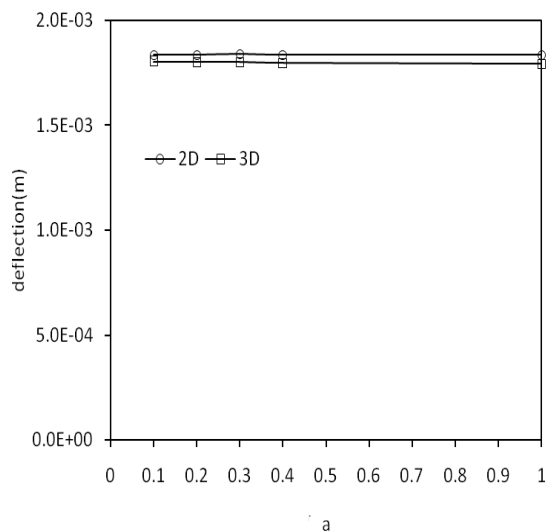


Fig 7 variation of deflection w.r.t 'a'

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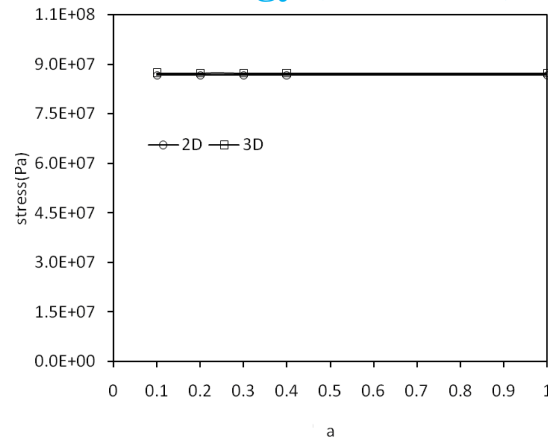


Fig 8 variation of stress w.r.t 'a'

From the above graphs it is observed that the deflection and stress of both the beams is almost same. There is no significant variation in the results of 2-D and 3-D approaches. Hence 2-D approach is sufficient for composite beams.

VI. CONCLUSIONS

Comparing 1D and 3D approaches, it can be seen that the percentage error decreases with increase in 's'. For the values of 's' below 20, 3-D analysis may be suggested as there is considerable variation in 1-D and 3-D results

As for the comparison of 2D and 3D approaches, there is no significant variation in the results of 2-D and 3-D approaches for both isotropic and orthotropic beams. Hence 2-D approach is sufficient

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