iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 Issue: VI Month of publication: June 2017

DOI:

**International Journal for Research in Applied Science & Engineering Technology (IJRASET)**

# Study of Simple Pendulum: Special Context to Hollow Bob

R. P. Zingre[1], B. U. Jadhav[2]

*[1,2] Dept. of Physics & Electronics, Shri Shivaji College, Vasmat Road, Parbhani -431401(M.S.) India*

***Abstract: In this research paper we will be formulating the expression for time period for varying length pendulum. The hollow bob suspended to non-stretchable string forms the system of our interest. The small solid ball of suitable radius is kept inside the bob. When this system is allowed to oscillate, the effective length of the pendulum will change with respective to its angular displacement. This will affect the time period of the system. In the present paper we will be attempting to give empirical formula for varying length pendulum under consideration.***
***Keywords— Simple pendulum, hollow bob, varying length pendulum.***

## I. INTRODUCTION

The problem of varying length pendulum is matter of interest for researchers**.** In most of the research problems, the length of the pendulum *i.e.* the distance of point of suspension from the bob is considered to be varying. As the bob is solid, its center of mass is at a fixed point. Most of the varying length pendulum problems deals with such solid bob and length of the attached string to be varying only.

In our proposed system, we approach the varying length pendulum with hollow bob attached to non-stretchable string and suspended to the rigid support. When this system of hollow bob pendulum is made to oscillate, the center of mass of the bob will not shift and the effective length of pendulum will be constant. If a solid ball of smaller radius than hollow bob is kept inside the hollow bob, the system becomes interesting. If we now allow this 'solid ball inside the hollow bob' system to oscillate, the center of mass of the hollow bob will change its position for each angular displacement Θ. In the expression of time period of simple pendulum,

$$T = 2\pi\sqrt{l/g}$$

the '$l$' i.e. the length of the pendulum is no longer constant. This will affect the time period of the pendulum.

Assuming the system to have small and non-chaotic oscillations, and using trigonometric constrictions, we may have the expression of 'T' for this varying length pendulum

The problem is dealt with many similarities with the child's swing by Anton belyakov & Alexander seyranian. This is the pendulum with periodically varying length which is also treated as a simple model of child's swing. Asymptotic expressions for boundaries of instability domains near resonance frequencies were derived. Domains for oscillation, rotation, and oscillation-rotation motions in parameter space were found analytically and compared with numerical study. Two types of transitions to chaos of the pendulum depending on problem parameters were investigated numerically [1].

This interesting physical system has been studied by some researchers using traditional perturbation methods, but due to the limitation of the conventional perturbation methods, the solutions are not valid for long-term prediction of the pendulum. Tianzhi Yang, Bo Fang, Song Li & Wenhu Huang used the homotopy analysis method to explore the approximate solution to this system [2]. Model of a swing where the swing is regarded as a pendulum with a variable length under the influence of friction is also analyzed to the optimal control of swing by Saebyok Bae & Yoon-Hwan Kang [3].

It is also necessary to measure the time period of the pendulum precisely. Neha Aggarwal, Nitin Verma and P Arun described an 8085 microprocessor interface developed to make reliable time-period measurements. The time period of each oscillation of a simple pendulum was measured using this interface. The variation of the time period with increasing oscillation was studied. This interface is useful in measuring the time period of varying length pendulum[4]

This interface can be used to verify the theoretical predictions.

## II. MATHEMATICAL FORMULATION FOR TIME PERIOD OF VARYING LENGTH PENDULUM

A heavy bob suspended to a stretch less string forms simple pendulum. The time period of the system is given by

$$T = 2\pi\sqrt{l/g}$$

Volume 5 Issue VI, June 2017
ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Where '$l$' is the distance of point of suspension to the center of mass of the bob. If the bob is solid, this distance is constant.

Consider a system of hollow bob with small solid ball in it. For this system the length is no longer constant during oscillations. The center of mass shifts and so there is change in the length of pendulum. The effective length of pendulum for arbitrary angular displacement can be considered to be '**L**`' and can be calculated with simple trigonometry.

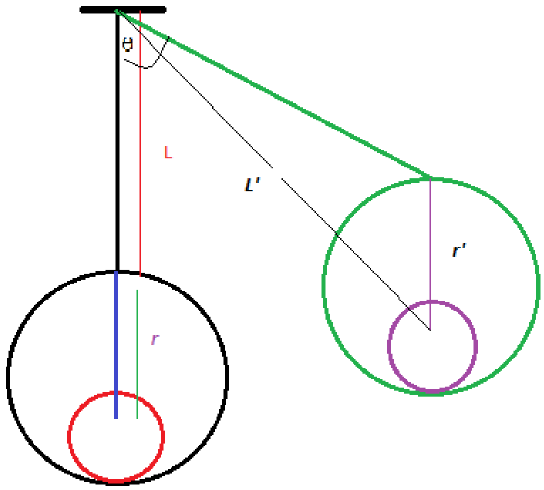


Fig.1

The displaced bob will have its own center of mass 'r`' due to the solid ball also have displaced in opposite direction of the displacement of the bob. The new center of mass will be,

$$r` = \frac{m(R-r)}{M+m}$$

Where,
'M' is the mass of the bob,
'm' is the mass of the solid ball,
'R' is the radius of the bob,
'r' is the radius of the solid ball.
The effective length 'L`' is found out to be,

$$L`^2 = L^2 + r`^2 - 2Lr` * \cos(180-\theta)$$

Which is,

$$L` = \{L^2 + r`^2 + 2Lr` * \cos(\theta)\}^{1/2}$$

Above expression holds for boundary conditions,

at $\Theta = 0$ L`=L+r`

at $\Theta = 90$ L`=( $L^2$+$r`^2$)

To find out the time period of the system over the angular displacement 0 to $\Theta_{max}$ we have to put the effective length in the expression of time period of simple pendulum,

$$\int_0^{\theta\ max} T = \int_0^{\theta\ max} 2\pi \{L^2 + r`^2 + 2Lr` * \cos(\theta)\}^{1/2} / g$$

This gives the modified formula for the time period of the proposed system. This empirical formula can be verified practically.

## III.CONCLUSION

Above derived formula gives us then empirical form of the system. 'The hollow bob with solid ball within' system can be used to understand the more complex systems. We are going to study the behaviour of the system with half filled hollow bob with viscous liquid. This system will also be undergoing the change in the length of the pendulum. We are going to change the liquid with different viscosities. We are also going to study the system with ½ filled hollow bob, ¾ filled hollow bob. The different degrees of freedom can be studied with the system such as temperature, resistance and buoyancy of air and so on. This study will put forward the relation between viscosity of the liquid and time period of this complex system.

## REFERENCES

[1] Anton Belyakov, Alexander Seyranian, Angelo Luongo, "Dynamics of the pendulum with periodically varying length, Physica D: Nonlinear Phenomena, Elsevier, Vol. 238 (16), pp.1589-1597,2009.
[2] Tianzhi Yang, Bo Fang1, Song Li and Wenhu Huang, IOP Publishing Ltd European Journal of Physics, Vol. 31, No. 5 ,2010.
[3] Saebyok Bae1 and Yoon-Hwan Kang, IOP Publishing Ltd European Journal of Physics, Vol. 27, No.1, 2005.
[4] Neha Aggarwal, Nitin Verma and P Arun, "Simple pendulum revisited", IOP Publishing Ltd European Journal of Physics, Vol. 26, 2005,