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# Development and Validation of Robotic Kinematic Solver for Industrial Application using Matlab 

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#### Abstract

This paper aims to formulate the robotic kinematic solver for performing forward kinematic analysis using the D-H parameters, for all types of robotic manipulators with varying degrees of freedom. This solver created in MATLAB R2014b evaluates the end-effector matrix resulting from the input given by the user. It also gives the overall transformation matrix in symbolic form which gives a better understanding of the relationship between various joints, links and the end-effector positions. The solver developed has been validated using Stanford manipulator and can hence be used for various industrial robotic applications.


Keywords: forward kinematics, Stanford manipulator, Denavit-Hartenberg parameters, industrial robotics, symbolic toolbox

## I. INTRODUCTION

With growing automation the robot technology is advancing rapidly. The industries are moving towards robotization to improve their productivity and efficiency. The current research goals indicate that industrial robots of the future would be more robust, efficient and will have many capabilities. Such a trend has made the study of robots important for all these further advancements. A robotic manipulator is designed to perform a task in the 3-D space. In order to perform the task, the end-effector is required to follow a planned trajectory and this requires control over the position and orientation of the end-effector. A kinematic model establishes the relationship between the end effector position and orientation and the spatial positions of the joints and links. The kinematic model is further classified as forward kinematics and inverse kinematics. In forward kinematic model, the end-effector position and orientation is found from the specified joint-link positions whereas in the inverse kinematic model, the joint-link positions are found from the specified end effector position and orientation.

## II. KINEMATIC ANALYSIS

This mathematical model is based on the Denavit and Hartenberg notation. A revolute joint is denoted by ' R ' and a prismatic joint by ' P '. The links of the manipulator are numbered from the immobile base as link 0 with the next moving body numbered as 1 and so on. The joints are numbered such that joint $n$ is formed between link $(\mathrm{n}-1)$ and n . A co-ordinate frame is attached to each link, namely frame \{i\} for link i. After the frame assignment, the four D-H parameters, two link parameters and two joint parameters with respect to frame $\{i-1\}$ and frame $\{i\}$ are defined as:
A. Link Length $\left(a_{i}\right)$

It is the distance measured along $\mathrm{X}_{\mathrm{i}}$ axis from the point of intersection of $\mathrm{x}_{\mathrm{i}}$ axis with $\mathrm{z}_{\mathrm{i}-1}$ axis to the origin of frame \{i\}.

## B. Link Twist $\left(\alpha_{i}\right)$

It is the angle between $\mathrm{z}_{\mathrm{i}-1}$ axis and $\mathrm{z}_{\mathrm{i}}$ axis measured about $\mathrm{x}_{\mathrm{i}}$ axis in the right hand sense.

## C. Joint Distance $\left(d_{i}\right)$

It is the distance measured along $\mathrm{z}_{\mathrm{i}-1}$ axis from the origin of frame $\{\mathrm{i}-1\}$ to the intersection of $\mathrm{x}_{\mathrm{i}}$ axis with $\mathrm{z}_{\mathrm{i}-1}$ axis.
D. Joint Angle $\left(\theta_{i}\right)$

It is the angle between $\mathrm{x}_{\mathrm{i}-1}$ and $\mathrm{x}_{\mathrm{i}}$ axis measured about the $\mathrm{z}_{\mathrm{i}-1}$ axis in the right hand sense.
E. The Transformation of $\{i-1\}$ to frame \{i\} consists of Four Basic Transformations

1) A rotation about $z_{i-1}$ axis by an angle $\theta_{i}$
2) Translation along $z_{i-1}$ axis by distance $d_{i}$
3) Translation by distance $a_{i}$ along $x_{i}$ axis
4) Rotation by an angle $\alpha_{i}$ about $x_{i}$ axis
F. The Transformation Matrix is thus given by
${ }^{\mathrm{i}-\mathrm{l}} \mathrm{T}_{\mathrm{i}}=\left[\begin{array}{cccc}C \theta_{i} & -S \theta_{i} C \alpha_{i} & S \theta_{i} S \alpha_{i} & a_{i} C \theta_{i} \\ S \theta_{i} & C \theta_{i} C \alpha_{i} & -C \theta_{i} S \alpha_{i} & a_{i} S \theta_{i} \\ 0 & S \alpha_{i} & C \alpha_{i} & d_{i} \\ 0 & 0 & 0 & 1\end{array}\right]$
Thus the overall transformation matrix between the base frame $\{0\}$ and the tool frame $\{n\}$ is given by

$$
\begin{equation*}
{ }^{0} \mathrm{~T}_{\mathrm{n}}={ }^{0} \mathrm{~T}_{1}^{1} \mathrm{~T}_{2} \ldots \ldots \ldots \ldots . .{ }^{\mathrm{n}-1} \mathrm{~T}_{\mathrm{n}} \tag{2}
\end{equation*}
$$

## III. METHODOLOGY

The forward kinematic solver was formulated using the knowledge of forward kinematics, D-H parameters and MATLAB R2014b software. The steps involved were:

## A. Formation of Overall Transformation Matrix

The first step involved understanding of forward kinematics and its application to various industrial manipulators. Frame assignment and D-H parameters were thoroughly studied. With the help of D-H parameters the overall transformation matrix was derived.

## B. Formulation of Solver

The forward kinematic solver was created in MATAB R2014b using the symbolic toolbox. The solver thus provided the individual link transformation matrices and end-effector transformation matrix in symbolic as well as numerical form based on the input from the user. The solver was designed to accommodate manipulators with varying degrees of freedom.

## C. Validation of the Results

Stanford manipulator was taken as an example for the validation of the solver. The overall transformation matrix for the Stanford manipulator was found analytically. The results obtained by the solver were observed to be exactly matching with the results found by analytical approach. Thus the solver was validated and can be universally applied to various industrial manipulators.

## IV.REALIZATION OF SOLVER AND VALIDATION

## A. Example

Formulate the forward kinematic model of a Stanford manipulator consisting of 6 degrees of freedom. Also find the end-effector position for $\theta_{1}=\theta_{2}=\theta_{4}=\theta_{5}=\theta_{6}=0^{\circ}, d_{3}=150$ units, $L_{2}=100$ units, $L_{6}=200$ units.


Fig. 1 Schematic of Stanford Manipulator

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Fig. 2 Frame Assignment for Stanford Manipulator

TABLE I
Joint-link Parameters for Stanford Manipulator

| Link $i$ | $a_{i}$ | $\alpha_{i}$ | $d_{i}$ | $\theta_{i}$ | $q_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | $-90^{\circ}$ | 0 | $\theta_{1}$ | $\theta_{1}$ |
| 2 | 0 | $90^{\circ}$ | $L_{2}$ | $\theta_{2}$ | $\theta_{2}$ |
| 3 | 0 | 0 | $d_{3}$ | 0 | $d_{3}$ |
| 4 | 0 | $-90^{\circ}$ | 0 | $\theta_{4}$ | $\theta_{4}$ |
| 5 | 0 | $90^{\circ}$ | 0 | $\theta_{5}$ | $\theta_{5}$ |
| 6 | 0 | 0 | $L_{6}$ | $\theta_{6}$ | $\theta_{6}$ |

The individual transformation matrices between successive links are calculated by substituting joint-link parameters in Eq. (1).
The six link transformation matrices are,

$$
\begin{align*}
& { }^{0} T_{1}=\left[\begin{array}{cccc}
C_{1} & 0 & -S_{1} & 0 \\
S_{1} & 0 & C_{1} & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{3}\\
& { }^{1} T_{2}=\left[\begin{array}{cccc}
C_{2} & 0 & S_{2} & 0 \\
S_{2} & 0 & -C_{2} & 0 \\
0 & 1 & 0 & L_{2} \\
0 & 0 & 0 & 1
\end{array}\right]
\end{align*}
$$

$$
{ }^{2} T_{3}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d_{3} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
{ }^{3} T_{4}=\left[\begin{array}{cccc}
C_{4} & 0 & -S_{4} & 0  \tag{6}\\
S_{4} & 0 & C_{4} & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
{ }^{4} T_{5}=\left[\begin{array}{cccc}
C_{5} & 0 & S_{5} & 0  \tag{7}\\
S_{5} & 0 & -C_{5} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

$$
{ }^{5} T_{6}=\left[\begin{array}{cccc}
C_{6} & -S_{6} & 0 & 0  \tag{8}\\
S_{6} & C_{6} & 0 & 0 \\
0 & 0 & 1 & L_{6} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The transformation of the tool frame with respect to the base frame is obtained by substituting the individual transformation matrices in Eq. (2) as
${ }^{0} T_{6}={ }^{0} T_{1} \cdot{ }^{1} T_{2} \cdot{ }^{2} T_{3} \cdot{ }^{3} T_{4} \cdot{ }^{4} T_{5} \cdot{ }^{5} T_{6}$
Substituting Eq. (3) to Eq. (8) in Eq. (9) and multiplying gives

$$
o_{T_{6}}=\left[\right]
$$

For home position $\theta_{1}=\theta_{2}=\theta_{4}=\theta_{5}=\theta_{6}=0^{\circ}$ and $d_{3}=150$, the end-effector matrix is

$$
{ }^{0} T_{6}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0  \tag{10}\\
0 & 1 & 0 & L_{2} \\
0 & 0 & 1 & 15 \nmid L_{6} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Substituting values of $L_{2}$ and $L_{6}$ from the given data, the final end-effector transformation matrix is calculated as
$T=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 350 \\ 0 & 0 & 0 & 1\end{array}\right]$

## C. Solution Obtained Using Solver

The inputs for the solver developed in MATLAB R2014b are summarized in the table below. The code takes degrees of freedom, link parameters, viz. link length and link twist, type of joint i.e. revolute or prismatic and joint parameters, viz. joint distance and joint angle from the user as input. It gives individual link transformation matrices and overall end-effector transformation matrix in symbolic as well as numerical form.

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TABLE Input for Solver

| Link $i$ | $a_{i}$ | $\alpha_{i}$ | Type of joint | $q_{i}$ | $q_{i}{ }^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | $-90^{\circ}$ | R | 0 | 0 |
| 2 | 0 | $90^{\circ}$ | R | 100 | 0 |
| 3 | 0 | 0 | P | 0 | 150 |
| 4 | 0 | $-90^{\circ}$ | R | 0 | 0 |
| 5 | 0 | $90^{\circ}$ | R | 0 | 0 |
| 6 | 0 | 0 | R | 200 | 0 |

The symbolic matrix obtained using solver was found to be exactly same as formulated by analytical approach.

```
The end-effector matrix is symbolic form is
[ 1, 0, 0, 0]
[ 0, 1, 0, 100]
[ 0, 0, 1, 350]
[ 0, 0, 0, 1]
```

Fig. 3 Output Given by Solver
It is observed that the end-effector matrix given by solver is in accordance with Eq. (11) calculated using analytical approach. Hence the proposed robotic kinematic solver is validated.

## V. CONCLUSIONS

Robotic kinematic solver can determine the end effector transformation matrix symbolically and numerically for any degree of freedom.
The Matlab code developed is generalized for all types of industrial manipulators having degrees of freedom less than or equal to Symbolic end-effector transformation matrix for Stanford manipulator was derived analytically. The results from the solver are in perfect match with the analytical results.
End-effector matrix when determined with numerical values was also matching perfectly with the results given by the solver.
It is thus concluded that developed robotic kinematic solver is validated. It is extremely useful for finding end-effector transformation matrix in all the running industrial scenarios.
The symbolic matrix obtained can be further utilized to perform inverse kinematic analysis of the same manipulator.

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