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# Fixed Point Theorem on Dislocated BMetric Spaces 

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#### Abstract

In this paper, we prove Fixed point theorem for cyclic contractions in dislocated b-metric spaces.This paper generalized many result in the current literature. Mathematics Subject Classification: 47H05; 47H10; 47J25 Keywords: Fixed point, Contraction mapping, Dislocated b-metric.


## I. INTRODUCTION AND PRELIMINARIES

Fixed point theory plays one of the important roles in Mathematical Analysis. Many authors [1-6]presented fixed point theorem in different ways. In Banach contraction principle was introduced in 1922 by Banach [7] as follows:
Let ( $\mathrm{X}, \mathrm{d}$ ) be a metric space and $\mathrm{T}: \mathrm{X} \rightarrow \mathrm{X}$.Then T is called a banach
contraction mapping if there exists $k \in[0,1)$ such that $d(T x, T y) \leq k d(x, y)$ for all
$x, y \in X$.
The concept of kannan mapping was introduced in 1969 by kannan[8] as follows: (ii) T is called a kannan
mapping if there exists $r \in\left[0,{ }^{1}\right.$ ) such that
$\overline{2}$
$d(T x, T y) \leq r d(x, T x)+r d(y, T y)$ for all $x, y \in X$
Now, we recall the definition of cyclic map.Let $A$ and $B$ be non-empty subsets of a metric space ( $\mathrm{X}, \mathrm{d}$ ) and $\mathrm{T}: \mathrm{A} \cup$ $\mathrm{B} \rightarrow \mathrm{A} \cup \mathrm{B} . \mathrm{T}$ is called a cyclic map iff $\mathrm{T}(\mathrm{A}) \subseteq \mathrm{B}$ and $\mathrm{T}(\mathrm{B}) \subseteq \mathrm{A}$.
In 2003, kirk etal.[9] introduced cyclic contraction as follows:
(iii) A cyclic map $T: A \cup B \rightarrow A \cup B$ is said to be cyclic contraction if there exists
$\alpha \in[0,1)$ such that
$\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}) \leq \alpha \mathrm{d}(\mathrm{x}, \mathrm{y})$ for all $\mathrm{x} \in \mathrm{A}$ and $\mathrm{y} \in \mathrm{B}$.
In 2010, Karapinar and Erhan[10] introduced kannan type cyclic contraction as follows:
(iv) A cyclic map $T: A \cup B \rightarrow A \cup B$ is called a kannan type cyclic contraction if there exists $b \in\left[0,{ }^{1}\right.$ ) such that
$\overline{2}$
$d(T x, T y) \leq b d(x, T x)+b d(y, T y)$ for all $x \in A$ and $y \in B$.
If (X, d) is a complete metric space, at least one of (i),(ii),(iii) and (iv) holds, then it has a unique fixed point[7-
10].Next, we discuss the development of space.
Definition 1.1. [7] Let $X$ be a nonempty set. Suppose that the mapping
$\mathrm{d}: \mathrm{X} \times \mathrm{X} \rightarrow[0, \infty)$ satisfies the following conditions:
(d1) $d(x, x)=0$ for all $x \in X$
(d2 ) $d(x, y)=d(y, x)=0$ implies $x=y$ for all $x, y \in X$
(d3 ) $d(x, y)=d(y, x)$ implies for all $x, y \in X$
(d4) $d(x, y) \leq[d(x, z)+d(z, y)]$ for all $x, y, z \in X$
If d satisfies conditions $\left(\mathrm{d}_{1}\right),\left(\mathrm{d}_{2}\right)$ and ( $\mathrm{d}_{4}$ ), then d is a called quasi-metric on X . If
d satisfies conditions $(\mathrm{d} 2),(\mathrm{d} 3)$ and ( d 4 ), then d is a called dislocated metric on X . If
d satisfies conditions ( $\mathrm{d}_{1}$ ), ( $\mathrm{d}_{3}$ ) and ( d 4 ),then d is a called a metric on X .
In 2005 the concept of dislocated quasi-metric, which is a new generalization of quasi-b-metric spaces and dislocated b-metric space, was introduced.By Definition
1.1,if setting conditions ( $\mathrm{d}_{2}$ ), and ( d 4 ) holds true, then d is called a dislocated quasi- metric on X .

## A. Remark

It is obvious that metric spaces are quasi-metric spaces and dislocated metric spaces, but the converse is not true. Definition 1.2. [11] Let X be a non-empty set. Suppose that the mapping
$\mathrm{b}: \mathrm{X} \times \mathrm{X} \rightarrow[0, \infty)$ such that the constant $\mathrm{s} \geq 1$ satisfies the following conditions:
(b1) $b(x, y)=b(y, x)=0 \Leftrightarrow x=y$ for all $x, y \in X$
(b2) $b(x, y)=b(y, x)=0$ for all $x, y \in X$
(b3) $b(x, y) \leq s[b(x, z)+b(z, y)]$ for all $x, y, z \in X$
The pair ( $\mathrm{X}, \mathrm{d}$ ) is then called a b -metric space.

## II. MAIN RESULTS

## A. The Following Theorem Generalizes Theorem

Theorem 2.1. Let ( $\mathrm{X}, \mathrm{d}$ ) be a complete dislocated b -metric with $\mathrm{s} \geq 1$. Let A and B be a non-empty closed subset of $X$. Let $T: A \cup B \rightarrow A \cup B$ be a self map such that $d(T x, T y) \leq a_{1} d(x, y)+a_{2} d(T x, x)$ $+\mathrm{a}_{3} \mathrm{~d}(\mathrm{~T} y, \mathrm{y})+\mathrm{a} 4 \mathrm{~d}(\mathrm{~T} y, x)+\mathrm{a} 5 \mathrm{~d}(\mathrm{y}, \mathrm{Tx})$ where $\mathrm{a}_{\mathrm{i}} \geq 0, \mathrm{i}=1,2,3,4,5$ and $\mathrm{a}_{1}+\mathrm{a}_{2}+\mathrm{a} 3+2 \mathrm{sa} 4+2 \mathrm{sa} 5<$ Then $T$ has a unique fixed point in $A \cap B$.

$$
\begin{aligned}
& \text { Proof. Let } T^{\mathbf{n}} \subseteq X,\left\{T^{2 n}\right\} \subseteq A \text { and }\left\{T^{2 n+1}\right\} \subseteq B \text {. Fix } x \in A \text {. } \\
& \mathrm{d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)=\mathrm{d}(\mathrm{~T}(\mathrm{~T} \mathrm{x}), \mathrm{Tx}) \\
& \leq a_{1} d(T x, x)+a_{2} d\left(T^{2} x, T x\right)+a_{3} d(T x, x)+a_{4} d(T x, T x)+a_{5} d\left(x, T^{2} x\right) \\
& \leq a_{1} d(T x, x)+a_{2} d\left(T^{2} x, T x\right)+a_{3} d(T x, x)+\operatorname{sa4}[d(T x, x)+d(x, T x)] \\
& { }^{+} \operatorname{sa5}\left[\mathrm{d}(\mathrm{x}, \mathrm{Tx})+\mathrm{d}\left(\mathrm{Tx}, \mathrm{~T}^{2} \mathrm{x}\right)\right] \\
& d\left(T^{2} x, T x\right) \leq a_{1} d(T x, x)+a_{2} d\left(T^{2} x, T x\right)+a_{3} d(T x, x)+2 s a 4 d(T x, x) \\
& +\operatorname{sa} 5 \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\operatorname{sa5} \mathrm{d}\left(\mathrm{Tx}, \mathrm{~T}^{2} \mathrm{x}\right) \\
& =\left(\mathrm{a}_{1}+\mathrm{a} 3+2 \mathrm{sa4}+\mathrm{sa5}\right) \mathrm{d}(\mathrm{~T} \mathrm{x}, \mathrm{x})+(\mathrm{a} 2+\mathrm{sa5}) \mathrm{d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right) \\
& \leq \frac{\left(\mathrm{a}_{1}+\mathrm{a} 3+2 \mathrm{sa} 4+\mathrm{sa5}\right.}{1-(\mathrm{a} 2+\mathrm{sa} 5)}{ }^{2}{ }_{\mathrm{d}(\mathrm{Tx}, \mathrm{x})} \\
& \left.d\left(T^{2} x, T x\right) \leq \operatorname{kd}(T x, x) \text {, where } k=\frac{\left(a_{1}+a_{3}+2 \mathrm{sa}_{4}+\mathrm{sa}_{5}\right.}{1-\left(\mathrm{a}_{2}+\mathrm{sa5}\right)}\right)
\end{aligned}
$$

Now,

$$
\begin{aligned}
& \mathrm{d}\left(\mathrm{~T}^{3} \mathrm{x}, \mathrm{~T}^{2} \mathrm{x}\right)=\mathrm{d}\left(\mathrm{~T}\left(\mathrm{~T}^{2} \mathrm{x}\right), \mathrm{T}(\mathrm{~T} \mathrm{x})\right) \\
& \leq a_{1} d\left(T^{2} x, T x\right)+a_{2} d\left(T^{3} x, T^{2} x\right)+a_{3} d\left(T^{2} x, T x\right)+a 4 d\left(T^{2} x, T^{2} x\right) \\
& +\mathrm{a} 5 \mathrm{~d}\left(\mathrm{Tx}, \mathrm{~T}^{3} \mathrm{x}\right) \\
& \leq \mathrm{a}_{1} \mathrm{~d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)+\mathrm{a}_{2} \mathrm{~d}\left(\mathrm{~T}^{3} \mathrm{x}, \mathrm{~T}^{2} \mathrm{x}\right)+\mathrm{a}_{3} \mathrm{~d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)+\mathrm{sa} 4\left[\mathrm{~d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)+\mathrm{d}\left(\mathrm{~T} x, T^{2} \mathrm{x}\right)\right] \\
& +\operatorname{sa5}\left[\mathrm{d}\left(\mathrm{Tx}, \mathrm{~T}^{2} \mathrm{x}\right)+\mathrm{d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{~T}^{3} \mathrm{x}\right)\right] \\
& \leq \mathrm{a}_{1} \mathrm{~d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)+\mathrm{a}_{2} \mathrm{~d}\left(\mathrm{~T}^{3} \mathrm{x}, \mathrm{~T}^{2} \mathrm{x}\right)+\mathrm{a} 3 \mathrm{~d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right) \\
& +2 \operatorname{san} d\left(T^{2} x, T x\right)+\operatorname{sa} 5 d\left(T x, T^{2} x\right)+a 5 d\left(T^{2} x, T^{3} x\right) \\
& d\left(T^{3} x, T^{2} x\right) \leq\left(a_{1}+\mathrm{a}_{3}+2 \mathrm{sa} 4+\operatorname{sa} 5\right) d\left(T^{2} \mathrm{x}, \mathrm{Tx}\right)+\left(\mathrm{a}_{2}+\mathrm{sa} 5\right) \mathrm{d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{~T}^{3} \mathrm{x}\right) \\
& (\mathrm{a} 1+\mathrm{a} \underline{3}+2 \mathrm{sa} \underline{4}+\mathrm{sa} \underline{5})_{\mathrm{d}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right)} \\
& \leq \quad 1-(\mathrm{a} 2+\mathrm{sa} 5)
\end{aligned}
$$

$$
\mathrm{d}\left(\mathrm{~T}^{3} \mathrm{x}, \mathrm{~T}^{2} \mathrm{x}\right) \leq \mathrm{kd}\left(\mathrm{~T}^{2} \mathrm{x}, \mathrm{Tx}\right) \text {, where } \mathrm{k}=\frac{\left(\mathrm{a}_{1}+\mathrm{a} 3+2 \mathrm{sa} 4+\mathrm{sa5}\right.}{1-(\mathrm{a} 2+\mathrm{sa} 5)} \quad 2
$$

## B. Fixed Point Theorem on Dislocated b-Metric Spaces

$\mathrm{d}\left(\mathrm{T}^{3} \mathrm{x}, \mathrm{T}^{2} \mathrm{x}\right) \leq \mathrm{k}[\operatorname{kd}(\mathrm{Tx}, \mathrm{x})]$
$\mathrm{d}\left(\mathrm{T}^{3} \mathrm{x}, \mathrm{T}^{2} \mathrm{x}\right) \leq \mathrm{k}^{2} \mathrm{~d}(\mathrm{~T} \mathrm{x}, \mathrm{x})$ ] induction,
$d\left(T^{\mathbf{n + 1}}{ }_{x,} T^{\mathbf{n}_{x}}\right) \leq k^{n^{n}(T x, x)}$
In general $\mathrm{n}, \mathrm{m} \in \mathrm{N}$ where $\mathrm{m}>\mathrm{n}$

$$
\begin{aligned}
& \leq \mathrm{sk}^{n} \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{s}^{2} \mathrm{k}^{\mathrm{n}+1} \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\ldots+\mathrm{s}^{\mathrm{m}_{\mathrm{k}} \mathrm{n}^{2}+\mathrm{m}-1} \mathrm{~d}(\mathrm{x}, \mathrm{Tx}) \\
& \leq \mathrm{sk}^{\mathrm{n}}\left[1+\mathrm{sk}+\ldots+(\mathrm{sk})^{\mathrm{m}-1}\right] \mathrm{d}(\mathrm{x}, \mathrm{Tx}) \\
& d\left(T^{n}, T^{n+m} \leq k^{n} \frac{1}{1} d(x, T x)\right. \\
& \mathrm{d}\left(\mathrm{~T}^{\left.\mathbf{n}_{\mathrm{x}}, \mathrm{~T}^{\mathbf{n +}+\mathrm{m}_{\mathrm{x}}}\right) \rightarrow 0 \text { as } \mathrm{n} \rightarrow \infty}\right.
\end{aligned}
$$

Hence $\left\{\mathrm{T}^{\mathbf{n}}\right\}$ is a Cauchy sequence.
Since $(X, d)$ is complete, Then $\left\{T^{\mathbf{n}}\right\}$ coverages to some point $x \in X$. Since $\left\{T^{2 n}\right\} \subseteq A$ and $\left\{\mathrm{T}^{2 \mathrm{n}+1}\right\} \subseteq \mathrm{B}$.
Thus $\mathrm{x} \in \mathrm{A} \cap \mathrm{B}$
We show that $T x=x$. Now,

$$
\begin{aligned}
& d(T x, x)=d\left(T x, T^{2 n} x\right) \\
& \leq \mathrm{a}_{1} \mathrm{~d}(\mathrm{x}, \mathrm{x})+\mathrm{a} 2 \mathrm{~d}(\mathrm{Tx}, \mathrm{x})+\mathrm{a}_{3} \mathrm{~d}\left(\mathrm{~T}^{2 n} \mathrm{x}, \mathrm{x}\right)+\mathrm{a} 4 \mathrm{~d}\left(\mathrm{~T}^{\left.2 n_{x}, x\right)}+\mathrm{a} 5 \mathrm{~d}(\mathrm{x}, \mathrm{Tx}) \text { Letting } \mathrm{n} \rightarrow \infty\right. \\
& d(T x, x) \leq a_{1} d(x, x)+a_{2} d(T x, x)+a_{3} d(x, x)+a 4 d(x, x)+a_{5} d(x, T x) \\
& \leq \mathrm{sa}_{1}[\mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{d}(\mathrm{Tx}, \mathrm{x})]+\mathrm{a} 2 \mathrm{~d}(\mathrm{Tx}, \mathrm{x})+\mathrm{sa} 3[\mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{d}(\mathrm{~T} x, \mathrm{x})] \\
& +\mathrm{a} 4[\mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{d}(\mathrm{Tx}, \mathrm{x})]+\mathrm{a} 5 \mathrm{~d}(\mathrm{x}, \mathrm{Tx}) \\
& \leq 2 \operatorname{sal} 1 \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{a}_{2} \mathrm{~d}(\mathrm{Tx}, \mathrm{x})+2 \mathrm{sa} 3 \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+2 \mathrm{sa} 4 \mathrm{~d}(\mathrm{x}, \mathrm{Tx})+\mathrm{a} 5 \mathrm{~d}(\mathrm{x}, \mathrm{Tx}) \\
& \leq\left(2 \mathrm{sa}_{1}+\mathrm{a}_{2}+2 \mathrm{sa} 3+2 \mathrm{sa} 4+\mathrm{a}_{5}\right) \mathrm{d}(\mathrm{x}, \mathrm{Tx}) \\
& {\left[1-\left(2 \mathrm{sa}_{1}+\mathrm{a}_{2}+2 \mathrm{sa} 3+2 \mathrm{sa} 4+\mathrm{a} 5\right)\right] \mathrm{d}(\mathrm{x}, \mathrm{Tx}) \leq 0} \\
& \text { Hence } \mathrm{d}(\mathrm{~T} \mathrm{x}, \mathrm{x})=0 \text {. } \\
& \text { This implies } T x=x \text {. Therefore, } T \text { has a fixed point. } \\
& \text { Uniqueness: Let } \mathrm{x} \text { and } \mathrm{y} \text { be two fixed points of } \mathrm{T} \text {, that is } \mathrm{T} x=\mathrm{x} \text { and } \mathrm{Ty}=\mathrm{y} \\
& (\mathrm{x}, \mathrm{y})=\mathrm{d}(\mathrm{Tx}, \mathrm{Ty}) \\
& \leq a_{1} d(x, y)+a_{2} d(T x, x)+a_{3} d(T y, y)+a_{4} d(T y, x)+a_{5} d(y, T x) \\
& \leq a_{1} d(x, y)+a_{2} d(x, x)+a_{3} d(y, y)+a_{4} d(y, x)+a 5 d(y, x) \\
& \leq \mathrm{a}_{1} \mathrm{~d}(\mathrm{x}, \mathrm{y})+\mathrm{a}_{2} \mathrm{~s}[\mathrm{~d}(\mathrm{x}, \mathrm{y})+\mathrm{d}(\mathrm{y}, \mathrm{x})]+\operatorname{sa3}[\mathrm{d}(\mathrm{y}, \mathrm{x})+\mathrm{d}(\mathrm{x}, \mathrm{y})]+\mathrm{a} 4 \mathrm{~d}(\mathrm{y}, \mathrm{x})+\mathrm{a} 5 \mathrm{~d}(\mathrm{y}, \mathrm{x}) \\
& \mathrm{d}(\mathrm{x}, \mathrm{y}) \leq(\mathrm{a} 1+2 \mathrm{sa} 2+2 \mathrm{sa3}+\mathrm{a} 4+\mathrm{a} 5) \mathrm{d}(\mathrm{y}, \mathrm{x})
\end{aligned}
$$

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$[1-(\mathrm{a} 1+2 \mathrm{sa} 2+2 \mathrm{sa3}+\mathrm{a} 4+\mathrm{a} 5)] \mathrm{d}(\mathrm{x}, \mathrm{y}) \leq 0$
Hence $\mathrm{d}(\mathrm{x}, \mathrm{y})=0$.
Therefore, $x=y$.
Hence, T has a unique fixed point.
Example 2.2. Let $X=R, A=[-2,0] ; B=[0,2]$. Define $d(a, b)={ }^{1}[|a-b|+|a|$
$+|b|]$
Then $d$ is the dislocated metric us define $T: A \cup B \rightarrow A \cup B$ by $T a=-a$
Then T is a cyclic mapping.Here ${ }^{0} 0^{0}$ is the unique fixed point.

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