



# **iJRASET**

International Journal For Research in  
Applied Science and Engineering Technology



---

# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume: 5      Issue: VIII      Month of publication: August 2017**

**DOI: <http://doi.org/10.22214/ijraset.2017.8001>**

**[www.ijraset.com](http://www.ijraset.com)**

**Call: ☎ 08813907089**

**E-mail ID: [ijraset@gmail.com](mailto:ijraset@gmail.com)**

# Triple Connected Domination Number of A Bipolar Fuzzy Graph

R.Jahir Hussain<sup>1</sup>, S.Satham Hussain<sup>2</sup>

<sup>1</sup>Associate Professor of Mathematics, Jamal Mohamed College, Trichy, Tamil Nadu, India,

<sup>2</sup>Ph.D. Research Scholar, Department of Mathematics, Jamal Mohamed College, Trichy, Tamil Nadu, India,

**Abstract:** Zhang introduced the concept of Bipolar fuzzy Graph as a generalization of fuzzy sets. Bipolar fuzzy sets have shown advantage in solving decision making problem than fuzzy sets. In this paper we introduce the concept of triple connected domination in bipolar fuzzy graph properties of triple connected domination number for some standard bipolar fuzzy graphs with an example. Nowadays, bipolar fuzzy theory received much attention and has fruitful application in many fields.

**Keywords:** Bipolar fuzzy graphs, Domination, Neighborhood triple connected domination number, Triple connected domination number,

## I. INTRODUCTION

Bipolar fuzzy sets are the generalization of fuzzy sets. The notion of fuzzy sets was introduced by Zadeh [16] as a way of representing uncertainty and vagueness. As a generalization of fuzzy sets, Zhang [15] introduced the concept of bipolar fuzzy sets. A bipolar fuzzy set has a pair of positive and negative membership values range is  $[-1, 1]$ . In a bipolar fuzzy set, the membership degree of an element is 0 means that the element is irrelevant to the corresponding property, the membership degree  $(0, 1]$  of an element indicates that the element somewhat satisfies the property, and the membership degree  $[-1, 0)$  of an element indicates that the element somewhat satisfies the implicit counter-property. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. In particular, fuzzy and probabilistic formalisms for bipolar information have been proposed in [6] because when we deal with spatial information in image processing or in spatial reasoning applications, this bipolarity also occurs.

In 1975, Rosenfeld [11] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs. Bhutani and Rosenfeld introduced the concept of M-strong fuzzy graphs in [5] and studied some of their properties. Many literatures and books are found in fuzzy graph theory and it has applications in many domains like engineering, statistics, graph theory, artificial intelligence, signal processing etc. (see [9,10,11] and references therein). Recently, bipolar fuzzy sets have received much attention to the authors. As a generalization of fuzzy set theory, bipolar fuzzy set theory gives more precision, flexibility and compatibility to the system as compared to the classical and fuzzy models.

Akram[2] introduced the notion of bipolar fuzzy graphs and describing various methods of their construction, the concept of isomorphism of these graphs, and investigate some of their important properties. Further, he introduced the notion of strong bipolar fuzzy graphs and studied some of their properties and discussed some propositions of self-complementary and self-weak complementary strong bipolar fuzzy graphs. Additionally, Akram and Dudek[3]. The complement and isomorphism on bipolar fuzzy graphs are established by Rashmanlou[13] studied regular bipolar fuzzy graphs, and Akram [4] also discussed bipolar fuzzy graphs with applications. Moreover, [7] discussed the product of bipolar fuzzy graphs and their degree. In 2013, Akram et al. introduces the domination in bipolar fuzzy graphs. Here we introduce concept of triple connected domination in bipolar fuzzy graph properties of triple connected domination number for some standard bipolar fuzzy graphs in the main section 3. Section 2 has some preliminary definitions and results needed for proving the main result.

## II. PRELIMINARIES

### A. Definition 2.1

A fuzzy subset  $\mu$  on a set  $X$  is a map  $\mu: X \rightarrow [0, 1]$ . Let  $X$  and  $Y$  be two sets and let  $\mu$  and  $\nu$  be fuzzy subsets of  $X$  and  $Y$  respectively. Then a fuzzy relation  $\rho$  from the fuzzy subset  $\mu$  into the fuzzy subset  $\nu$  is a fuzzy subset  $\rho$  of  $X \times Y$  such that  $\rho: X \times Y$  such that  $\rho(x, y) \leq \min(\mu(x), \nu(y))$  for all  $x \in X$  and  $y \in Y$ . When  $Y=X$  and  $\mu$  is a fuzzy subset of  $X$ , a map  $\rho: X \times X \rightarrow [0, 1]$  is called a fuzzy relation on  $\mu$  if  $\rho(x, y) \leq \min(\mu(x), \mu(y))$  for all  $x, y \in X$ . Fuzzy relation  $\rho$  is called symmetric if  $\rho(x, y) = \rho(y, x)$  for all  $x, y \in X$ .

### B. Definition 2.2

A graph G is said to be triple connected if any three vertices lie on a path in G.

### C. Definition 2.3

Let X be a non empty set. A bipolar fuzzy set M in X an object having the form  $M = \{(x, \mu^+(x), \mu^-(x)) | x \in X\}$  where,  $\mu^+(x): X \rightarrow [0,1]$  and  $\mu^-(x): X \rightarrow [-1,0]$  are mappings.

### D. Definition 2.4

Let  $A = (\mu_1^+, \mu_1^-)$  and  $B = (\mu_2^+, \mu_2^-)$  be a Bipolar fuzzy set X. then  $A = (\mu_1^+, \mu_1^-)$  is called a bipolar relation on  $B = (\mu_2^+, \mu_2^-)$  if  $\mu_1^+(x, y) \leq \min(\mu_2^+(x), \mu_2^-(y))$  and  $\mu_1^-(x, y) \geq \max(\mu_2^-(x), \mu_2^-(y))$  for all  $x, y \in X$ .

### E. Definition 2.5

A bipolar fuzzy graph (BFG) is of the form  $G = (V, E)$  where (i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1^+: X \rightarrow [0,1]$  and  $\mu_2^-: X \rightarrow [-1,0]$  such that,  $\mu_2^+(v_i, v_j) \leq \min(\mu_1^+(v_i), \mu_1^+(v_j))$  and  $\mu_2^-(v_i, v_j) \geq \max(\mu_1^-(v_i), \mu_1^-(v_j))$  for all  $(v_i, v_j) \in E$ .

1) Example.2.1:  $(v_1 = (0.6, -0.5)), (v_2 = (0.7, -0.5)), (v_3 = (0.5, -0.8)), (v_4 = (0.5, -0.5))$  and  $(v_1, v_2 = (0.6, -0.4)), (v_2, v_3 = (0.5, -0.4)), (v_3, v_4 = (0.1, -0.6)), (v_1, v_4 = (0.4, -0.5)), (v_2, v_4 = (0.5, -0.5))$ .

### F. Definition 2.6

A path in a bipolar fuzzy graph G is a sequence of vertices and edges  $v_1 e_1 v_2 e_2 \dots v_n$  such that either one of the following conditions is satisfied.

- 1)  $\mu_2^+(v_i, v_j) > 0$  and  $\mu_2^-(v_i, v_j) = 0$  for some  $v_i, v_j$ .
- 2)  $\mu_2^+(v_i, v_j) = 0$  and  $\mu_2^-(v_i, v_j) < 0$  for some  $v_i, v_j$ .
- 3)  $\mu_2^+(v_i, v_j) > 0$  and  $\mu_2^-(v_i, v_j) < 0$  for some  $v_i, v_j$ .

### G. Definition 2.7

A bipolar fuzzy graph  $G = (V, E)$  is connected if any two vertices are joined by a path. Then  $\mu^+$  is strength of path P, if  $v_1, v_2 \dots v_n$  is defined as  $\min(\mu_2^+(v_i, v_j))$  for all  $i, j$  and is denoted by  $S\mu^+$ . Thus  $\mu^-$  is strength of path P, if  $v_1, v_2 \dots v_n$  is defined as  $\max(\mu_2^-(v_i, v_j))$  for all  $i, j$  and is denoted by  $S\mu^-$ . If the same possesses both  $\mu^+$  is strength and  $\mu^-$  is strength value, then it is the strength of a path P. The strength of a path is  $(\mu_2^+(v_i, v_j), \mu_2^-(v_i, v_j)) = S\mu^+, S\mu^-$

### H. Definition 2.8

Let G be a bipolar fuzzy graph. The  $\mu^+$  strength of the path connecting two vertices  $v_i, v_j$  is defined as  $\max(\mu_2^+)$  and is denoted by  $(\mu_2^+(v_i, v_j))^\infty$ . The strength of the path connecting any two vertices  $v_i, v_j$  is defined  $\min(\mu_2^-)$  and is denoted by  $(\mu_2^-(v_i, v_j))^\infty$ . If the same possesses of the strongest path P and it is denoted by  $S_p = ((\mu_2^+(v_i, v_j))^\infty, (\mu_2^-(v_i, v_j))^\infty)$ . For all  $i, j = 1, 2, \dots, n$ .

### I. Definition 2.9

An edge  $(u, v)$  is said to be strong edge in BFG,  $G = (V, E)$  if  $(\mu_2^+)(u, v) \geq (\mu_2^+(u, v))^\infty$  and  $(\mu_2^-)(u, v) \leq (\mu_2^-(u, v))^\infty$  where if  $(\mu_2^+(u, v))^\infty = \max\{(\mu_2^+)^k(u, v) / k = 1, 2, 3, 4, \dots, n\}$  and  $(\mu_2^-(u, v))^\infty = \min\{(\mu_2^-)^k(u, v) / k = 1, 2, 3, 4, \dots, n\}$   $(\mu_2^+)^k(u, v) = \sup\{\mu_2^+(u, v_1) \wedge \mu_2^+(v_1, v_2) \dots \wedge \mu_2^+(v_{k-1}, v) / (u_1, v_1, \dots, v_{k-1} \in V)\}$  and  $(\mu_2^-)^k(u, v) = \inf\{\mu_2^-(u, v_1) \vee \mu_2^-(v_1, v_2) \dots \vee \mu_2^-(v_{k-1}, v) / (u_1, v_1, \dots, v_{k-1} \in V)\}$ .

### J. Definition 2.10

A vertex  $u \in V$  of a BFG  $G = (V, E)$  is said to be isolated vertex if  $\mu_2^+(u, v) = 0$  and  $\mu_2^-(u, v) = 0$  for all  $u \in V, u \neq v$ . That is  $N(u) = \emptyset$ . Thus on isolated vertex does not dominated any other vertex of G.

### K. Definition 2.11

Let  $G = (V, E)$  be a BFG on V. Let  $u, v \in V$ . We say that u dominates v in G if there exists a strong edge between them.

- a) For any  $u, v \in V$  if u dominates v then v dominates u and hence domination is symmetric relation on V.
- b) For any  $v \in V$ ,  $N(u)$  is precisely the set of all vertices in V which are dominated by v.
- c) If  $\mu_2^+(u, v) < \mu_2^+(u, v)^\infty$  and  $\mu_2^-(u, v) > \mu_2^-(u, v)^\infty$  for all  $u, v \in V$  then the dominating set of G is V.
- d)

**L. Definition 2.12**

A subset  $S$  of  $V$  is called a dominating set in  $G$  if for every  $v \in V - S$  there exists  $u \in S$  such that  $u$  dominates  $v$ .

**M. Definition 2.13**

A dominating set  $S$  of a BFG is said to be minimal dominating set if no proper subsets of  $S$  is a dominating set.

**N. Definition 2.14**

Minimum cardinality among all minimal dominating set is called lower domination number of  $G$  and is denoted by  $d_B(G)$ ,

**1) Example 2.2** Consider a BFG,  $G = (V, E)$ , such that

$V = \{v_1, v_2, v_3, v_4, v_5\}$ ,  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_1), (v_1, v_4), (v_2, v_4)\}$ ;  $(v_1, v_2) = (0.2, -0.3)$ ,  $(v_2, v_3) = (0.2, -0.2)$ ,  $(v_3, v_4) = (0.1, -0.4)$ ,  $(v_4, v_5) = (0.1, -0.4)$ ,  $(v_1, v_5) = (0.2, -0.4)$ ,  $(v_1, v_4) = (0.0, -0.3)$ ,  $(v_2, v_4) = (0.1, -0.2)$ . By example  $\{v_1, v_2, v_4, v_5\}$ ,  $\{v_2, v_4\}$ ,  $\{v_2, v_4, v_3\}$  are dominating sets,  $\{v_4, v_3\}$  is not a dominating set,  $\{v_1, v_3\}$ ,  $\{v_1, v_4\}$ ,  $\{v_4, v_2\}$ ,  $\{v_2, v_3, v_5\}$  are minimal dominating sets,  $\{v_1, v_4\}$  has minimum cardinality among all minimal dominating sets and  $d_B(G) = 0.5$ ,  $\{v_2, v_5, v_3\}$  has maximum cardinality among all minimal dominating sets and  $D_B(G) = 1.5$ .

**O. Definition 2.15**

A bipolar fuzzy graph  $G$  is said to be triple connected if any three vertices lie on a path in  $G$ .

**1) Example 2.3:** All path, cycles, complete graphs some standard example of triple connected graph.

**2) Remark: 2.1:** Clearly, every triple connected graph is a connected graph.

**P. Definition 2.16** A dominating set  $S$  of a connected graph  $G$  is said to be connected dominating set of  $G$  if the induced subgraph  $\langle S \rangle$  is connected.

### III. MAIN RESULT

**A. Triple Connected Domination Number In Bipolar Fuzzy Graph**

**1) Definition 3.1.1:** A subset  $S$  of  $V$  is said to be triple connected dominating set, if  $S$  is a dominating set and the induced subgraph  $\langle S \rangle$  is triple connected, minimum cardinality set is denoted by  $\gamma_{BFTC}(G)$ .

**2) Example 3.1.1:** By above example 2.2 triple connected dominating sets  $S_1 = \{v_1, v_4, v_3\}$ ,  $S_2 = \{v_1, v_4, v_5\}$ . Therefore  $S_1$  has minimum cardinality among all dominating sets.  $\gamma_{BFTC}(S_1) = 0.95$

**3) Observation 3.1.1 :** Triple connected Bipolar fuzzy graph dominating set (BFTcd) does not exist for all graph and if exists, then  $\gamma_{BFTC}(G) \geq 3$ .

**4) Observation 3.1.2 :** The complement of the BFG triple connected dominating set need not be BFG Triple connected dominating set.

**5) Example: 3.1.2:** Consider a BFG,  $G = (V, E)$  such that  $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ ,  
 $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_1, v_6)\}$ ,  $v_1 = (0.7, -0.4)$ ,  $v_2 = (0.2, 0.3)$ ,  $v_3 = (0.2, -0.5)$ ,  $v_4 = (0.4, -0.7)$ ,  $v_5 = (0.5, -0.4)$ ,  $v_6 = (0.6, -0.9)$   $\{v_1, v_2\} = (0.1, -0.3)$ ,  $v_2, v_3 = (0.2, 0.3)$ ,  $v_3, v_4 = (0.1, -0.5)$ ,  $v_2, v_5 = (0.2, -0.3)$ ,  $v_5, v_6 = (0.5, -0.4)$ ,  $v_1, v_6 = (0.1, -0.3)$  by example  $D_1 = \{v_2, v_3, v_5\}$   $|D_1| = 0.5 + 0.4 + 0.35 = 1.3$ ,  $D_1 = \{v_2, v_3, v_5\}$  form a BFG Triple connected but the complement  $V - S = \{v_1, v_4, v_6\}$  not a BFG triple connected.

**6) Observation 3.1.3:** Every BFG triple connected dominating set is a dominating set but not conversely.

**7) Observation 3.1.4:** If the induced subgraph of each connected BFG dominating set of  $G$  has more than two pendant vertices, then  $G$  does not contain BFG triple connected.

**8) Example: 3.1.3:** Consider a BFG,  $G = (V, E)$  such that  $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ ,  
 $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_4, v_7), (v_7, v_8)\}$   
 $v_1 = (0.7, -0.4)$ ,  $v_2 = (0.1, -0.3)$ ,  $v_3 = (0.2, -0.5)$ ,  $v_4 = (0.4, -0.7)$ ,  $v_5 = (0.2, -0.3)$ ,  $v_6 = (0.4, -0.7)$   $\{v_1, v_2\} = (0.1, -0.3)$ ,  $v_2, v_3 = (0.2, 0.3)$ ,  $v_3, v_4 = (0.1, -0.5)$ ,  $v_2, v_5 = (0.2, -0.3)$ ,  $v_5, v_6 = (0.5, -0.4)$ ,  $v_4, v_7 = (0.1, -0.4)$ ,  $v_7, v_8 = (0.5, -0.4)$  for the graph  $S = \{v_2, v_3, v_4, v_5, v_7\}$  minimum connected BFG dominating set  $\gamma_{BFTC}(G) = 5$ . Here we notice that the induced subgraph of  $S$  has three pendant vertices and hence  $G$  does not contain BFG triple connected dominating set.

**9) Observation 3.1.5.** If a spanning subgraph  $H$  of a graph  $G$  has a BFG triple connected dominating set then  $G$  also has a triple connected dominating set.

10) *Example 3.1.4:* Consider a BFG,  $G = (V, E)$ , such that  $G_1 = (V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_5), (v_5, v_3), (v_4, v_1)\})$   $v_1 = (0.2, -0.4)$ ,  $v_2 = (0.3, -0.4)$ ,  $v_3 = (0.8, -0.3)$ ,  $v_4 = (0.3, -0.6)$ ,  $v_5 = (0.4, -0.7)$ ,  $v_6 = (0.7, -0.8)$ ,  $v_7 = (0.5, -0.3)$   $\{v_1 v_2 = (0.1, -0.1), v_2 v_3 = (0.3, 0.4), v_3 v_4 = (0.3, -0.5), v_4 v_5 = (0.1, -0.5), v_5 v_6 = (0.3, -0.5), v_6 v_7 = (0.4, -0.2), v_7 v_5 = (0.2, -0.2), v_5 v_3 = (0.4, -0.5), v_1 v_4 = (0.2, -0.4)\}$   $G_2 = (V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}, E = \{(v_1, v_2), (v_2, v_3), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_5), (v_5, v_3), (v_4, v_1)\})$   $v_1 = (0.2, -0.4), v_2 = (0.3, -0.4), v_3 = (0.8, -0.3), v_4 = (0.3, -0.6), v_5 = (0.4, -0.7), v_6 = (0.7, -0.8), v_7 = (0.5, -0.3)$   $\{v_1 v_2 = (0.1, -0.1), v_2 v_3 = (0.3, 0.4), v_4 v_5 = (0.1, -0.5), v_5 v_6 = (0.3, -0.5), v_6 v_7 = (0.4, -0.2), v_7 v_5 = (0.2, -0.2), v_5 v_3 = (0.4, -0.5), v_1 v_4 = (0.2, -0.4)\}$  spanning subgraph of  $G_1$ . BFG Dominating set  $S_1 = \{v_1, v_5\}, S_2 = \{v_2, v_5\}, S_3 = \{v_3, v_5, v_4\}$  so BFG triple connected dominating set  $D_1 = \{v_1, v_5, v_4\}$ .

11) *Remark 3.1.1:* Some exact value for some standard strong bipolar fuzzy graph.

1) For any bipolar fuzzy graph path of order  $p \geq 3$

$$\gamma_{BFTC}(p_p) = \begin{cases} 3, & p < 5 \\ p - 2, & p \geq 5 \end{cases}$$

2) For any bipolar fuzzy cycle of order  $p \geq 3$ ,

$$\gamma_{BFTC}(c_p) = \begin{cases} 3, & p < 5 \\ p - 2, & p \geq 5 \end{cases}$$

3) For any complete bipolar fuzzy graph of order  $p \geq 3$ ,

$$\gamma_{BFTC}(k_p) = 3.$$

12) *Theorem 3.1.1 :* If  $G$  has a cut vertex  $v$  such that  $\omega(G - v) \geq 3$  then  $G$  is not BFG triple connected.

Proof: Let  $v$  be a cut vertex of  $G$  such that  $\omega(G - v) \geq 3$ . Let  $G_1, G_2, G_3$  be any three components of  $G - v$ . let  $x \in V(G_1)$ ,  $y \in V(G_2)$ ,  $z \in V(G_3)$ , then any path connecting two vertices of  $x, y$  and  $z$  must pass through  $v$  and hence any two such path have at least two vertices in common. [By observation three vertices  $u, v$  and  $w$  lie on a path if and only if  $p$  is the union of two path, with exactly one common vertex.] therefore  $x, y$  and  $z$  do not lie on a path. Hence BFG is not triple connected.

13) *Theorem 3.1.2 :* A tree  $T$  is triple connected if and only if  $T \cong P_n, n \geq 2$ .

Proof: Let  $T$  be a tree. Assume that  $T$  is BFG triple connected. Suppose  $T \cong P_n, n \geq 2$ , then there exists a vertex  $v$  such that  $d(v) \geq 3$ , clearly  $v$  is a cut vertex of  $T$  and  $\omega(T - v) \geq 3$ . hence by above theorem  $T$  is not triple connected, which is a contradiction. Thus  $T \cong P_n, n \geq 2$ , converse is obvious.

14) *Theorem 3.1.3:* For any connected BFG graph  $G$  with  $p$  vertices,  $\gamma_{BFTC}(G) = p - 1$  if and only if  $G \cong P_4, C_4, K_4, K_{1,3}, K_4 - \{e\}, C_3(P_2)$ .

Proof: Suppose  $G \cong P_4, C_4, K_4, K_{1,3}, K_4 - \{e\}, C_3(P_2)$ . Then  $\gamma_{BFTC}(G) = p - 1 = 3$ .

Conversely, let  $G$  be a connected graph with  $p$  vertices such that  $\gamma_{BFTC}(G) = p - 1$ , let  $S = \{u_1, u_2, \dots, u_{p-1}\}$  be a  $\gamma_{BFTC}(G)$  set of  $G$ . Let  $x$  be in  $V - S$ . Since  $S$  is a dominating set, there exists a vertex  $v_i$  in  $S$  such that  $v_i$  adjacent to  $x$ . If  $p \geq 5$  by taking the vertex  $v_i$ , we can construct a triple connected dominating set  $S$  with fewer element then  $p - 1$  which is a contradiction. Hence  $p \leq 5$ , since  $\gamma_{BFTC}(G) = p - 1$  by observation (for any connected BFG with  $p$  vertices.  $\gamma_{BFTC}(G) = p - 1$  if and only if  $G \cong P_3$  or  $C_3$ ) we have  $p = 4$ , let  $S = \{v_1, v_2, v_3\}$  and  $V - S = \{v_4\}$ . Since  $S$  is a  $\gamma_{BFTC}$  set of  $G$ .

Case-(i)  $\langle S \rangle = P_3 = v_1, v_2, v_3$  since  $G$  is connected,  $v_4$  is adjacent to  $v_1$  (or  $v_3$ ) or  $v_4$  is adjacent to  $v_2$  hence  $G \cong P_4$

Case-(ii)  $\langle S \rangle = C_3 = v_1 v_2 v_3 v_1$ . Since  $G$  is connected,  $v_4$  is adjacent to  $v_1$  (or  $v_2$  or  $v_3$ ). Hence  $G \cong C_3(P_2)$  now by adding edges to  $P_4, K_{1,3}$  or  $C_3(P_2)$  without affecting  $\gamma_{BFTC}(G)$ . We have  $G \cong C_4, K_4 - \{e\}, K_4$ .

## B. Neighborhood Domination Number Of A Bipolar Fuzzy Graph

1) *Definition: 3.2.1 :* A subset  $S$  of  $V$  of a non trivial BFG graph  $G$  is said to be neighbourhood triple connected domination set if  $S$  is a domination set and the induced sub graph  $\langle N(S) \rangle$  is triple connected. The minimum cardinality taken over all the neighbourhood triple connected domination sets is called neighbourhood triple connected domination number and it is denoted by  $\gamma_{Btc}(G)$ .

2) *Definition 3.2.2 :* Let  $u$  be a vertex in a BFG  $G = (V, E)$  then  $N(u) = \{v \in V \text{ and } (u, v) \text{ is a strong edge in } G\}$  is called neighbourhood of  $u$  in  $G$ .

3) *Example 3.2.1 :* For example neighbourhood triple connected domination in BFG.

Consider a BFG,  $G = (V, E)$ , such that  $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ ,

$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_1, v_6), (v_3, v_6), (v_6, v_2)\}; v_1 = (0.6, -0.6), v_2 = (0.4, 0.6), v_3 = (0.9, -0.1), v_4 = (0.8, -0.2), v_5 = (0.7, -0.7), v_6 = (0.1, -0.8)$   
 $(v_1, v_2) = (0.3, -0.5), (v_2, v_3) = (0.1, -0.4), (v_3, v_4) = (0.6, -0.2), (v_4, v_5) = (0.2, -0.1), (v_5, v_6) = (0.5, -0.5), (v_1, v_6) = (0.1, -0.6), (v_2, v_6) = (0.1, -0.5), (v_6, v_3) = (0.08, -0.5)$ .  
 $S_1 = \{v_2, v_4\}, S_2 = \{v_2, v_5\}$  is the minimum neighbourhood triple connected domination set. and  $\gamma_{Btc}(G) = 0.95$ .  $\langle N(S) \rangle = (v_1, v_6, v_3, v_4)$

- 4) *Observation 3.2.1:* Neighbourhood triple connected dominating set does not exist for all bipolar fuzzy graphs.
- 5) *Remark 3.2.1:* Throughout this section, we consider only connected BFG for which neighbourhood triple connected domination set exists.
- 6) *Observation 3.2.2:* The complement of a neighbourhood triple connected domination set  $S$  need not be a neighbourhood triple connected domination set.
- 7) *Observation 3.2.3:* Every neighbourhood triple connected domination set is a domination set but not conversely.
- 8) *Remark 3.2.2:* The neighbourhood triple connected domination number of BFG,
  - a) For a complete graph,  $p \geq 4$ ,  $\gamma_{Btc}(K_p) = 1$ .
  - b) For a complete bipartite graph  $m, n \geq 4$  and  $m + n = p$ ,  $\gamma_{Btc}(K_{m,n}) = 2$ .

#### IV. CONCLUSION

In this paper, we have investigated the triple connected domination number in the bipolar graphs which give new ideas in this field. Also neighbourhood domination numbers of a bipolar fuzzy graph have been discussed. Further these results can be extended to the fields of vague graphs and intuitionistic fuzzy graphs.

#### REFERENCES

- [1] N. O., Alshehri, M. Akram, Cayley bipolar fuzzy graphs. The Scientific World Journal, 2013.
- [2] M. Akram, Bipolar fuzzy graphs, Information Sciences 181 (2011) 5548 - 5564.
- [3] M. Akram and W. A. Dudek, Regular bipolar fuzzy graphs, Neural Computing and Applications, 1 (2012) 197 - 205.
- [4] M. Akram, Bipolar fuzzy graphs with applications, Knowledge-Based Systems 39, (2013), 1 - 8.
- [5] K. R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sciences 152 (2003), 319 - 322
- [6] D. Dubois, S. Kaci and H. Prade, Bipolarity in reasoning and decision, an introduction, Int. Con. on Inf. Pro. Man. Unc. IPMU04, (2004) 959 - 966.
- [7] R. Hossein, S. Samanta, M. Pal, and R. A. Borzooei. "Product of bipolar fuzzy graphs and their degree." International Journal of General Systems 45, no. 1 (2016): 1-14.
- [8] M. G Karunambigai, M. Akram, K. Palanivel Sivasankar, S. Domination in bipolar fuzzy graphs. In FUZZ-IEEE. (2013).
- [9] A. Nagoorgani, K. Radha, Isomorphism on fuzzy graphs, International Journal of Computational and Mathematical Sciences 2 (2008) pp.190 - 196
- [10] A. Nagoorgani, Akram, M., Anupriya, S. Double domination on intuitionistic fuzzy graphs. Journal of Applied Mathematics and Computing, (2015) 1-14.
- [11] A. Rosenfeld, Fuzzy graphs, L.A. Zadeh, K.S. Fu, M. Shimura, Fuzzy Sets and their Applications, Academic Press, New York, (1975) 77 - 95.
- [12] N., Sarala, Kavitha, T. Strong (Weak) Triple Connected Domination Number of a Fuzzy Graph. International Journal of Computational Engineering Research (IJCER), Vol 05, (2015), 11.
- [13] A.A., Talebi, Rashmanlou, H. Complement and isomorphism on bipolar fuzzy graphs. Fuzzy Information and Engineering, 6(4), (2014) 505-522.
- [14] H. L., Yang, Li, S. G., Yang, W. H., Lu, Y. Notes on bipolar fuzzy graphs. Information Sciences, 242, (2013) 113-121.
- [15] W.R., Zhang, Bipolar fuzzy sets, Proc. of FUZZ-IEEE, 1998, 835-840.
- [16] L.A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 1a, 338-353.



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)