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Design of Efficient Linear Phase Quadrature Mirror Filter Bank Using Eigenvector Approach

M. Gopala Krishna[1], B. Bhaskara Rao [2]
[1]M. Tech Student, [2]Assistant Professor, Dept. of ECE, University College of Engineering and Technology
Acharya Nagarjuna University, Guntur, AP, India

Abstract: ***Two-channel linear phase quadrature mirror filter(QMF) is designed using eigen vector approach. Proposed QMF design using polyphase type-1 representation of the filter to reduce the computational complexity. filter coefficients are optimized to minimize an objective function using eigenvector approach, As compared to the existing design techniques, The proposed technique gives better performance in terms of peak reconstruction error(PRE) and stopband attenuation. The design examples gives effectiveness of the proposed method.***
Keywords: ***Sub-band coding, Polyphase decomposition, Perfect reconstruction.***

I. INTRODUCTION

The concept of QMF bank was first introduced in multirate filter banks by Croiser et al in 1976[1], and the Esteband and Galand[2] applied this filter bank in voice coding scheme, the term quadrature mirror filter bank means quadrature(4 filters) i.e, two filters in analysis section(H_0 and H_1) and two filters in synthesis section(F_0 and F_1). Designing of any one of the 4 filters and remaining 3 filters are mirror image of that filter (i.e, low-pass filter) this QMF bank is also called as two channel filter bank . QMF bank has extensively used for splitting a signal into two or more sub bands in frequency domain, so each sub band signal can be processed in an independent manner and sufficient compression may be achieved. This sub band signals are recombined in properly reconstructed to form the signal very nearer to the original signal.

By using QMF technique, the average number of bits per sample is reduced, even though the average number of samples per unit time is unchanged, Advances in filter banks have provided a new generation of subband coders for image compression[3], digital multiplexers used in FDM/TDM conversion[4], design of wavelet bases[5], discrete multitone modulation systems[6], digital audio system industry[7], ECG signal compression[8], equalization of wireless communication channels[9] and analog voice privacy systems[10] etc… QMF banks can be designed either perfect reconstruction(PR) or near perfect reconstruction(NPR) property, filter bank section can be cascaded in tree structure to generate multilevel or multichannel filter banks. There are two types of tree structures, namely, uniform filter banks and non uniform filter banks. In uniform structure or M-channel filter bank(full grown tree), at every level, the low pass and high pass channels are divided into two parts, whereas, only low pass channel divided into two parts in non uniform or octave filter banks.

The structural representation of two channel filter bank is shown in fig(1), the discrete input signal x(n) is splits into two sub band signals having equal bandwidths of low pass and high pass analysis filters H_0(Z) and H_1(Z) respectively. These sub band signals are decimated by a factor of two to achieve signal compression to reduce the processing complexity. The decimated signals are typically coded and transmitted. At the synthesis section, the two sub band signals are decoded and the interpolated by a factor of two and finally processed through low pass and high pass filters F_0(Z) and F_1(Z) respectively. The out puts of synthesis filters are combined to obtain the reconstructed signal y(n)= $\hat{x}(n)$.

$\hat{x}(n)$ suffers from three type of errors, those are aliasing error, amplitude distortion or peak reconstructed error and phase distortion. Therefore the researchers are to design QMF banks, mainly focus to eliminate these three distortion to get a perfect reconstructed (PR) or nearly perfect reconstructed (NPR) system.

Now, ideally, the filters at the synthesis section should be ideal low pass and ideal high pass with a cutoff at ***fs/2,*** i.e., one-half the Nyquist frequency. With real filters, however, there is nonzero energy in the stopband, which gets reflected back into the passband during the interpolation process at the receiver. In a QMF bank, these reflected images can be exactly canceled during reconstruction (i.e., at the summer in Fig. 1) In designing process aliasing can be cancelled completely by selecting the synthesis filters are same as analysis filters, whereas phase distortion can be eliminated by using linear phase FIR filters, but amplitude distortion can be minimized by optimizing the filter tap weights of the low pass analysis filters using computer aided techniques because of mirror image symmetry constraints, by using computer-aided optimization technique to satisfy the perfect reconstruction

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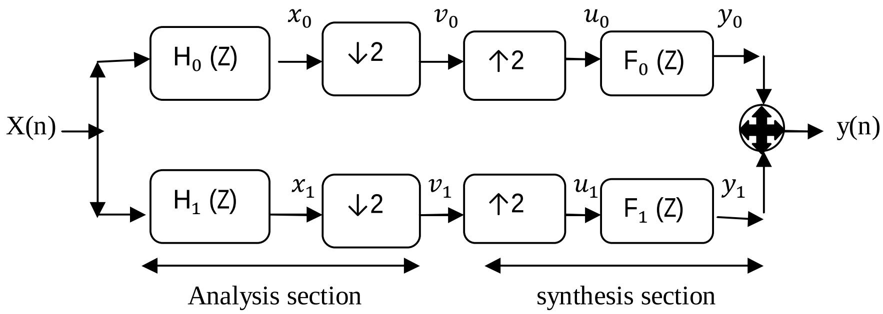
condition nearly. These types of filter banks are known as nearly perfect reconstruction QMF bank. The overall transfer function of aliasing and phase distortion free system to be a function of filter tap coefficients of low pass analysis filters only, where high pass filter is related to the mirror image symmetry condition around the quadrature frequency π/2 .



[Fig. 1:Two Channel- Filter Bank]

II.TWO CHANNEL QMF BANK

For a two-channel QMF bank, as shown in Fig. 1, the reconstructed output signal is defined as

$$Y(z) = Y_0(z) + Y_1(z) \quad (1)$$

$$Y(z) = \frac{1}{2}[H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + \frac{1}{2}[H_0(-z)F_0(z) + H_1(-z)F_1(z)]X(-z) \quad (2)$$

$$Y(z) = T(z)X(z) + A(z)X(-z) \quad (3)$$

Where T(z)=distortion transfer function
A(z)=aliasing distortion
where, Y(z) is reconstructed signal and X(z) is original input signal. For perfect reconstruction of original input signal, all the three types of distortions need to be eliminated.
The perfect reconstruction property means the signal at the output of the analysis/synthesis filter bank y(z) is delayed version of the original signal x(n), i.e,

$$y(n) = x(n-k) \quad (4)$$

The QMF bank is free from aliasing effect, amplitude distortion and phase distortion then it is a perfect reconstructed QMF bank.
In the above equation (1), T(z) is distortion transfer function and A(z) is aliasing distortion, which completely eliminated with use of the condition given by

$$H_1(z) = H_0(-z), F_0(z) = 2H_1(-z) \text{ and } F_1(z) = -2H_0(-z) \quad (5)$$

Therefore all the four filters are completely determined by the low-pass analysis filter $H_0(z)$ only. By using Eq.2, the expression for the alias free reconstructed signal, the perfect reconstructed (PR) condition is given by

$$T(z) = \frac{1}{2}[H_0{}^2(z) - H_1{}^2(z)] \quad (6)$$

$$= \frac{1}{2}[H_0{}^2(z) - H_0{}^2(-z)] \quad (7)$$

The perfect reconstruction can be achieved when the linear phase analysis filters are power complementary. The QMF bank with linear phase filters has no phase distortion, but the amplitude distortion will always exits. Hence,

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 \approx 1 \quad (8)$$

III. DESIGN METHODOLOGY

An efficient implementation of 2-channel QMF bank is obtained using polyphase decomposition and the noble identities. The analysis and synthesis filter banks can be redrawn as in Fig.2. The down samplers are shifted to the left of the polyphase components of $H_0(z)$, namely $E_0(z)$ and $E_1(z)$, so that the entire analysis bank requires only about L/2 multiplications per unit sample and L/2 additions per unit sample, where L is the length of $H_0(z)$.

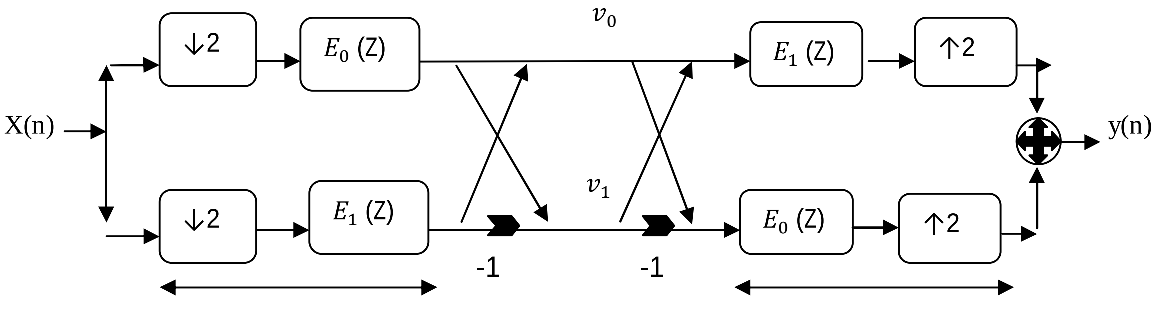


Fig. 1:Two Channel- Filter Bank with poly phase structure

Type-1 polyphase representation of analysis filters is given by

$$H_0(Z) = E_0(Z^2) + Z^{-1}.E_1(Z^2) \quad (9)$$

$$H_1(Z) = E_0(Z^2) - Z^{-1}.E_1(Z^2) \quad (10)$$

Where

$$E_0(z) = \sum_{n=0}^{M} h_0(2n)z^{-n} \text{ and } E_1(z) = \sum_{n=0}^{M} h_0(2n+1)z^{-n} \quad (11)$$

The impulse response h_0(n) of the prototype filter H_0 (z) is symmetric, this also reflects into the polyphase components E_0 (z) and E_1 (z). Due to the symmetry of h_0(n), the impulse response e_1 (n) = $h_0(2n+1)$ is the mirror image of $e_0(n) = h_0(2n)$, then e_0 (n) & e_1 (n) are symmetric sequences. This further impact on computational complexity, we obtain a factor of two additional saving in multiplication rate.

$$e_0(n) = e_1((N-2)/2-n) = e_1(M-n) \quad (12)$$

Where $M = (N-2)/2$
then, $E_1(z)$ can be expressed as

$$E_1(z) = Z^{-(\frac{N-2}{2})}E_0(Z^{-1}) \quad (13)$$

By substituting $E_1(z)$ from (13) into (9), we obtain

$$H_0(Z) = E_0(Z^2) + Z^{-1}.Z^{-(N-2)}E_0(Z^{-1}) \quad (14)$$

The frequency response of prototype filter is given by

$$H_0(e^{j\omega}) = E_0(e^{2j\omega}) + e^{-j\omega(N-1)}E_0(e^{-2j\omega}) \quad (15)$$

$$E_0(e^{2j\omega}) = \sum_{n=0}^{(N-2)/2} e_0(n)e^{-2j\omega} \quad (16)$$

$$H_0(e^{j\omega}) = \sum_{n=0}^{(N-2)/2} e_0(n)[e^{-2j\omega n} + e^{-j\omega(N-1)}e^{2j\omega n}] \quad (17)$$

Compering Eq. (17) with

$$H_0(e^{j\omega}) = e^{-\frac{j\omega(N-1)}{2}}H(\omega) \quad (18)$$

Yields

$$H(\omega) = \left[\sum_{n=0}^{(N-2)/2} a_n cos\omega\left(2n - \frac{N-1}{2}\right)\right] \quad (19)$$

$$= b^T c(\omega) \quad (20)$$

Where

$$b = [b_0\ b_1\ b_2\ \dots\dots b_M]^T \quad (21)$$

$$c(\omega) = \left[cos\omega\left(0 - \frac{N-1}{2}\right)\ cos\omega\left(2 - \frac{N-1}{2}\right)\dots\dots cos\omega\left(\frac{N-1}{2} - 1\right)\right]^T \quad (22)$$

The desired response of the LPF, the "stopband error" can be formulated by using least-square (LS) approach[11],

$$E_S = \frac{1}{\pi}\int_{\omega_s}^{\pi}[D(\omega) - H(e^{j\omega})]^2\, d\omega \quad (23)$$

$$= \frac{1}{\pi}\int_{\omega_s}^{\pi} b^T c(\omega)c(\omega)^T b\; d\omega \quad (24)$$

$$= b^T P_s b \quad (25)$$

Where

$$P_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} c(\omega)c(\omega)^T\, d\omega \quad (26)$$

Similarly, The passband error measure Ep, from zero frequency response($\omega = 0$) of H, so positive-valued (quadratic) error measure for the passband can be

$$E_P = \frac{1}{\pi}\int_{0}^{\omega_p}((1-c)^T b)^2\, d\omega \quad (27)$$

$$= \frac{1}{\pi}\int_{0}^{\omega_p} b^T(1-c)(1-c)^T b\; d\omega \quad (28)$$

$$= b^T P_p b \quad (29)$$

Where

$$P_p = \frac{1}{\pi}\int_{0}^{\omega_p}(1-c)(1-c)^T\; d\omega \quad (30)$$

The total error measure to be minimized as

$$E = b^T P b \quad (31)$$

Where

$$P = (1-\alpha)P_p + \alpha P_s \quad (32)$$

The quantity α, which is in the range $0 \le \alpha \le 1$, controls the relative accuracies of approximation in the pass and stopbands. Notice that the elements of P are given by

$$P(n,m) = \frac{1-\alpha}{\pi}\left[\int_{0}^{\omega_p}(1-\cos(n+\tfrac{1}{2})\omega)(1-\cos(m+\tfrac{1}{2})\omega)d\omega\right] + \frac{\alpha}{\pi}\left[\int_{\omega_s}^{\pi}(\cos(n+\tfrac{1}{2})\omega)(\cos(m+\tfrac{1}{2})\omega)d\omega\right] \quad (33)$$

In summary, we have been able to formulate the linear phase low-pass FIR design problem in the form of Eigen problem, for the given band edges ω_p , ω_s and parameter α, Then only matrix P can be compued. It is easy to obtain closed-form expressions for the integrals in (Eq.33), and, hence, the elements P(n, m) are easily computed once ω_p , ω_s and α are known.

It then remains only to compute the eigenvector of a real, symmetric, and positive-definite matrix corresponding to the smallest eigenvalue. The resulting filter is guaranteed to have linear phase because the vector b rather than the vector h is directly involved in the optimization problem. The eigenvector b can be used to obtain the filter coefficients of simple prototype lowpass filter.

The overall objective function to be minimized for the QMF bank design is therefore

$$E = (1-\alpha)E_p + \alpha E_s \quad (34)$$

IV. RESULTS AND CONCLUSIONS

Design example show the efficiency of this method and is calculated in terms of following powerful parameters:

Peak reconstruction error (PRE)

$$\text{PRE} = \max\left\{10\log\left(\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)})\right|^2\right)\right\} \quad (35)$$

Stopband attenuation (As)

$$\text{As} = -20 log|H_0(\omega)|\; at\; \omega = \omega_s \quad (36)$$

Example 1*:* A two-channel QMF bank is designed with For N = 32, ωs = 0.6π, ωp = 0.4π, α = 0.001, the resulting performance parameters obtained are:

Peak reconstruction error (PRE)= 0.0086dB ; Stopband attenuation(As)= 29.05dB;

Passband error (E_P)= 1.143×10^{-07} ; stopband error (E_S) = 7.2625×10^{-07}

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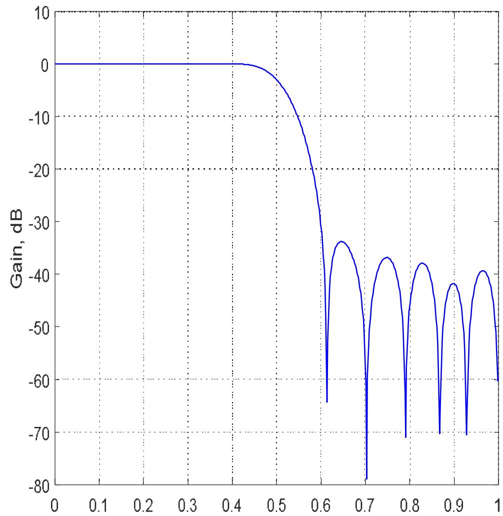
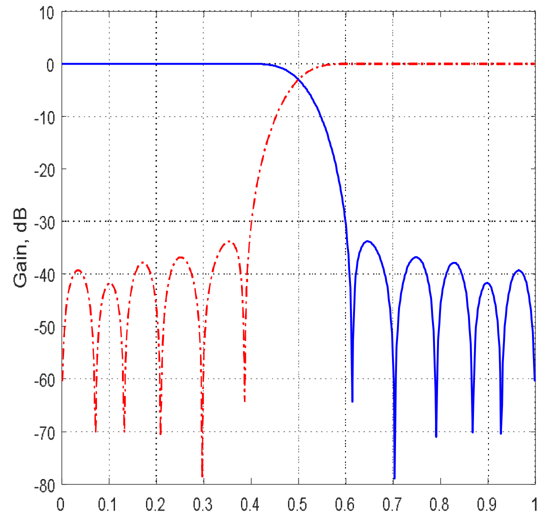
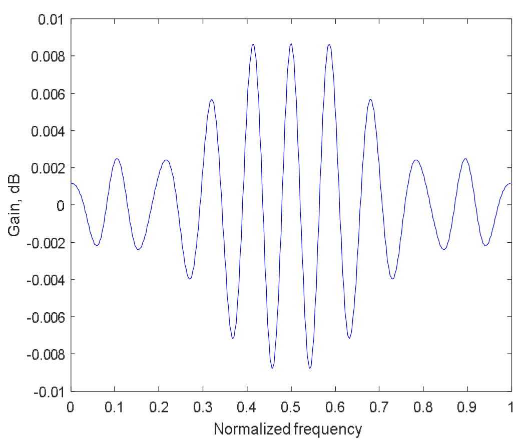


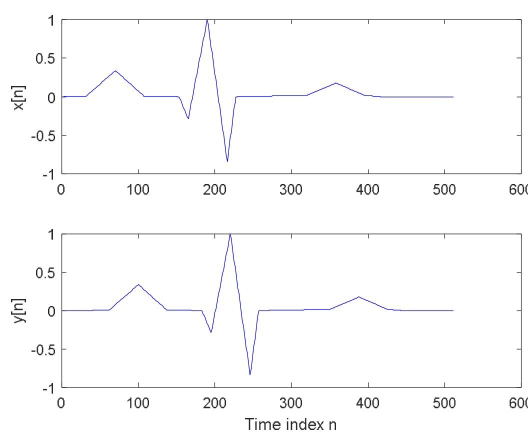
Fig(a): Amplitude response of the prototype LPF in dB



Fig(b): Amplitude response of the analysis filters in dB



Fig(c): reconstruction error in dB



Fig(d): input and output wave forms of the ecg signal

Performance comparisons of proposed method

TABLE I

Techniques	Ep	Es	Et	As(dB)	PRE(dB)
Levenberg-Marquardt optimization[13]	7.42×10^{-9}	1.27×10^{-6}	6.63×10^{-9}	36.59	0.0102
optimal fractional derivative constraints[15]	1.74×10^{-7}	2.89×10^{-5}	9.24×10^{-27}	43.27	0.0045
lagrange multipliers method[16]	5.23×10^{-8}	2.99×10^{-6}	1.23×10^{-23}	35.48	0.0141
An Efficient Algorithm[12]	1.16×10^{-8}	7.48×10^{-5}	–	25.06	0.0139
Artificial bee colony algorithm[14]	2.12×10^{-8}	5.08×10^{-6}	–	34.95	0.010
Proposed work	$\mathbf{1.14 \times 10^{-7}}$	$\mathbf{7.26 \times 10^{-7}}$	–	**29.05**	**0.0086**

V. CONCLUSION

In this paper, we have formulated linear-phase FIR design problems in the form of eigen problems. The filter coefficients are computed from eigenvectors of certain matrices which represent the specification requirements. The simulation results clearly indicate that proposed technique gives improved performance in terms of smallest peak reconstruction error and also exhibits better results for larger tap QMF banks. The extension of this approach for designing more than two-band QMF bank is under investigation. This type of design methods may be suitable various engineering fields such as image and speech compression .However, many issues remain to be investigated in the future.

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