International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue VII, July 2017- Available at www.ijraset.com

Basic Bilateral Analogue of Norlund's Continued Fraction

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Abstract: ***The continued fraction of a real number x is very efficient process for finding the best rational approximation of x. Moreover, continued fractions are very versatile tool for solving problems with movements involving two different periods. In this paper we establish the basic bilateral analogue Norlund's continued fraction.***
Keywords: Basic hypergeometric series, Bilateral analogue, Continued fraction, Convergence, Periods

I. INTRODUCTION

The expression of the form

$$= a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}$$

Where the $a_i s$ are integers, is called the continued expression of a real number.
The third part of Entry 21 of Ramanujan's [4] second notebook, generalized the Norlunds' continued fraction by establishing q-analogue.

II. NOTATIONS AND DEFINITIONS

A basic bilateral hypergeometric series is defined as.

$$ {}_2\Psi_2 \begin{bmatrix} a, b; x \\ c, d \end{bmatrix} = \sum_{n=-\infty}^{\infty} \frac{[a]_n [b]_n}{[c]_n [d]_n} x^n $$

$$\text{for } \left|\frac{cd}{ab}\right| < |x| < 1$$

It is oblivious that ${}_2\psi_2$ reduces to ${}_2\phi_1$ if any of the denominator parameters reduces to q.

Also, for $|q| < 1$ and arbitrary a

$$[a]_n = (1-a)(1-aq)(1-aq^2)\dots(1-aq^{n-1}), \quad [a]_0 = 1$$

Where $n > 0$

III.MAIN RESULT

In this paper, we shall establish the following result,

$$\frac{{}_2\Psi_2 \begin{bmatrix} aq, bq; x \\ cq, d \end{bmatrix}}{(1-c)\ {}_2\Psi_2 \begin{bmatrix} a, b; x \\ c, d \end{bmatrix}} = \frac{1}{\alpha_0 +} \frac{\beta_0\gamma_0}{\alpha_1} \frac{\beta_1\gamma_1}{\alpha_2 +} \dots \qquad (3.1)$$

Where, for $i = 0, 1, 2\dots$

$$\alpha_i = (1-cq^i) + \{(1+q)abq^{2i} - aq^i - bq^i\}x$$

$$\beta_i = \frac{(cq^i - abq^{2i+1}x)(1-ab^{i+1})(1-ab^{i+1})(d-bq^{i+1}x)}{aq^i(d-bq^{i+2})(1-bq^{i+1}x)}$$

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$$\gamma_i = \frac{(1-aq^i)(1-bq^i)(d-bq^{i+1}x)}{d-bq^{i+1}} - (1-aq^i) - \{(1+q)abq^{2i} - aq^i - bq^i\}\, x$$

PROOF: It is easily verified that

$$(1-cq^i)\; {}_2\Psi_2\begin{bmatrix} aq^i, bq^i; x \\ cq^i, d \end{bmatrix} = \alpha_i \; {}_2\Psi_2\begin{bmatrix} aq^{i+1}, bq^{i+1}; x \\ cq^{i+1}, d \end{bmatrix} + \frac{\beta_i\gamma_i}{1-cq^{i+1}}\; {}_2\Psi_2\begin{bmatrix} aq^{i+2}, bq^{i+2}; x \\ cq^{i+2}, d \end{bmatrix} \tag{3.2}$$

Where α_i, β_i and γ_i are given in (3.1). From (3.2) we have

$$\frac{(1-cq^i)\; {}_2\Psi_2\begin{bmatrix} aq^i, bq^i; x \\ cq^i, d \end{bmatrix}}{{}_2\Psi_2\begin{bmatrix} aq^{i+1}, bq^{i+1}; x \\ cq^{i+1}, d \end{bmatrix}} = \alpha_i + \frac{\beta_i\gamma_i}{(1-cq^{i+1})\dfrac{{}_2\Psi_2\begin{bmatrix} aq^{i+1}, bq^{i+1}; x \\ cq^{i+1}, d \end{bmatrix}}{{}_2\Psi_2\begin{bmatrix} aq^{i+2}, bq^{i+2}; x \\ cq^{i+2}, d \end{bmatrix}}} \tag{3.3}$$

Repeated application of (3.3) yields.

$$\frac{(1-c)\; {}_2\Psi_2\begin{bmatrix} a, b; x \\ c, d \end{bmatrix}}{{}_2\Psi_2\begin{bmatrix} aq, bq; x \\ cq, d \end{bmatrix}} = \alpha_i + \frac{\beta_0\gamma_0}{(1-cq)\dfrac{{}_2\Psi_2\begin{bmatrix} aq, bq; x \\ cq, d \end{bmatrix}}{{}_2\Psi_2\begin{bmatrix} aq^2, bq^2; x \\ cq^2, d \end{bmatrix}}}$$

$$= \alpha_0 + \cfrac{\beta_0\gamma_0}{\alpha_1 + \cfrac{\beta_1\gamma_1}{\alpha_2 + \cfrac{\beta_2\gamma_2}{\alpha_3 + \ddots}}}$$

$$or \quad \frac{{}_2\Psi_2\begin{bmatrix} aq, bq; x \\ cq, d \end{bmatrix}}{(1-c)\; {}_2\Psi_2\begin{bmatrix} a, b; x \\ cd, q \end{bmatrix}} = \frac{1}{\alpha_0 +}\frac{\beta_0\gamma_0}{\alpha_1 +}\frac{\beta_1\gamma_1}{\alpha_2 +}\cdots$$

This Proves (3.1).

The Convergence condition of the continued fraction appearing in (3.1) though complicated, can be worked out with the help of Worpitzky's theorem the left side of (3.1) is valid for $d = q^m (m \epsilon N)$ and $|x| < 1$.

We shall discuss some very interesting special cases of our result.

If we replace d by q in (3.1) we obtain the following result

$$\frac{{}_2\Psi_1\begin{bmatrix} aq, bq; x \\ cq \end{bmatrix}}{(1-c)\; {}_2\Psi_1\begin{bmatrix} a, b; x \\ c \end{bmatrix}} = \frac{1}{a_0 +}\frac{b_1}{a_1 +}\frac{b_2}{a_2 +}\cdots \tag{3.4}$$

Where for $i = 1,2,3..$

$$a_{i-1} = 1 - cq^{i-1} + \{(1+q)abq^{2i-2} - aq^{i-1} - bq^{i-1}\}\, x$$

$$b_i = x(cq^{i-1} - abxq^{2i-1})(1-aq^i)(1-bq^i)$$

If $q \to 1-0$ in (3.4) we get the following result due to Norlund

$$\frac{{}_2F_1\begin{bmatrix}1+a,1+b;x\\1+c,d\end{bmatrix}}{c\,{}_2F_1\begin{bmatrix}a,b;x\\c,d\end{bmatrix}}$$
$$=\frac{1}{c-(1+a+b)x+}\ \frac{(a+1)(b+1)(x-x^2)}{(c+1)-(3+a+b)x+}\ \frac{(a+2)(b+2)(x-x^2)}{(c+2)-(5+a+b)x+\cdots}$$

IV. CONCLUSION

Basic Bilateral Analogue of Norlund's continued fraction can be used to evaluate two centre problems in wave mechanics. The systematic use of N & C- terminal deletions can promote production and structural studies of recombination system. It can be used to study the nature of high energy radiation damage in iron and finds a wide range of application in electrical network, musical notes and in designing a planetarium etc.

V. ACKNOWLEDGMENT

My thanks are due to Dr. G.C Chaubey Ex Associate Professor & Head department of Mathematics TDPG College Jaunpur,
Professor B. Kunwar Department of Mathematics IET, Lucknow and Dr. S.N Pandey Associate Professor department of Mathematics MUIT, Lucknow for their encouragement and for providing necessary support. I am extremely grateful for their constructive support.

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