

Image-Centroid Tracking and Fusion with Square-Root Filtering Algorithms

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Abstract: *Image-based detection and tracking of moving objects are very important aspects for many aerospace and aviation applications, including satellite-based imagery, as well as road traffic management. Normally, Kalman filter is used for centroid tracking. We discuss the application of three square-root filtering algorithms for the image-centroid tracking. We also, consider the direct fusion of the centroids of two images, being tracked, using the square-root information filter (SRIF), and the square-root eigenfactor filter, the latter is known as VD (SRVD) filter, and this application to the image-centroid tracking and fusion is first of its kind. We also, propose a new image-centroid tracking-cum-fusion algorithm, called VDSRIF that has several merits and eliminates certain demerits of the SRVD and SRIF algorithms. Certain parametric studies and the performance metrics are evaluated for these fusion algorithms by utilizing synthetic images and the implementation has been done in MATLAB.*

Keywords: *Target/image-centroid tracking, centroid features, Kalman filtering, U-D filter, SRIF, SRVD filter, VDSRIF filter.*

I. INTRODUCTION

The target-image tracking is very important aspect of locating moving objects in real-time using some online and appropriate filtering algorithm. This filtering algorithm would utilize each image-frame that arrives at the processing centre, and output the location of the moving object. This process has two basic aspects: i) detection of the moving object/target in each frame, and ii) tracking-cum-filtering of thus detected object in each sequential frame. Conventionally, Kalman filter (KF) is used for target-tracking, however, we concentrate on square root type algorithms for image-centroid tracking because the latter are computationally more efficient, accurate and stable compared to the conventional KF. These special features are very important for online/real-time applications in many military and civilian applications to deploy an automated system for video-based observation and surveillance, and robotics that use vision sensors.

In many such situations the acquired image would often be cluttered, dim, spurious and noisy. This aspect might be due to the fact that the distance to the target from the sensing sites and centres is relatively very large, and thus weaken the signal, and accentuate the noise, relatively. The tracking problem involves processing of measurements (obtained from the sensors/radars) for a target of interest and producing at each time-step, an estimate of the target's current states. These states are: position, velocity, and even acceleration. The uncertainties present, are modelled as additive random noise in the measurements and the corresponding uncertainties in the target states. There would be additional uncertainty regarding the origin of the acquired image-data, which may or may not include actual measurements from the targets. The latter aspect might be due to some random clutter, say false alarms. This would lead to the need for data association, i.e. which measurement actually originated from which target? Hence, the detection and tracking of moving object is a reasonably difficult problem in forward-looking infrared (FLIR) image sequences. This is more so because of: i) low signal-to-noise (SNR) ratio, in the acquired image/s, ii) low (intensity) contrast, iii) presence of background clutter and false alarms, and/or iv) a partial occlusion of the target/image. This necessitates the use of efficient, accurate, and numerically stable filtering algorithms for image-centroid tracking and even image fusion. Effect of a few such parameters is evaluated for these new applications on the performance of the algorithms for tracking and fusion.

Several useful aspects on the target-image tracking are: i) correlation trackers for structured targets [1], ii) image-centroid tracking using the conventional least square (LS) linear method for weld pool application [2], iii) image-template matching application [3], iv) square root algorithms for estimation of certain classes of large scale interconnected systems [4,5], and v) cooperative tracking approach using the square root sigma point information filter (SRSPIF) [6]. However, for such purposes the use of efficient and numerically stable centroid tracking algorithms has been very limited. Although, several studies on square root type factorization filtering algorithms for state estimation and target tracking have been carried out, so far there has been no concrete evaluative study for the problem of image-centroid tracking, beyond [7,8].

Hence, we present certain parametric studies and performance results of image-centroid tracking and fusion using SRIF and SRVD algorithm, the study being first of its kind. Also, we propose a new image-centroid tracking-cum-fusion algorithm that has several merits and that eliminates some demerits of the SRVD and SRIF algorithms. These algorithms are implemented in MATLAB.

II. IMAGE-CENTROID FEATURES

An object, being not a point-mass, it should be assigned a correct coordinates or position of the considered image. This is because an object-image is wide, deep, and spread across an area or several pixels. In order to determine the coordinate of the object-image, the center of area (COA) is chosen as the representative position; and this is estimated by the center of mass (COM) or the so called centroid of the object [9], and hence, it is highly preferable to determine the COM or the centroid. The concept of estimation/filtering in image processing relates to the evaluation of image parameters, like the centroid. This is considered to be relevant to the characterization of the objects in the image; and the image analysis would involve measurements of certain characteristics of the image: i) intensity, ii) geometric features, and iii) centroid. The geometric features are: i) Length, L of a line in a discrete image is the distance between the centers of the pixels $L = d - 1$, here, d is the number of pixels the line covers, for if an object occupies one pixel, its length is zero; $L=1-1=0$; ii) Perimeter, P is equal to the sum of the side lengths; and iii) Area, A is equal to the sum of all the pixels covered by the object, i.e. area of an object in a digital image is the number of points in the object, thus one can compute the area of the object by $A = \text{total number of pixels}$. If an object is larger than one pixel, better is the area measurement, thus, for better centroid estimation the image should be spread over 2 or more pixels.

Two methods for determining the centroid of a star-object are: a) profile (or point spread function, PSF) fitting, and b) the image moment analysis; here, when a set of values has a tendency to cluster around some particular value, then it would be useful to characterize this set by a few numbers that are related to its moments (the sums of integer powers of the values themselves) [9]; that is if an object in an image is defined by the function $I(x,y)$. Then the moments generated by this function give interesting features of the object; and for digital images the $(n+m)$ th order is defined as

$$I_{nm} = \sum_x \sum_y x^n y^m I(x, y) \tag{1}$$

The moments' values would depend on the intensity or grey level; and image moments include center of mass, variance and orientation; and for, $n=m=0$, we get the I_{00} as the total intensity of the image as can be clearly seen from (1). If one considers the intensity of grey level $I(x,y)$ at each point (x,y) of the given image, I , as the mass of (x,y) , then one can define the centroid, the COM and other moments of I . In the 2-D case, the COM is given as (I_{10}, I_{01}) ; the normalized values of which are given as

$$I_{10} = \sum_x \sum_y xI(x, y) / I_{00}; \quad I_{01} = \sum_x \sum_y yI(x, y) / I_{00} \tag{2}$$

The variance is given as $\sigma_x^2 = I_{20} - I_{10}^2$; $\sigma_y^2 = I_{02} - I_{01}^2$; and it characterizes the spread or extension of the object-image in x - and y -directions. The orientation is defined as the angle of axis of the least moment of inertia (MOI)

$$\tan(2\theta) = \frac{2I_{11}}{I_{20} - I_{02}}; \quad \text{for } I_{11} \neq 0; \text{ and } I_{20} \neq I_{02} \tag{3}$$

The centroid of a cluster (in the normalized way) can also be determined using non-convolution method as

$$(x_c, y_c) = \frac{1}{\sum_{i=1}^n \sum_{j=1}^m I_{ij}} \left(\sum_{i=1}^n \sum_{j=1}^m iI(i, j), \sum_{i=1}^n \sum_{j=1}^m jI(i, j) \right) \tag{4}$$

In (4), I_{ij} is the intensity of the pixel and n, m are the dimensions of the cluster. One can use the 'regionprops' in MATLAB.

III. IMAGE-CENTROID TRACKING ALGORITHMS

In a (image-) tracking system one integrates the signal processing units for sensor signals and the data processing units for target tracking, in turn requiring a real time data processing capability. Such a system needs target-image-centroid tracking algorithm with lower computational cost in filtering and efficient data association schemes, also we need numerically accurate and stable algorithms. Hence, we now discuss five important centroid tracking-cum-filtering algorithms (CTA) of which four are of square-root type, and one of the latter is the newproposed algorithm. In a CTA, the determination of a moving object's position and velocity

from a noisy time series of images captured by image sensors constitutes a statistical estimation problem, often represented as a linear problem. A suitable state space model for centroid representation is then given by

$$x(k + 1) = \phi x(k) + Gw(k) \tag{5}$$

$$z(k + 1) = Hx(k) + v(k) \tag{6}$$

In (5), and (6), x is a state vector that contains the image-centroid coordinates of a target, z is the vector of observables (image-centroid measurements), and $w(\cdot)$, and $v(\cdot)$ are process and measurement noises with zero means and covariance matrices Q , and R_m respectively; often these noise processes are assumed to be white and Gaussian, and their statistics are assumed known and given, as also other matrices in (5), and (6) are known. The discrete time KF is given here for the sake of completion and comparison with other square root filtering algorithms.

A. The Discrete KF

The filtering algorithm is given as

1) State Propagation:

State estimate $\tilde{x}(k + 1) = \phi \hat{x}(k) \tag{7}$

Covariance (a priori) $\tilde{P}(k + 1) = \phi \hat{P}(k) \phi^T + GQG^T \tag{8}$

2) Measurement/Data Update:

Residuals/innovations $e(k + 1) = z(k + 1) - H \tilde{x}(k + 1) \tag{9}$

Kalman Gain $K = \tilde{P}H^T (H\tilde{P}H^T + R_m)^{-1} \tag{10}$

Filtered estimate $\hat{x}(k + 1) = \tilde{x}(k + 1) + K e(k + 1) \tag{11}$

Covariance (a posteriori) $\hat{P} = (I - KH)\tilde{P} \tag{12}$

The CTAKF (CTA based on KF) was investigated in [7].

B. UD Factorization Filter

At times, implementation of KF on a finite word length computing machine could pose a problem; and the effects would be greatly reduced by implementing it in a factorized form; such factorization implicitly preserves the symmetry and ensures the non-negativity of the covariance matrix P [10]. Such requirement would be very useful for online/real time implementation for the centroid tracking algorithm; because for a large scale problem, and heavy computational needs, the KF might diverge. One such widely used form of the algorithm is the UD factorization filter; here, U and D , are matrix factors of the covariance matrix P of the KF, where U is a unit upper triangular matrix (with 1's on diagonal elements) and D is a diagonal matrix. The major advantage from UD filter (UDF) comes from the fact that the square-root type algorithm processes square roots of the covariance matrices and hence, they essentially use half the word length normally required by the conventional KFs; i.e. in the UDF, the covariance update formulae of the conventional KF, (8), and (12); and the estimation recursion are reformulated. Specifically, we use recursions for U and D factors of covariance matrix $P = UDU^T$. The U-D filtering algorithm is given in two parts like the KF.

1) Time Propagation: We have for the covariance matrix propagation from KF recursion, see (8)

$$\tilde{P}(k + 1 | k) = \phi \hat{P}(k) \phi^T + GQG^T \tag{13}$$

Given $\hat{P} = \hat{U}\hat{D}\hat{U}^T$ (a priori factors are assumed known), and Q as the process noise covariance matrix, the time propagated factors \tilde{U} , \tilde{D} are obtained by utilizing the modified Gram-Schmidt orthogonalization process: we define, $V = [\phi\hat{U} | G]$ and $\bar{D} = \text{diag}[\hat{D}, Q]$, and $V^T = [v_1, v_2, \dots, v_n]$, then P is reformulated as $\tilde{P} = \tilde{V}\tilde{D}\tilde{V}^T$; then the new U and D factors of $\tilde{V}\tilde{D}\tilde{V}^T$ are computed by the following recursions; for $j = 1, \dots, n$; we evaluate [10]:

$$\tilde{D}_j = \frac{\langle v_j, v_j \rangle}{D} ; \tilde{U}_{ij} = (1/\tilde{D}_j) \langle v_i, v_j \rangle \quad i = 1, \dots, j - 1 \tag{14}$$

$$v_i = v_i - \tilde{U}_{ij} v_j \tag{15}$$

In (14), $\langle v_i, v_j \rangle_D = v_i^T \tilde{D} v_j$ is the weighted inner product between v_i and v_j ; the time propagation algorithm directly and efficiently produces the required U, D factors, taking the effect of previous U, D factors; transition matrix, and the process noise matrix Q , and also preserves the symmetry of the P matrix.

2) *Measurement/Data Update:* This data update process in KF combines a priori estimates \tilde{x} and error covariance \tilde{P} (obtained from the time propagation) with, say a scalar observation $z = cx + v$ to construct an updated estimate and covariance as

$$s = c\tilde{P}c^T + r; K = \tilde{P}c^T / s; \hat{x} = \tilde{x} + K(z - c\tilde{x}) \tag{16}$$

$$\hat{P} = \tilde{P} - Kc\tilde{P} \tag{17}$$

In (16), $\tilde{P} = \tilde{U}\tilde{D}\tilde{U}^T$; c is the (scalar) measurement matrix, r is the measurement noise variance, and z is the vector of noisy measurements; here, the processing is done in a scalar manner to avoid direct matrix inversion as occurring in (10). Kalman gain K , and updated covariance factors \hat{U} and \hat{D} can be obtained from the following equations [10]:

$$g = \tilde{U}^T c^T; g^T = (g_1, \dots, g_n); w = \tilde{D}g \tag{18}$$

$$\hat{d}_1 = \tilde{d}_1 R / s_1; s_1 = R + w_1 g_1 \tag{19}$$

For $j = 2, \dots, n$; compute the following

$$s_j = s_{j-1} + w_j g_j; \hat{d}_j = \hat{d}_{j-1} s_{j-1} / s_j \tag{20}$$

$$\hat{u}_j = \tilde{u}_j + \lambda_j K_j; \lambda_j = -g_j / s_{j-1} \tag{21}$$

$$K_{j+1} = K_j + w_j \tilde{u}_j; \tilde{U} = [\tilde{u}_1, \dots, \tilde{u}_n] \tag{22}$$

Then, the Kalman gain is given by

$$K = K_{n+1} / s_n \tag{23}$$

In (20), \tilde{d}_j is the predicted diagonal element, and \hat{d}_j is the updated diagonal element of the D matrix. The time propagation and measurement update for the state vector, x , are just similar to KF, as in (7), and (11).

C. Square Root Information Filter

Information filtering (IF) is the more direct way of dealing with the target tracking and multi sensor data fusion problems than the conventional covariance based KF. It has a special merit in tracking algorithms because the IF provides a direct interpretation of track observation and contribution in terms of information from multi sensor systems. However, the IF, if implemented as is, could be sensitive to computer round-off/quantization errors, like the KF. This would degrade the tracking performance of the filter. This is crucial if the algorithm is used for target tracking in a real time-online environment. The square root information filter (SRIF) offers a solution to this problem of numerical accuracy and stability of the filtering algorithm. Consider a linear algebraic measurement model, equation (6), in a simplified form for the sake of illustration

$$z = Hx + v \tag{24}$$

The least square (LS) solution for x is generally obtained by minimizing the leastsquare observation error (as the sum of squares of the errors)

$$J(x) = (z - Hx)^T (z - Hx) \tag{25}$$

Here, the idea is to estimate x (x could be an unknown parameter vector, if needed). In addition to the linear system, we assume that a priori unbiased estimate \tilde{x} (which we call covariance state, as against information state) of x and a priori information

matrix P^{-1} (inverse of the covariance matrix of the Kalman filter, P) form a priori state-information matrix pair $(\tilde{x}, \tilde{P}^{-1})$; then, the cost function J can be modified by inclusion of the a priori information pair to obtain [10]

$$J_a(x) = (z - Hx)^T (z - Hx) + (x - \tilde{x})^T \tilde{P}^{-1} (x - \tilde{x}) \quad (26)$$

The information matrix P^{-1} (being square of some quantity) can be factored as

$$\tilde{P}^{-1} = \tilde{R}^T \tilde{R} \quad (27)$$

So, in (27), we have the R matrix as the square root of the information matrix. After substituting (27) in (26), and a couple of simple algebraic (without any approximations) the steps we get

$$J_a(x) = (z - Hx)^T (z - Hx) + (\tilde{y} - \tilde{R}x)^T (\tilde{y} - \tilde{R}x) \quad (28)$$

Here, $\tilde{y} = \tilde{R}\tilde{x}$, and y is the information state associated with the square root of information matrix (this is not the same as the information state associated with the information matrix, i.e. \tilde{P}^{-1} , (27)). The second term of equation (28) can be written as

$$\tilde{y} = \tilde{R}x + v \quad (29)$$

Thus, the interpretation and inclusion of the a priori information (in terms of R, and y) as additional observations is the main step in obtaining the square root information filter (SRIF); and obtains the measurement/data update part of the SRIF.

1) *Measurements/Data Update* : Now, combining (29) and using (24), the composite measurements/data-system can be equivalently represented as

$$T(k) \begin{bmatrix} \tilde{R}(k-1) & \tilde{y}(k-1) \\ H(k) & z(k) \end{bmatrix} = \begin{bmatrix} \hat{R}(k) & \hat{y}(k) \\ 0 & e(k) \end{bmatrix}; \quad k = 1, \dots, \quad (30)$$

In (30), T(.) is an orthogonal transformation (say, Householder transformation matrix), and the process obtains the updated estimates of R, and y, on the right hand side of (30). Here, y is the information state (associated with R), and if required the covariance state can be obtained by (since, in real life situation one sees and uses only states in the covariance form!):

$$\hat{x} = \hat{R}^{-1} \hat{y} \quad (31)$$

Because of the structure of (30), the measurement update part of SRIF, one can see that this filter can be easily extended to include many imaging-sensor channels for multi sensor image-data fusion and target tracking, and since, estimates of y, and x are directly available from (30), and (31), Kalman gain is not required.

2) *Time Propagation of SRIF*: We assume that the transition matrix (5), and Q are non-singular. The process noise w(.) can sometimes represent the effects of un-modelled parameters, and errors due to linearization. Here, Q is also factored as

$$Q = R_w^{-1} R_w^{-T} \quad (32)$$

We assume that some a priori information is given in the form of data equation

$$\begin{aligned} y_w(0) &= R_w w(0) + v_w \\ \tilde{y}(0) &= \tilde{R}(0)x(0) + \tilde{v}(0) \end{aligned} \quad (33)$$

The variables v(.) are assumed to be zero mean, independent and with unity variances. Then, by introducing the effect of the state transition, (5), the time propagation part of the SRIF is given as [10]

$$\begin{bmatrix} \tilde{R}_w(k+1) & \tilde{R}_{w\alpha}(k+1) & \tilde{y}_w(k+1) \\ 0 & \tilde{R}(k+1) & \tilde{y}(k+1) \end{bmatrix} = T(k+1) \begin{bmatrix} \hat{R}_w(k) & 0 & \hat{y}_w(k) \\ -\hat{R}(k)\phi^{-1}G & \hat{R}(k)\phi^{-1} & \hat{y}(k) \end{bmatrix} \quad (34)$$

In (34), the terms in the right hand are provided as initial conditions (to start with), or are available from the previous cycle of the measurement update, (30). The left hand side terms in (34) come from the application of the HH transformation onto the right hand side block matrix of (34), then the relevant and required terms from (34) are inserted in (30), and the cycle is repeated.

D. Eigenvalue-Eigenvector Factorization Filtering

We study SVD (singular value decomposition)-based filtering algorithm for image-centroid tracking and fusion, we call this as SRDL filter. Such an SVD-based algorithm might be of some importance in certain applications where continuous monitoring of the eigenfactors is necessary in order to reveal singularities which might occur during the running of the algorithm; this would be an added merit in a large scale multi-dimensional fusion processing systems. This also helps to identify the states that are nearly dependent [11]. Here, V is eigenvector matrix and D is the diagonal matrix with the diagonal elements as the singular values of the given original matrix.

1) *V-D Discrete Time Measurement Update:* We use (6) as the measurement equation. The idea is to obtain the a posteriori eigenfactors given the a priori eigenfactors, the latter can also be called square root factors. Given the time propagated factors $V(k+1/k)$ and $D^{1/2}(k+1/k)$ of $P(k+1/k)$ (or the factors from initial conditions), the measurement matrix $H(k+1)$, and the measurement covariance matrix $R_m(k+1/k)$, we define the augmented matrix as

$$A(k+1) = [V(k+1/k)D^{1/2}(k+1/k) \quad H^T R_m^{-1/2}(k+1/k)] \quad (35)$$

Then perform an SVD of $A(\cdot)$ to obtain

$$A(k+1) = Y(k+1)[S(k+1) \quad 0]Z^T(k+1) \quad (36)$$

In (36), $Y(\cdot)$, and $Z(\cdot)$ are actually the eigenvector matrices, with the columns as the respective eigenvectors. Then, we obtain the measurement updated spectral factors as

$$\begin{aligned} V(k+1/k+1) &= Y(k+1) \\ D^{1/2}(k+1/k+1) &= S^{-1}(k+1) \end{aligned} \quad (37)$$

In (37), $Y(\cdot)$ is the an $n \times n$ orthogonal matrix, and $S(k+1)$ is $n \times n$ diagonal matrix with elements as the singular values of $A(k+1)$, i.e. the positive square-roots of the eigenvalues of matrix $A(k+1)A^T(k+1)$. The Kalman gain is obtained by defining the $M(\cdot)$ matrix as

$$M(k+1/k) = W^T(k+1/k)H^T(k+1) \quad (38)$$

$$W(k+1/k) = V(k+1/k)D^{1/2}(k+1/k) \quad (39)$$

$$K(k+1) = W(k+1/k)M(k+1/k)[M^T(k+1/k)M(k+1/k) + R_m]^{-1} \quad (40)$$

2) *V-D Time Propagation :* Given the measurement updated factors $V(k/k)$ and $D^{1/2}(k/k)$ of $P(k/k)$, the state transition matrix, the input gain matrix $G(k)$, and $Q(k)$, we define the composite matrix as

$$A(k) = [\phi(k)V(k/k)D^{1/2}(k/k) \quad G(k)Q^{1/2}(k)] \quad (41)$$

Then, we perform the SVD to decompose (41) into

$$A(k) = Y(k)[S(k) \quad 0]Z^T(k) \quad (42)$$

Then, we obtain the time propagated factors as

$$\begin{aligned} V(k+1/k) &= Y(k) \\ D^{1/2}(k+1/k) &= S(k) \end{aligned} \quad (43)$$

These square-root factors are used in measurement part of the filtering algorithms and we then obtain the complete filtering algorithm.

E. Eigenvalue-Eigenvector Factorization - Square Root Information Filtering- A New Algorithm

We propose a new algorithm for image-centroid tracking-cum-fusion that is based on the eigenfactor V-D filtering (SRVD) and the SRIF. The time propagation part is the same as the V-D filter and the measurement/data update part is the same as that of the SRIF, and we call it as eigenfactor-SRIF, or VDSRIF. All the initial values of P, Q, R, ϕ, H, G, R_m are the same as SRVD filter and the SRIF. For more clarity, the algorithmic steps are given for the combined covariance-information domain of filtering and the direct measurement level fusion of the centroids of the two input images (in fact more images can be easily fused as in the case of SRIF). Now, since these initial values are known, we proceed as follows:

- 1) *Step 0:* Initial part with initial conditions; use some guesstimate of the fused state, the same as one used in the SRVD, and SRIF filters, and compute the covariance matrix P(0):

$$P(0) = \{x_f(0) - \hat{x}_f(0)\} \{x_f(0) - \hat{x}_f(0)\}^T \quad (44)$$

Then, obtain SVD of P(0):

$$[Y,S,Z]=\text{svd}(P); \text{ since } P \text{ is a } n \times n \text{ matrix, } S \text{ will be so.} \quad (45)$$

Here, $P \rightarrow Y^*S^*Z'$. Then, assign the svd factors as follows

$$V=Y; D12=S \quad (46)$$

Also, obtain initial

$$R(0)=\text{sqrtm}(\text{inv}(V^*D12^*Z')); \rightarrow R(0)=\text{sqrtm}(\text{inv}(P)) \quad (47)$$

This is the square root of information matrix, $\text{inv}(P)$. For simplicity \hat{R}_f is denoted as R, \hat{x}_f is denoted as x, and \hat{y}_f is denoted as y in the sequel. Then, we get

$$y(0)=R(0)*x(0) \quad (48)$$

Thus, at this stage R, and y, the information pair (similar to the SRIF) as required for the time propagation of the state estimate are available.

- 2) *Step 1: Time propagation:* The time propagated information state is obtained as follows

$$y(\text{new})=R^* \phi^* \text{inv}(R)^* y(\text{previous}) \quad (49)$$

In fact (49) is the same as $x_f(\text{new})=\phi x_f(\text{previous})$. In (49) we use R from (47), and y from (48) for the start; then, in the next cycle these will be available from the output of the measurement/data part. Then, form the following A matrix using the values from (45)

$$A=[\phi^*V^*D12 \ G^*\text{sqrtm}(Q)] \quad (50)$$

This means that the previous eigenfactors, V, D12 are now to be augmented with the new information, i.e. by using, ϕ, G and Q.

Then, the new time propagated factors are obtained as

$$[Y,S1,Z]=\text{svd}(A); \text{ Here, } A \rightarrow Y^*[S \ 0]^*Z'; \text{ and } S1=[S \ 0] \quad (51)$$

Then, assign the factors as follows

$$V=Y; D12=S \quad (52)$$

Now, since A in (50) is non-square matrix, S will be also non-square and hence, assign only the first sub-matrix of S that is square/here. Next, obtain the R matrix needed in the measurement/data update part of SRIF:

$$R=\text{sqrtm}(\text{inv}(A^*A')) \quad (53)$$

In this R matrix, the effects of G and Q are included, by virtue of (50). Hence, matrix R is the time propagated factor. So, at this stage we have R from (53), and y from (49), as the time propagated information pair required for the measurement/data update, the Step 2.

- 3) *Step 2: Measurement/Data Update Part :* Since, R, and y are available as mentioned above from the output of the time propagation part, say at k-1; form the following composite matrix, i.e. (30), modified for the fusion of two inputs (image-centroids), here:

$$T(k) \begin{bmatrix} \tilde{R}_f(k-1) & \tilde{y}_f(k-1) \\ H_1(k) & z_1(k) \\ H_2(k) & z_2(k) \end{bmatrix} = \begin{bmatrix} \hat{R}_f(k) & \hat{y}_f(k) \\ 0 & e(k) \end{bmatrix}; \quad k = 1, \dots, \quad (54)$$

In (54), we have direct measurement level fusion of the centroids of the two input images. By applying the orthogonal transformation as in (54), we get the updated R and y (fused only) information pair. These are used in equation (49). At this stage, since we need x_f (for each time step), we can use

$$x_f=\text{inv}(R_f)y_f \quad (55)$$

Now, we need the V, D12 factors required in Step 1, in (50), hence, formulate A using V and D12 factors from (52) as follows

$$A=[V^*\text{inv}(D12) \ H^*\text{inv}(\text{sqrtm}(R_m'))]; \quad (56)$$

In (56), the R_m is the covariance matrix of the measurement noise. Then obtain the svd of A as

$$[Y,S1,Z]=\text{svd}(A); \quad A \rightarrow Y^*[S \ 0]^*Z'; \text{ Here, } S1=[S \ 0] \quad (57)$$

Then, assign the factors as:

$$V=Y; D12=inv(S); \tag{58}$$

Use (58) in (50) and repeat the cycle by going to the Step 1.

4) *The Merits of the Proposed Filtering Algorithm, VDSRIF are:*

- a) In the time propagation part, it does not need inversion of the state transition matrix as required in the SRIF, as in (34).
- b) It does not need specification of the information state y_w , related to the process noise, as in (34). This is because it uses the eigenfactors/svd of the composite matrix.
- c) In the measurement/data update part, it does not need the computation of the Kalman gain, since, now it uses the measurement/data update part of the SRIF, and hence, it is the gain free filter. The main reason is that the information state y_f is now directly available from the orthogonal transformation.
- d) It is the hybrid algorithm based on the eigenfactors and SRIF, a combination of the covariance and the information filter (SRIF).
- e) It retains the merits of the two filters: a) SRVD filter, and b) SRIF.
- f) The new filter eliminates some demerits of both the filters.

IV. EVALUATION OF THE ALGORITHMS

A set of image-frames is generated synthetically; 50 frames of the images are generated that represent target environment. For centroid computation formula (4) is used. In the present case we have: a) the image of dimension 64 x 64; b) the target size is fixed with a dimension of 9 x 9; c) the image consists of an object and its surrounding along with noise that is uniformly distributed; d) the image would have intensity in the range 0 to 255; and e) the target intensity value and its background have a certain mean and variance. A 2-D array of pixels is considered where each pixel is represented by a single index $i=1, \dots, m$ and the intensity of pixel is given by $I_i = s_i + n_i$; wherein, s_i is the target/background intensity and n_i is the noise intensity in pixel 'i', this noise is assumed to be Gaussian with zero mean and covariance σ^2 . The centroid dynamics are given by (5) and (6), whereas the measurements of the centroid of the given synthetic image are determined by (4). The input parameters for the tracking algorithms are: i) Measurement model/matrix: $H = [1 \ 0 \ 0 \ 0; 0 \ 0 \ 1 \ 0]$; ii) State transition matrix, (5) 'phi': $[1 \ T \ 0 \ 0; 0 \ 1 \ 0 \ 0; 0 \ 0 \ 1 \ T; 0 \ 0 \ 0 \ 1]$; iii) Measurement noise variance: $R_m=0.5$ (could be varied based on the study); iv) Process noise coefficient matrix: $G = [T^2/2 \ 0; T \ 0; 0 \ T^2/2; 0 \ T]$; and iv) Process noise co-variance: $Q = 0.00001$ (can be varied). Other image related parameters are: a) target image mean and std: (100, 10); b) Target background mean and STD: (50, 50) (can be varied based on the study; TGBSTD); c) Track scan (sampling interval/period, T): 1 sec.; d) The initial states $\{x(0), y(0)\} = (10, 10)$, with constant initial velocity of 1 m/s in both the coordinates; and e) Target noise std (TGNSTD) can be varied. All the algorithms are written and implemented in MATLAB. The performance metrics are evaluated as follows:

$$PFE = \% \text{fit error} = \text{norm}(\text{state or measurement error}) * 100 / \text{norm}(\text{true signal}). \tag{59}$$

$$RSSPE = \text{sqrt}(\text{mean}(x_{\text{perr}}^2 + y_{\text{perr}}^2)); \text{ similarly for the velocity state variable.} \tag{60}$$

A. Centroid Tracking-CTUDF

Figure 1 shows a screen shot of a typical run of a CTA. Table 1 gives the performance metrics for different target image noise STDs (TGNSTDs) for the filter. It is seen that there is not much of trend of the performance metrics wrt the STDs. However, it was established earlier that CTUDF performed somewhat better than CTKF in a similar centroid tracking task [12], and that the position and velocity state errors (time histories) were found to lie within their theoretical bounds as predicted by the CTUDF.

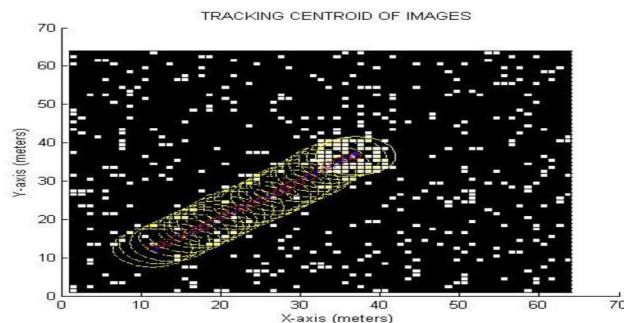


Figure 1 A screen shot of a typical run of the

image-centroid tracking algorithm.

Table 1: Performance metrics of image-centroid tracking algorithm using UDF

Parameter (*) Metrics (%fit errors)	TGNSTD		
	1	3	5
PFE _x	0.264	0.274	0.277
PFE _y	0.372	0.262	0.319
RMSPE	0.175	0.145	0.162
RMSVE	0.102	0.093	0.096

(*: Q=0.00001; TGBSTD=50)

B. Centroid Tracking-CTSRIF

Table 2 gives the performance metrics for different target image noise STDs for the filter. It is seen that there is a slight upward trend of certain metrics wrt the STDS; however, the performance is mostly robust. The position and velocity state errors (time histories) were found to lie within their theoretical bounds as predicted by the SRIF [13].

Table 2: Performance metrics of image-centroid tracking algorithm using SRIF

Parameter (*) Metrics (%fit errors)	TGNSTD		
	1	3	5
PFE _x	0.496	0.566	0.566
PFE _y	0.572	0.589	0.618
RMSPE	0.289	0.312	0.321
RMSVE	0.0092	0.0093	0.0098

(*: Q=0.001; TGBSTD=50)

C. Centroid Tracking and Fusion-CTSRIF

Now, since satisfactory tracking performance has been established, we consider the application of the SRIF to image-centroid tracking and fusion. The fusion is carried out by direct measurement level fusion, MLF. The target background is set at (mean=50, std=50, 100, 150). Two images are considered with target image set as (mean=100, std=10), and with variation in the target noise standard deviation as 1 (image 1, CTSRIF1), and 3 (image 2, CTSRIF2); these images are considered two-at-a-time for centroid tracking-cum-fusion. The performance metrics are shown in Table 3. Two of such results are plotted in Figure 2.

Table 3 Performance metrics of image-centroid tracking-cum-fusion using SRIF

Filter	target noise STD 1 & 3 (SB=50)			target noise STD 1 & 3 (SB=100)			target noise STD 1 & 3 (SB=150)		
	PFE _x	PFE _y	RMSPE	PFE _x	PFE _y	RMSPE	PFE _x	PFE _y	RMSPE
CTSRIF1 (std=1)	0.5837	0.7373	0.3604	0.3678	0.6552	0.2879	0.3386	0.5880	0.2600
CTSRIF2 (std=3)	0.6116	0.6173	0.3330	0.3862	0.3599	0.2023	0.3558	0.2765	0.1727
CTSRIMLF	0.5986	0.6459	0.3375	0.3805	0.4869	0.2368	0.3472	0.4076	0.2052

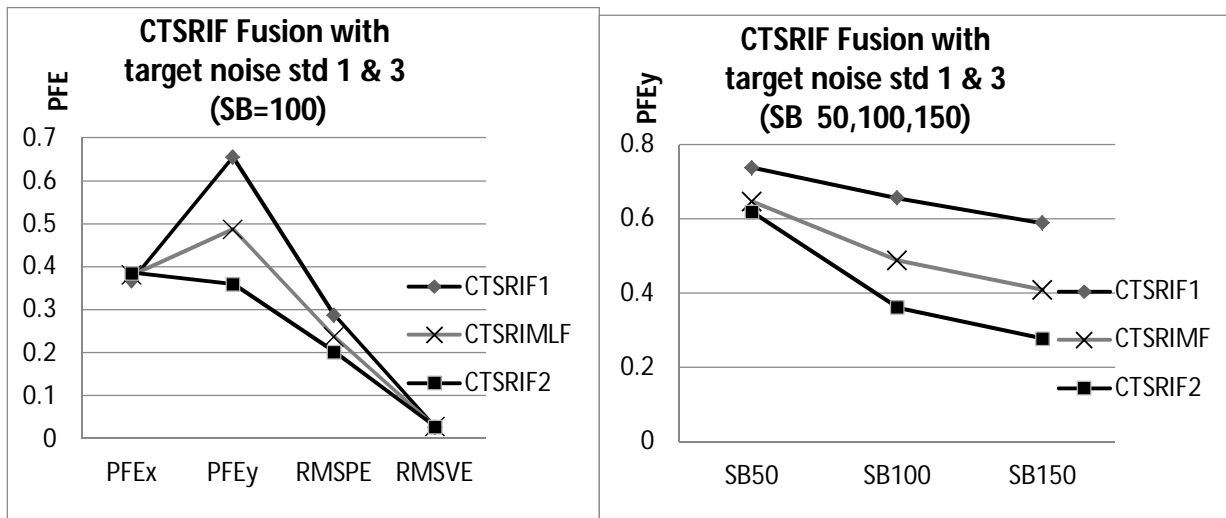


Figure 2 Fusion performance with SRIF with target noise std as 1 & 3.

Also, the fusion results with tgnstd 1 & 5; and tgnstd 5 & 9 have been obtained for fusion of centroid of two images, using SRIF, these extensive tables are not presented here. Most of the trends of these results were also found to be similar to those in Table 3, and Figure 2; the trends of various performance metrics across these combinations (images with tgnstd 1 & 5; and tgnstd 5 & 9) were found to be almost similar with minor variations.

D. Centroid Tracking and Fusion-CTSRVD Filter

Similar procedure as in 5.3 for the SRIF has been used here also. The results with tgnstd 1 & 3; and tgnstd 1 & 5; and 5 & 9 have been obtained for fusion of centroid of two images, using SRVD filter. These extensive results as generated for SRIF have also been generated here, but are not tabulated for brevity. However, Figure 3 depicts one such result for this case. In the case of this filter the variation of PFE/swrt SB, after the value of SB=100 was found to increase slightly, however, up to SB=100, it was consistent with that of SRIF.

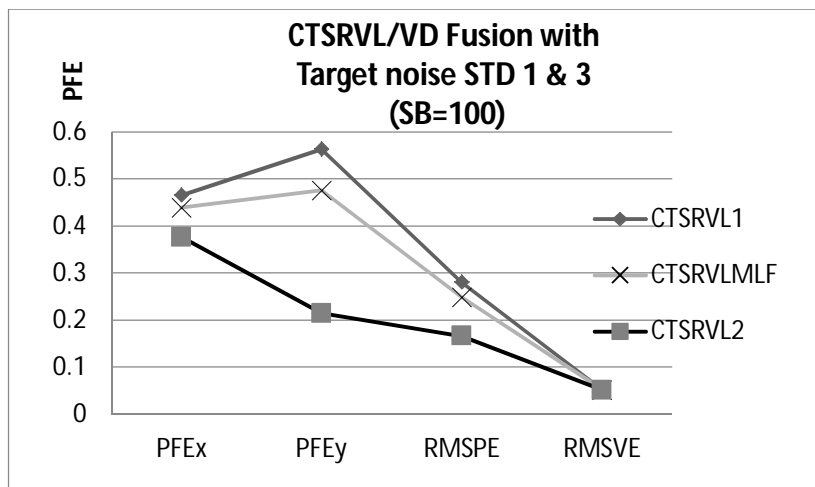


Figure 3 Fusion performance with SRVL/VD with target noise std as 1 & 3.

Here again, the trends of various performance metrics across these fusion combinations (images with tgnstd 1 & 3; tgnstd 1 & 5; and 5 & 9) for this filter were also found to be almost similar with minor variations.

E. Centroid Tracking and Fusion with the New CTVDSRIF Algorithm

Similar procedure as in 5.3 for the SRIF has been used here also. Since, this is the new filter implemented for image-centroid tracking its tracking performance was been evaluated first. One such result from Table 4 is plotted in Figure 4.

Table 4 Performance metrics of image-centroid tracking using VDSRIF

CTVDSRIF Filter	with target noise STD 1			with target noise STD 3			with target noise STD 5		
	PFE _x	PFE _y	RMSPE	PFE _x	PFE _y	RMSPE	PFE _x	PFE _y	RMSPE
SB=50	0.2087	0.5923	0.2404	0.2190	0.2615	0.1306	0.1679	0.4414	0.1808
SB=100	0.1454	0.5689	0.2248	0.1525	0.2289	0.1053	0.1444	0.3831	0.1567

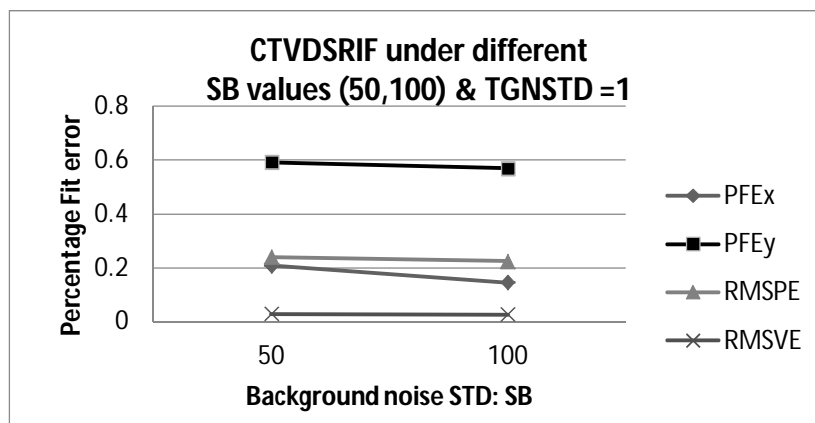


Figure 4 Tracking performance with VDSRIF with target noise std as 1.

We see that the trend of the PFE wrt SB variation is similar to that of with SRIF. Now, for the fusion, the results with tgnstd 1 & 5; and 5 & 9 have also been obtained for fusion of centroid of two images using the new VDSRIF filter. The extensive results as generated for SRIF and SRVL/VD have also been generated here, but are not tabulated for brevity. However, Table 5 shows one such result.

Table 5 Performance metrics of image-centroid tracking-cum-fusion using VDSRIF

Filter	with target noise STD 1 & 5 (SB=50)			with target noise STD 1 & 5 (SB=100)		
	PFE _x	PFE _y	RMSPE	PFE _x	PFE _y	RMSPE
CTVDSRIF1 (std=1)	0.2087	0.5923	0.2404	0.1554	0.5689	0.2248
CTVDSRIF2 (std=5)	0.1679	0.4414	0.1808	0.1444	0.3831	0.1567
CTVDSRIFM	0.2027	0.5466	0.2232	0.1520	0.5139	0.2052

Here again, the trends of various performance metrics across these fusion combinations (images with tgnstd 1 & 5; and 5 & 9) for this filter were also found to be almost similar with minor variations. Also, one such result is plotted in Figure 5, and the fusion performance is found to be satisfactory.

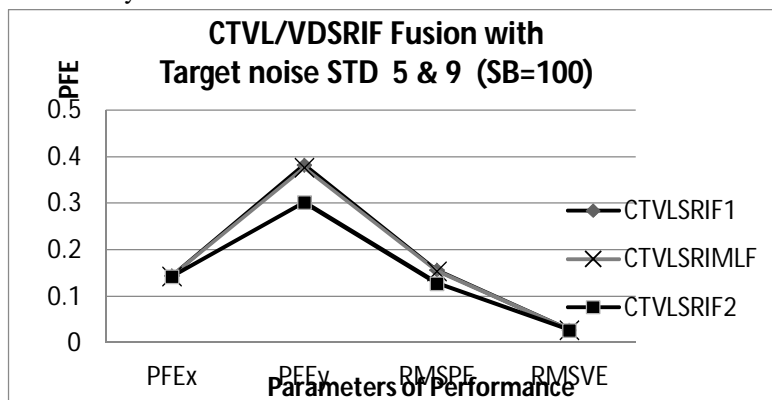


Figure 5 Fusion performance with VDSRIF with target noise std as 5 & 9.

F. Inferences from the Study Carried Out

We make the following observations from the presented and other results (not presented here) for the image-centroid tracking and fusion:

- 1) The CTUDF was found to perform somewhat better than CTKF.
- 2) The tracking performances of CTUDF and CTSRIF were nearly similar.
- 3) There was somewhat upward trend of the PFEs wrt the increase in TGNSTDs, however largely the performance was robust.
- 4) The most PFEs wrtan increase in SB (the standard deviation of the background) showed a downward trend, and this can be viewed as somewhat increase in the contrast, since the target image parameters were kept invariant, and the increase in the SB can be looked upon as the dispersion of the background. However, here, also, somewhat robust performance has been observed in most of the PFEs and across various filters studied in the present paper.
- 5) The image-centroid tracking performance has been found to be very satisfactory across all the square root type filters.
- 6) The image-centroid tracking-cum-fusion performance also has been found to be in accordance with the well-established theory of general data fusion, for the square root filtering algorithms studied here.
- 7) As seen from Table 5, the performance of the new filter for image-centroid tracking-cum-fusion is found to be somewhat better than its counter-part filters: SRIF, and SRVD algorithms. Since, the new algorithm has several merits compared to its parent filters, and that it eliminates certain demerits, the new filter provides numerically stable and accurate viable alternative algorithm for image-centroid tracking and fusion. It could serve as a good candidate for online-real time applications in aviation and robotics problems which use vision sensors in multi-dimensional image/data fusion processing tasks.

G. Concluding Remarks

We have considered the image-centroid tracking and fusion using several square root type filtering algorithms and proposed a new algorithm for the same. We specifically considered centroid tracking/and or fusion using UD factorization filtering, square root filtering, eigenfactor filtering and the new algorithm based on combination of eigenfactor&SRIF algorithms, and evaluated their performances with synthetic image/s generated using MATAB. Based on the performance metrics and plots, it has been found that these algorithms gave very satisfactory performance in tracking and fusion. Although, we observed certain trends in the percentage fit errors wrt target noise standard deviation and the background standard deviation largely the performance of the square root type filtering algorithms has remained nearly robust. The proposed new algorithm with several merits can be considered as a viable alternative for online-real time applications for variety of image/target tracking and multi-sensor data fusion tasks.

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