

Performance Analysis of Non Uniform Quadrature Mirror Filter Bank Using Fir Filters

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Abstract: In this paper, a well-organized procedure is proposed for the design of multi-channel nearly perfect reconstructed non-uniform filter bank (NUFB). The proposed system is very simple, direct in nature, and easy to implement. The method employs the FIR technique to design the prototype filter for filter banks with novelty of exploiting a new perfect reconstruction condition of the non-uniform filter banks instead of using complex objective functions. It was found that the projected methodology performs better as compared to earlier reported results in terms of reconstruction error (RE). For nonuniform filter banks, there is a lack of efficient design methods. The NUFBs are constructed in a direct structure. With this condition, The NUFBs can be designed simply and it works efficiently.

Keywords: Filter banks, QMF, Sub-band coding, Non-uniform filter bank (NUFB), Tree-structured.

I. INTRODUCTION

Numerous applications have made multirate filter banks design a growing field of research approach. Multirate filter banks are so named because they effectively alter the sampling rate of a digital system, as indicated by the decimators (down samplers) following the analysis filters, H_0 and H_1 , and the expanders (up samplers) preceding the synthesis filters, F_0 and F_1 .

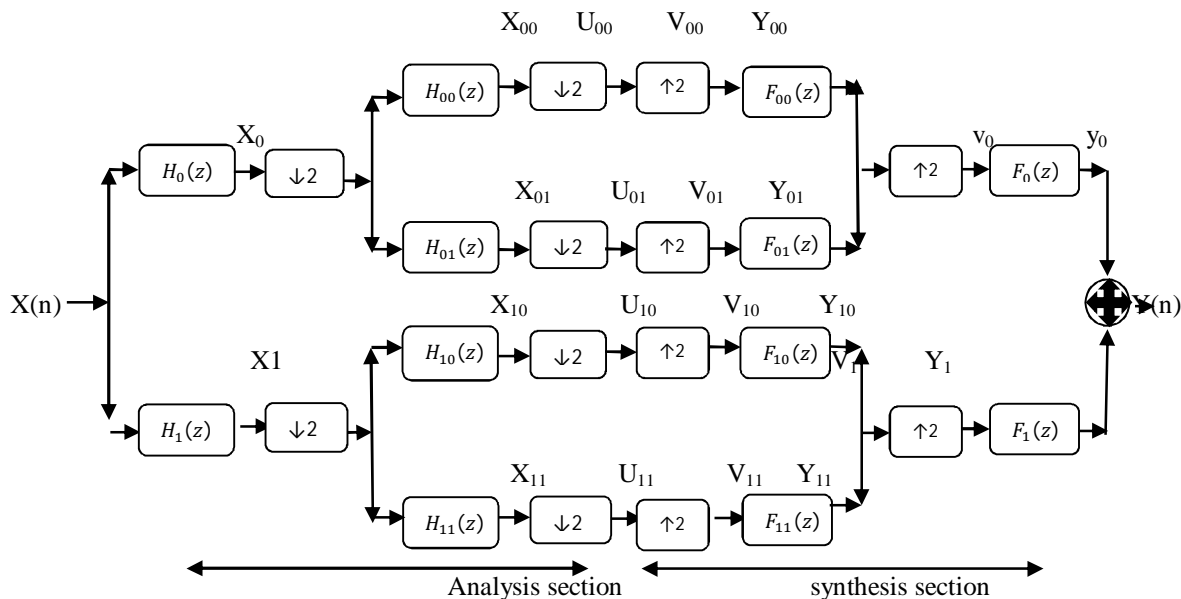


Fig. 1: Four Channel- Filter Bank

Properly designed analysis and synthesis filters combined with the properties of decimation and expansion allow filter banks to partition a wideband input signal into multiple frequency bands (often called sub-bands or channels) and to recombine these sub-band signals back into the original signal. The analysis filters, H_0 and H_1 , (high pass and low pass filters respectively) are typically complementary (mirror) to each other about the digital frequency, $\pi/2$, as shown in Figure 1. Such filters are often called quadrature mirror filters (QMF). The quadrature mirror filters are widely used in various applications. M-channel maximally decimated and interpolated filter banks are widely used in different applications. A basic multirate filter bank is shown in figure. Filter banks divide a digital signal into sub-bands of different frequency. The uneven distribution of energy of signal in sub bands different filter banks provides the basis for signal compression. The very necessity is the frequency selectivity of the individual filters which is further

defined as small passband ripple, narrow transition bandwidth and stop band attenuation. The researches were mainly focused only for designing of a two channel quadrature mirror filter (QMF) bank [1] in early days, which was then extended to design M-channel filter banks [2,3]. Since then, several techniques [4,5] were developed to enhance the performance of filter banks in different engineering grounds. specific applications include sub-band coding like audio coding, speech coding, data and image compression [1] and also in Uniform filter banks have many constraints like integer and uniform decimation in each sub-band, and limited time frequency resolution. These constraints catalyze the significance of non-uniform filter banks (NUFBs). Over the past few years, a number of design methods [6-8] have been proposed by different authors for the design of multi-channel filter banks. Such an application oriented technique was defined in [9]. The quadrature mirror filters have the objective to split the input signal into its sub-band components, one high-frequency sub-band and one low-frequency sub-band. The optimized quadrature mirror filters satisfy partially the power complementary restriction and the perfect reconstruction is not obtained. The proposed method gives good solution for the physical realizations of the QMF banks. The amplitude distortion can be minimized by using continuous polynomials. In many applications a reduced error is important. The use of the continuous polynomials in the filter design allows the design of optimal FIR filters in the least-squares sense, having as effect a reduced approximation error.

II. NON UNIFORM FILTER BANK

The desired magnitude response of the filter is specified as an optimization objective In this paper, we present a simple method for designing nonuniform NPR filter banks having integer decimation factors In addition, NUFBs are able to provide any sort of rational decimation in each channel, to any extent of time–frequency resolution as per requirement of the application, less quantization error, and low computational complexity. several design methods [12,13] have been proposed and valued for designing the non-uniform filter bank based on optimization and non-optimizations. The non-uniform filter bank with integer decimation and linear phase is realized with the help of tree structured techniques, which is based on building the two-channel filter bank as basic building blocks [1,2,16]. The generalized structure of Four-channel filter banks based on tree structure approach is depicted in Fig. 1. For M-channel NUFB having decimation $M_0, M_1, M_2, \dots, M_{M-1}$ for each band, then decimation factors described [1].

$$\sum_{k=0}^{M-1} \frac{1}{Mk} = 1 \tag{1}$$

and the reconstructed signal $\hat{X}(z)$ is

$$\hat{X}(z) = \sum_{k=0}^{M-1} F_k(z) \frac{1}{Mk} = \sum_{l=0}^{Mk-1} X(zW_{Mk}^1) F_k(zW_{Mk}^1) \tag{2}$$

Where $H_k(z)$ and $F_k(z)$ are the analysis and synthesis filters respectively, while $W_L = e^{j2\pi/L}$.

The terms $H(zW_{Mk}^1)$ and $X(zW_{Mk}^1)$ are aliasing terms introduced for decimation /interpolation. For the perfect reconstruction (PR), the error can be eliminated by the below condition.

$$F_1(z) = -2H_0(-z) \quad \text{and} \quad F_0(z) = 2H_1(-z). \tag{3}$$

In general, 2^P sub-bands can be obtained by repeating the same decomposition process P times and at each of the P stages of decomposition process, the number of two-channel QMF bank structure required, is 2^P-1 . On the other side, the process of reconstructing the original input signal can be seen as mirror image of the decomposition process at analysis side. After resolving the tree structured nonuniform filter bank into its parallel forms, the following relations can be deduced

$$\begin{aligned} H_0(z) &= H_L(z), H_L(z^2), & F_0(z) &= F_L(z), F_L(z^2) \\ H_1(z) &= H_L(z), H_H(z^2), & F_1(z) &= F_L(z), F_H(z^2) \\ H_2(z) &= H_H(z), & F_2(z) &= F_H(z) \end{aligned} \tag{4}$$

In multi-channel non-uniform filter bank [1,2], the perfect reconstruction is possible if

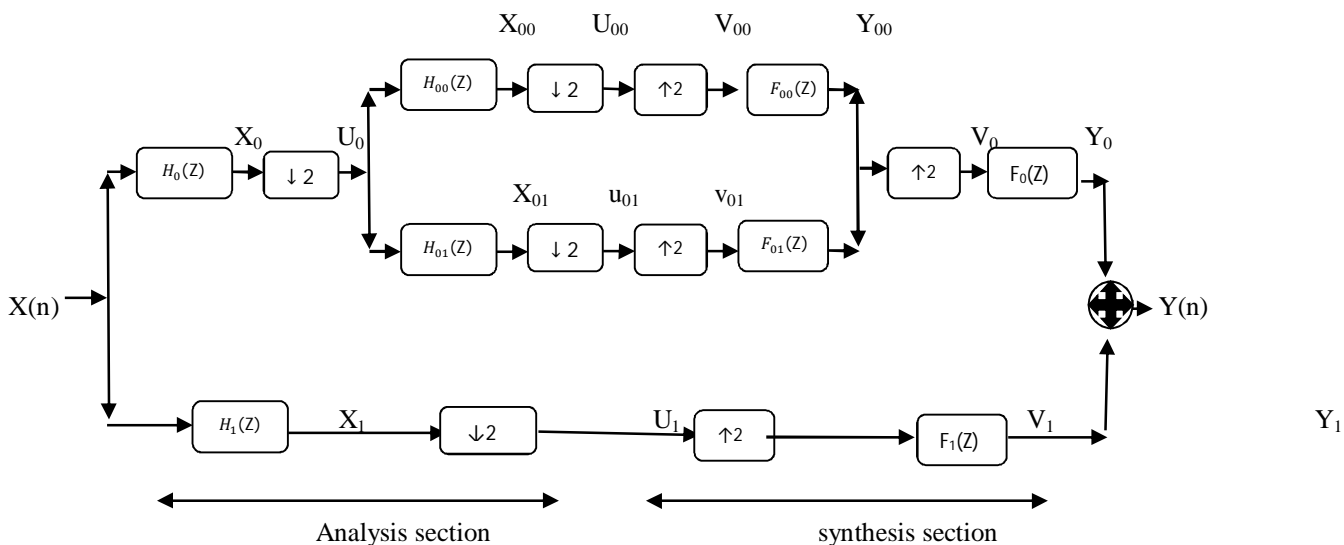


Fig.2: Two Channel- Three Stage Filter Bank

$$\sum_{k=0}^{M-1} |H_k(e^{j\omega})|^2 = 1 \text{ for } 0 < \omega \leq \frac{\pi}{M} \tag{5}$$

For M=3 the equation can be written as

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 + |H_2(e^{j\omega})|^2 = 1 \text{ for } 0 \leq \omega \leq \frac{\pi}{3} \tag{6}$$

The above Eq.(6) is further shown in fig.2 and simplified using the prototype filter $H_L(e^{j\omega})$ of a two-channel QMF bank, then it is reduced to

$$|H_L(z)H_L(z^2)|^2 + |H_L(z)H_H(z^2)|^2 + |H_H(z)|^2 = 1 \tag{7}$$

As the energy of the original signal and its up-sampled version will be same as in up sampled signal. In up sampling, L-1 equidistant zero valued samples are placed between two successive samples of the original signal. Then,

$$|H_L(z^2)|^2 = |H_L(z)|^2 \tag{8}$$

Similarly,

$$|H_H(z^2)|^2 = |H_H(z)|^2 \tag{9}$$

Using the above equations and the quadrature mirror filter condition ($H_H(z) = H_L(-z)$), Eq. (9) is redefined as

$$|H_L(z)|^4 + |H_L(z)|^2 |H_L(-z)|^2 + |H_L(-z)|^2 = 1 \tag{10}$$

In frequency domain, Eq. (10) is reduced to

$$2|H_L(e^{j\pi/2})|^4 + |H_L(e^{j\pi/2})|^2 = 1 \tag{11}$$

Let $|H_L(e^{j\pi/2})| = x$, then Eq. (11) is simplified as

$$2x^4 + x^2 - 1 = 0 \tag{12}$$

Using simple factorization, the roots are

$$x = \pm \frac{1}{\sqrt{2}} = \pm 0.7072 \text{ and } x = \pm i \tag{13}$$

The imaginary and negative roots are not possible, so neglect them and then, it leads to Eq. (14), giving a new PR condition.

$$x = |H_L(e^{j\pi/2})| = 0.7072 \tag{14}$$

Similarly, in 4-channel NUFB with decimation factors (8, 8, 4, 2), the perfect reconstruction can be achieved, if Eq. (15) is satisfied

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 + |H_2(e^{j\omega})|^2 + |H_3(e^{j\omega})|^2 = 1 \text{ for } 0 \leq \omega \leq \frac{\pi}{4} \tag{15}$$

Where

$$H_0(z) = H_L(z)H_L(z^2)H_L(z^4) \quad \text{and} \quad H_1(z) = H_L(z)H_L(z^2)H_H(z^4) \tag{16}$$

$$H_2(z) = H_L(z)H_H(z^2) \quad \text{and} \quad H_3(z) = H_H(z) \tag{17}$$

Using Eq. (16), Eq. (17), then Eq. (15) turns out to be

$$|H_L(z)H_L(z^2)H_L(z^4)|^2 + |H_L(z)H_L(z^2)H_H(z^4)|^2 + |H_L(z)H_H(z^2)|^2 + |H_H(z)|^2 = 1 \quad (18)$$

As discussed earlier, the energy of original signal and its up sampled version has same value, therefore

$$|H_L(z^4)|^2 = |H_H(z^2)|^2 = |H_L(-z)|^2 \quad \text{and} \quad |H_H(z^4)|^2 = |H_H(z^2)|^2 = |H_H(z)|^2 \quad (19)$$

Using the above equation and $(H_H(z) = H_L(-z))$, Eq. (19) is further refined as

$$|H_L(z)|^6 + |H_L(z)|^4 + |H_L(-z)|^2 + |H_L(z)|^2|H_L(-z)|^2 + |H_L(-z)|^2 = 1 \quad (20)$$

and in frequency domain, it leads to

$$|H_L(e^{j\omega})|^6 + |H_L(e^{j\omega})|^4 + |H_L(e^{j\omega-\pi})|^2 + |H_L(e^{j\omega})|^2|H_L(e^{j\omega-\pi})|^2 + |H_L(e^{j\omega-\pi})|^2 = 1 \quad (21)$$

At $\omega = 0.5\pi$

$$2|H_L(e^{j\pi/2})|^6 + |H_L(e^{j\pi/2})|^4 + |H_L(e^{j\pi/2})|^2 = 1 \quad (22)$$

$$\text{Let } H_L(e^{j\pi/2}) = x$$

$$2x^6 + x^4 + x^2 - 1 = 0 \quad (23)$$

The roots of this equation are found using simple factorization approach, these are $x = \pm 0.7071$,

$0.5 \pm 0.866i$ and $0.5 \pm 0.866i$.

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 + |H_2(e^{j\omega})|^2 + |H_3(e^{j\omega})|^2 + |H_4(e^{j\omega})|^2 = 1 \text{ for } 0 \leq \omega \leq \frac{\pi}{5} \quad (24)$$

Where

$$\begin{aligned} H_0(z) &= H_L(z)H_L(z^2)H_L(z^4)H_L(z^8) & H_1(z) &= H_L(z)H_L(z^2)H_L(z^4)H_H(z^8) \\ H_2(z) &= H_L(z)H_L(z^2)H_H(z^4) & H_3(z) &= H_L(z)H_H(z^2) \\ \text{and } H_4(z) &= H_H(z) \end{aligned} \quad (25)$$

Using Eq. (25), then Eq. (24) turns out to be

$$|H_L(z)H_L(z^2)H_L(z^4)H_L(z^8)|^2 + |H_L(z)H_L(z^2)H_L(z^4)H_H(z^8)|^2 + |H_L(z)H_L(z^2)H_H(z^4)|^2 + |H_L(z)H_H(z^2)|^2 + |H_H(z)|^2 = 1 \quad (26)$$

Since

$$|H_L(z^8)|^2 = |H_L(z^4)|^2 = |H_L(z^2)|^2 = |H_L(z)|^2 \quad (27)$$

and

$$|H_H(z^8)|^2 = |H_H(z^4)|^2 = |H_H(z^2)|^2 = |H_H(z)|^2 \quad (28)$$

Using the above equations and $H_H(z) = H_L(-z)$ Eq. (26) is further refined as

$$|H_H(z)|^8 + |H_H(z)|^6|H_H(-z)|^2 + |H_H(z)|^4|H_H(-z)|^2 + |H_H(z)|^2|H_H(-z)|^2 + |H_H(-z)|^2 = 1 \quad (29)$$

and it leads to Eq. (30) in frequency domain

$$|H_L(e^{j\omega})|^8 + |H_L(e^{j\omega})|^6|H_L(e^{j(\omega-\pi)})|^2 + |H_L(e^{j\omega})|^4|H_L(e^{j(\omega-\pi)})|^2 + |H_L(e^{j\omega})|^2|H_L(e^{j(\omega-\pi)})|^2 + |H_L(e^{j(\omega-\pi)})|^2 = 1 \quad (30)$$

At $\omega = 0.5\pi$

$$2|H_L(e^{j\pi/2})|^8 + |H_L(e^{j\pi/2})|^6 + |H_L(e^{j\pi/2})|^4 + |H_L(e^{j\pi/2})|^2 = 1 \quad (31)$$

Let $H_L(e^{j\pi/2}) = x$

$$2x^8 + x^6 + x^4 + x^2 - 1 = 0 \quad (32)$$

The roots of this equation are found using simple factorization approach and the roots are

$$x = 0.7072 = |H_L(e^{j\pi/2})| = 0.7072 \quad (\text{Since negative and imaginary roots are not possible,})$$

To derive a design criterion for the NUFB, the filter coefficients values at quadrature frequency are approximately equal to 0.707, which is derived in above equations. Several design examples are included to demonstrate the effectiveness of this designing non-uniform filter bank (NUFB) in the comparison table. The decimation factors we focused on are integers. That shows a requirement with respect to the magnitude responses of two adjacent filters as so to reduce the amplitude distortion. This can be done by using the available filter design tools. Tree-structure can be used to build LP NUFBs with integer decimation factors by cascading NUFBs. Whereas system delay is not a critical issue in some applications, a lower system delay is not only a desired property but also a need in some delay-sensitive applications, such as radar signal processing and voice service in wireless communication. Employed in many signal processing applications, mainly to their flexibility of partitioning sub bands.

III. PROPOSED METHODOLOGY

We employ the Parks-McClellan[PM], Least Square, Remez, and some other algorithms, these algorithms are described as different methods in the following, In Method 1 Parks-McClellan designs a linear-phase FIR [Finite impulse response] filter using the Parks-McClellan algorithm .The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with an optimal fit between the desired and actual frequency responses. The filters are optimal in the sense that the maximum error between the desired frequency response and the actual frequency response is minimized. Filters designed this way exhibit an equiripple behavior in their frequency responses and are sometimes called equiripple filters firpm exhibits discontinuities at the head and tail of its impulse response due to this equiripple nature. returns row vector *b* containing the *n*+1 coefficients of the order *n* FIR filter whose frequency-amplitude characteristics match those given by vectors *f* and *a*.

The output filter coefficients/taps in ‘*b*’ obey the symmetry relation:

$$b(k)=b(n+2-k), \quad k=1,\dots,n+1 \tag{33}$$

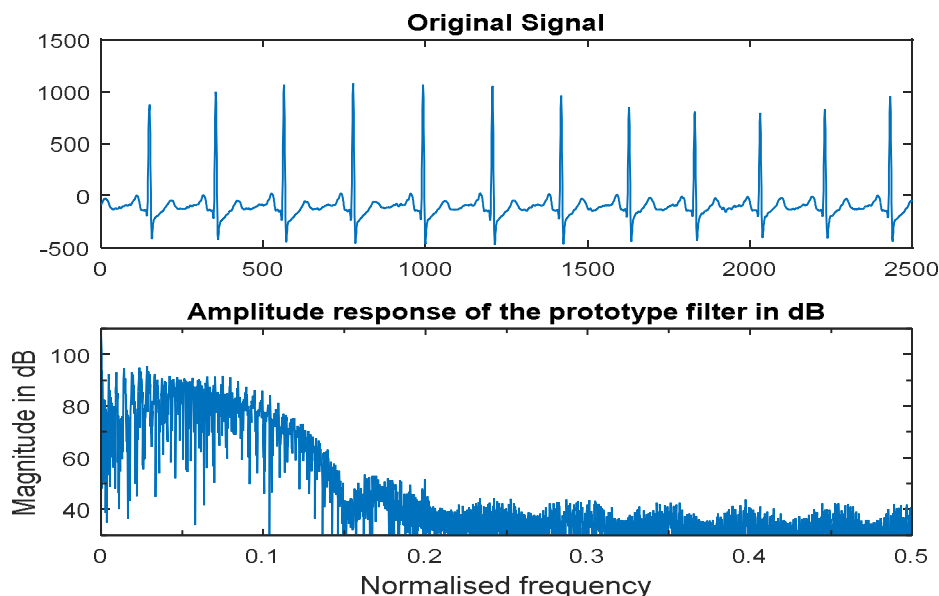
similarly in method 2 called firfs designs a linear-phase FIR filter that minimizes the weighted, integrated squared error between an ideal piecewise linear function and the magnitude response of the filter over a set of desired frequency bands which is used for setting the particular energy levels using non-uniform filter bank techniques. In method 3 firgr is a minimum, maximum filter design procedure use to design linear phase filters of even order and odd filter (symmetric, antisymmetric) of real FIR filters. Which is used to calculate Minimum phase, Maximum phase, Minimum order (even or odd), and Extra ripple, Maximal ripple etc., In method 4 fir2 uses frequency sampling to design filters. The function interpolates the desired frequency response. To obtain the filter coefficients, the function applies an inverse fast Fourier transform to the given variables. In method 5 we use firband which is similar to the above firgr all these techniques are compared in a table. All these methods realizes an efficient control of the transition bandwidth authors have been used the evolutionary programming algorithms to design the optimized prototype filter for all the real signals have uneven distribution of energy in different bands or energy is dominantly concentrated in a particular region of frequency.

IV. RESULTS AND CONCLUSION

In this section, the proposed methods has been implemented on MATLAB and used for designing non-uniform filter bank using FIR technique. To illustrate the ability of this method, many examples with different design specifications are included. The performance of all these methods are evaluated in terms of reconstruction error (PRE) given by

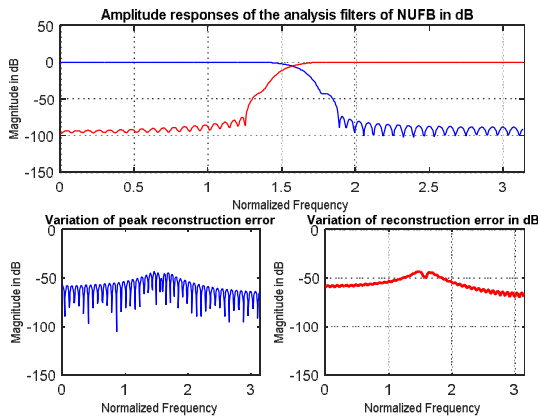
$$PRE = \max\{\sum_{k=0}^{M-1}|H_k(e^{j\omega})|^2\} - \{min \sum_{k=0}^{M-1}|H_k(e^{j\omega})|^2\} \tag{34}$$

All the values are compared in the below table, By comparing all the values the better values are taken into consideration for the better performance. The better value is taken into account better results in the future work scope. Here ‘*N*’ describes the value of filter taps used for effective outputs which are observed in the below waveforms and comparison table

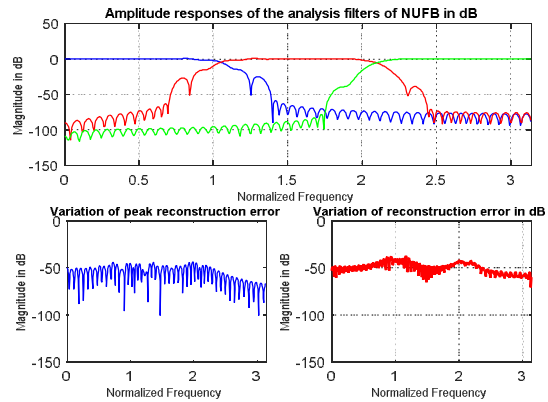


Fig(a): original input and amplitude response of the signal

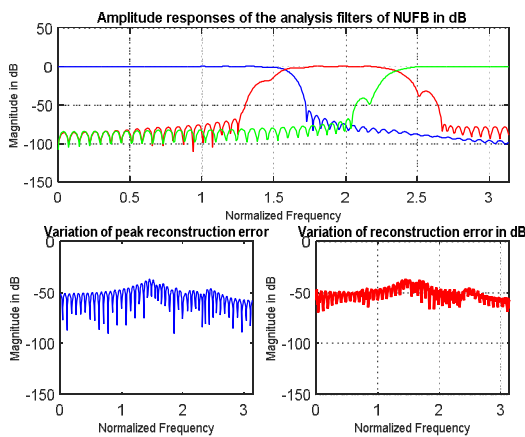
Obtained outputs for different sub bands are shown below



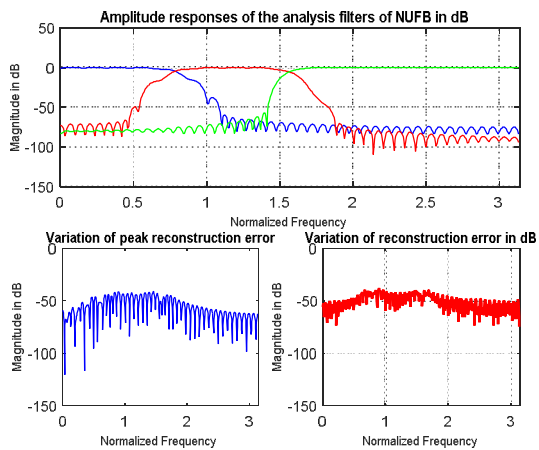
Fig(b): Two Channel Two Stage Output(2,2)



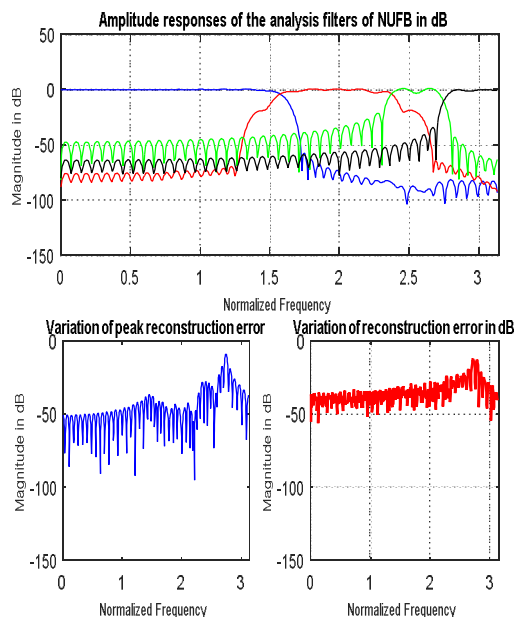
Fig(c): Three Channel Three Stage Output(3,3,3)



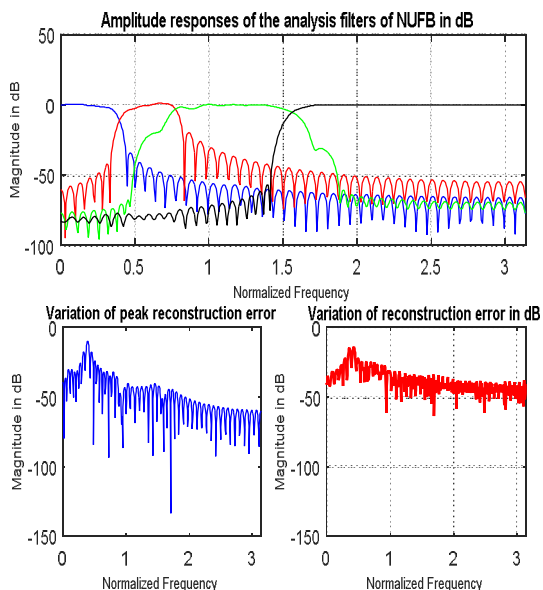
Fig(d): Two Channel Three Stage Output (2,4,4)



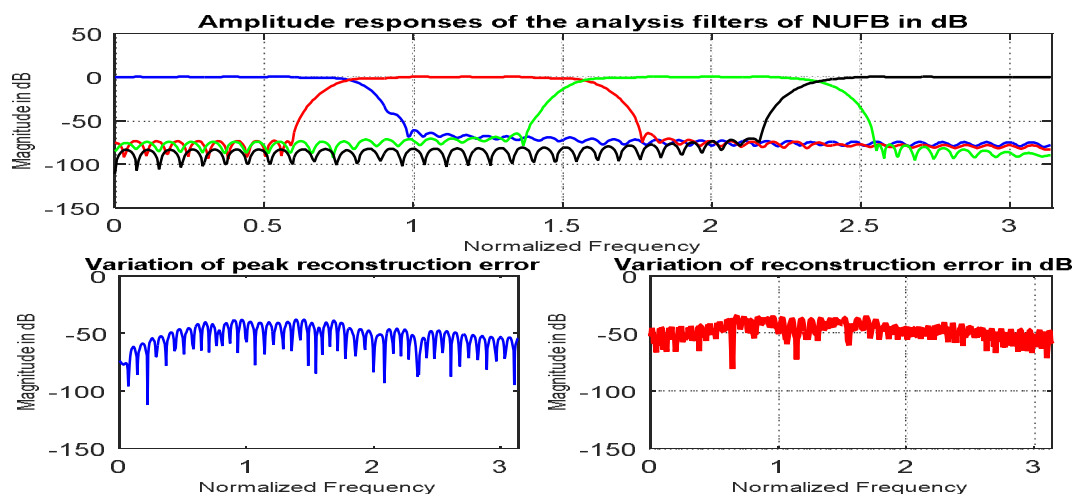
Fig(e): Two Channel Three Stage Output(4,4,2)



Fig(f): Two Channel Four Stage Output (2,4,8,8)



Fig(g): Two Channel Four Stage Output(8,8,4,2)



Fig(f): Two Channel Four Stage Output (4,4,4,4)

A. Comparison Table of PRE

METHOD	[2,2]	[2,4,4]	[4,4,2]	[2,4,8,8]	[8,8,4,2]	[3,3,3]	[4,4,4,4]
FIR2 N=84	0.09818	0.081258	0.084826	0.019581	0.024393	0.080041	0.077052
FIRLS N=84	0.085728	0.07921	0.083293	0.017988	0.022187	0.086353	0.080096
FIRPM N=84	0.085651	0.078827	0.083482	0.018055	0.02223	0.085993	0.080351
FIRCBA N=85	0.076236	0.071293	0.071927	-----	-----	0.070788	0.072179
FIRGR N=85	0.076236	0.071293	0.071927	-----	-----	0.070788	0.072179

V. CONCLUSION

The concept and design of perfect-reconstruction uniform-band QMF banks have been studied. In this paper we propose a simple design methods for nonuniform filter banks. Energy associated with the particular region should be calculated using this method. The approach is to transform the nonuniform filter bank design problem to a corresponding uniform filter bank design problem. The analysis and synthesis filters are simple combinations of finite impulse response modulated versions of the prototype filter.

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