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Certain Transformations Involving q-Series

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Abstract : In this paper, certain transformations for basic hypergeometric series have been established by making use of a known identity.
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I. INTRODUCTION

In the following identity $\sum_{m=0}^{n}\delta_m\sum_{r=0}^{m}\alpha_r=\sum_{r=0}^{n}\alpha_r\sum_{m=0}^{n}\delta m-\sum_{r=0}^{n-1}\alpha_{r+1}\sum_{m=0}^{r}\delta m$ (1.1)

If we take $\alpha_j=z^j$, (1.1) takes the following form :

$$\sum_{m=0}^{n}\delta_m z^m=\sum_{m=0}^{n}\delta_m+(1-z)\sum_{r=0}^{n-1}z^r\sum_{m=0}^{r}\delta m \tag{1.2}$$

II. DEFINITIONS AND NOTATIONS

For real or complex $q(|q|<1)$, we get

$$(\lambda;q)_\infty=\prod_{j=0}^{\infty}(1-\alpha q^j) \tag{2.1}$$

and let $(\lambda;q)_\mu$ be defined by

$$(\lambda;q)_\mu=\frac{(\lambda;q)_\infty}{(\lambda q^\mu;q)_\infty} \tag{2.2}$$

For arbitrary parameters λ and μ, so that

$$(\lambda;q)_n=\begin{cases}1\ , & n=0\\ (1-\lambda)(1-\lambda q)......(1-\lambda q^{n-1}), & n\in(1,2,3..)\end{cases} \tag{2.3}$$

A truncated basic hypergeometric series is defined as

$${}_r\Phi_s\left[\begin{matrix}a_1,a_2,.....a_r;q;z\\ b_1,b_2....b_s\end{matrix}\right]_N=\sum_{n=0}^{N}\frac{\prod_{i=1}^{r}[a_i;q]_n z^n}{\prod_{j=1}^{s}[b_j;q]_n(q;q)_n} \tag{2.4}$$

Where $|q|<1,|z|<1$ and no zero appear in the denominator.

The truncated polybasic hypergeometric series of one variable is defined as

$${}_r\Phi_s\left[\begin{matrix}a_1,a_2,.....a_r;c_{1,1},.....c_{m,1};...c_{m,r_m};q,q_1,....q_m;z\\ b_1,b_2....b_s:d_{1,1},......,d_{1s_1};....;d_{ms_1};...d_{ms_m}:\end{matrix}\right]_N=\sum_{n=0}^{N}\frac{[a_1,a_2,.....a_r;q]_n z^n}{[q,b_1,b_2,.....b_s;q]_n}\prod_{j=1}^{m}\frac{[c_{j,1},.....a_r;q_j]_n}{[d_{j,1},.....d_{j,s_1};q_j]_n} \tag{2.5}$$

The series (2.5) converges for $|q||q_1|,.....|q_m|<1,|z|<1$.

The other notations appearing in this paper shall stand for their usual meaning.

We shall use the following summations of truncated series in our analysis

$${}_2\Phi_1\left[\begin{matrix}a,y;q\\ ayq\end{matrix}\right]_n=\frac{[aq,yq;q]_n}{[q,ayq;q]_n} \tag{2.6}$$

$$ {}_4\Phi_3\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, e; q; \frac{1}{e} \\ \sqrt{a}, -\sqrt{a}, \quad \frac{aq}{e} \end{matrix}\right]_n = \frac{[aq, eq; q]_n}{[q, \frac{aq}{e}; q]_n e^n} \tag{2.7} $$

$$ {}_6\Phi_5\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, d; q; q \\ \sqrt{a}, -\sqrt{a}, \quad \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d} \end{matrix}\right]_n = \frac{[aq, bq, cq, dq; q]_n}{[q, \frac{aq}{b}, \frac{aq}{c}, \frac{aq}{d}; q]_n} \tag{2.8} $$

Provided a= bcd.

$$ \sum_{k=0}^{n} \frac{(1-ap^k q^k)[a;p]_k [c;q]_k c^{-k}}{(1-a)[q;q]_k \left[\frac{ap}{c}; p\right]_k} = \frac{[ap;p]_n [cq;q]_n c^{-n}}{[q;q]_n [ap/c;p]_n} \tag{2.9} $$

$$ \sum_{k=0}^{n} \frac{(1-ap^k q^k)(1-bp^k q^{-k})[a,b;p]_k \left[c, \frac{a}{bc}; q\right]_k q^k}{(1-a)(1-b)\left[q, \frac{aq}{b}; q\right]_k \left[\frac{ap}{c}, bcp; p\right]_k} = \frac{[ap, bp; p]_n \left[cq, \frac{aq}{bc}; q\right]_n}{\left[q, \frac{aq}{b}; q\right]_n \left[\frac{ap}{c}, bcp; p\right]_n} \tag{2.10} $$

$$ \sum_{k=0}^{n} \frac{(1-adp^k q^k)(1-\frac{b}{d}p^k q^{-k})[a,b;p]_k \left[c, \frac{ad^2}{bc}; q\right]_k q^k}{(1-ad)(1-\frac{b}{d})\left[dq, \frac{adq}{b}; q\right]_k \left[\frac{adp}{c}, \frac{bcp}{d}; p\right]_k} $$

$$ = \frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{d(1-ad)\left(1-\frac{b}{d}\right)\left(1-\frac{c}{d}\right)\left(1-\frac{ad}{bc}\right)} \times \left[\frac{[ap, bp; p]_n \left[cq, \frac{ad^2 q}{bc}; q\right]_n}{\left[dq, \frac{adq}{b}; q\right]_n \left[\frac{adp}{c}, \frac{bcp}{d}; p\right]_n} - \frac{(b-ad)(c-ad)(d-bc)(1-d)}{d(1-a)(1-b)(1-c)(bc-ad^2)} \right] \tag{2.11} $$

Which is m=0 case of [Gasper and Rahman 3; App. II (II.36)]

$$ \sum_{k=0}^{n} \frac{(1-adp^k q^k P^k Q^k)\left(1-\frac{d}{c}\frac{P^k Q^k}{p^k q^{-k}}\right)\left(1-\frac{b}{d}\frac{p^k P^k}{q^k Q^k}\right)\left(1-\frac{ad}{bc}\frac{p^k Q^k}{q^k P^k}\right)}{(1-ad)\left(1-\frac{d}{c}\right)\left(1-\frac{b}{d}\right)\left(1-\frac{ad}{bc}\right)} $$

$$ \times \frac{[a, p^2]_k [c, q^2]_k [b, P^2]_k \left[\frac{ad}{bc}; Q^2\right]_k q^{2k}}{\left[d\frac{qPQ}{p}; \frac{qPQ}{p}\right]_k \left[\frac{ad}{c}\frac{pPQ}{q}; \frac{pPQ}{q}\right]_k \left[\frac{ad}{b}\frac{pqQ}{P}; \frac{pqQ}{P}\right]_k \left[\frac{bc}{d}\frac{pqP}{Q}; \frac{pqP}{Q}\right]_k} $$

$$ = \frac{(1-a)(1-b)(1-c)\left(1-\frac{ad^2}{bc}\right)}{(1-ad)(c-d)\left(1-\frac{b}{d}\right)\left(1-\frac{ad}{bc}\right)} $$

$$ \times \left[\frac{[ap^2; p^2]_n [cq^2; q^2]_n [bP^2; P^2]_n \left[\frac{ad^2}{bc} Q^2; Q^2\right]_n}{\left[d\frac{qPQ}{p}; \frac{qPQ}{p}\right]_n \left[\frac{ad}{c}\frac{pPQ}{q}; \frac{pPQ}{q}\right]_k \left[\frac{ad}{b}\frac{pqQ}{P}; \frac{pqQ}{P}\right]_k} \frac{1}{\left[\frac{bc}{d}\frac{pqP}{Q}; \frac{pqP}{Q}\right]_k} \times -\frac{(b-ad)(c-ad)(d-bc)(1-d)}{d(1-a)(1-b)(1-c)(bc-ad^2)} \right] $$

(2.12)

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Which is m = 0 case of [Agarwal et. al.[4] ; (18) p. (89)]

III.MAIN RESULTS

If we take $\delta_m = \dfrac{[a,y;q]_m q^m}{[q,ayq;q]_m q^m}$ in (1.2) and make use of (2.6) we get :

$$ {}_2\Phi_1\left[\begin{matrix} a,y;q;qz \\ ayq \end{matrix}\right]_n = \frac{z^n[aq,yq;q]_n}{[q,ayq;q]_n} + (1-z)\,{}_2\Phi_1\left[\begin{matrix} aq,yq;q;z \\ ayq \end{matrix}\right]_{n-1} \quad , \quad |z|<1 \tag{3.1} $$

As $n \to \infty$ (3.1) yields the following transformation

\

$$ {}_2\Phi_1\left[\begin{matrix} a,y;q;qz \\ ayq \end{matrix}\right] = (1-z)\,{}_2\Phi_1\left[\begin{matrix} aq,yq;q;z \\ ayq \end{matrix}\right] , \quad |z|<1 \tag{3.2} $$

(i) Taking $\delta_m = \dfrac{\left[a,q\sqrt{a},-q\sqrt{a};e;q\right]_m}{[q,\sqrt{a},-\sqrt{a},aq/e;q]_m e^m}$ in (1.2) and making use of (2.7) we get

$$ {}_4\Phi_3\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},e;z/e \\ \sqrt{a},-\sqrt{a} \quad aq/e \end{matrix}\right]_n = \frac{z^n[aq,eq;q]_n}{e^n[e,aq/e;q]_n} + (1-z)\,{}_2\Phi_1\left[\begin{matrix} aq,eq,q;q;z/e \\ e \quad aq/e \end{matrix}\right]_{n-1} \tag{3.3} $$

For $|z|<1$ and $|e|>1$ (1.1) yields the following transformation, when $n\to\infty$,

$$ {}_4\Phi_3\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},e;z/e \\ \sqrt{a},-\sqrt{a} \quad aq/e \end{matrix}\right] = (1-z)\,{}_2\Phi_1\left[\begin{matrix} aq,eq,q;q;z/e \\ e \quad aq/e \end{matrix}\right] \tag{3.4} $$

(ii) Taking $\delta_m = \dfrac{\left[a,q\sqrt{a},-q\sqrt{a},b,c,a/bc;q\right]_m}{[q,\sqrt{a},-\sqrt{a},aq/b,aq/c,bcq;q]_m}$ in (1.2) and making use of (2.8) we get

$$ {}_6\Phi_5\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},b,c,a/bc;q;zq \\ q,\sqrt{a},-\sqrt{a} \quad aq/b,aq/c,bcq \end{matrix}\right]_n = \frac{[aq,bq,cq,aq/bc;q;z]_n z^n}{[q,aq/b,aq/c,bcq;q]_n} + (1-z)\,{}_2\Phi_1\left[\begin{matrix} aq,bq,cq,aq/bc;q;z \\ aq/b,aq/c,bcq \end{matrix}\right]_{n+1} ,|z|<1 \tag{3.5} $$

As $n\to\infty$, (3.5) yield the following transformation

$$ {}_6\Phi_5\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},b,c,a/bc;q;zq \\ q,\sqrt{a},-\sqrt{a} \quad aq/b,aq/c,bcq \end{matrix}\right] = (1-z)\,{}_4\Phi_3\left[\begin{matrix} aq,bq,cq,aq/bc;q;z \\ aq/b,aq/c,bcq \end{matrix}\right] \tag{3.6} $$

(iii) Taking $\delta_m = \dfrac{[a,pq;pq]_m[a;p]_m[c;q]_m c^{-m}}{[a;pq]_m[a;q]_m[ap/c;p]_m}$ in (1.2) and making use of (2.9) we get

$$ {}_3\Phi_2\left[\begin{matrix} c;a;apq;q,p,pq,z/c \\ -;ap/c;a; \end{matrix}\right]_n = \frac{z^n\,[ap;p]_n\,[cq;q]_n}{c^n\,[q;q]_n\,[ap/c;p]_n}(1-z)\,{}_2\Phi_1\left[\begin{matrix} cq,ap;p;z/c \\ -;ap/c \end{matrix}\right]_{n-1} \tag{3.7} $$

For $|z|<1$ and $|c|>1$ and $n\to\infty$, (3.7) yields

$$ {}_3\Phi_2\left[\begin{matrix} c;a;apq;q,p,pq,z/c \\ -;ap/c;a; \end{matrix}\right] = (1-z)\,{}_2\Phi_1\left[\begin{matrix} cq,ap;p;z/c \\ -;ap/c \end{matrix}\right] \tag{3.8} $$

(iv) Taking $\delta_m = \dfrac{[apq;pq]_m[bp/q;p/q]_m[a,b;p]_m[c,a/bc;q]_m q^m}{[a;pq]_m[b;p/q]_m[q,aq/b;q]_m[ap/c,bcp;p]_m}$ in (1.2) and making use (2.10), we get

$$ {}_6\Phi_5\left[\begin{matrix} c;aq/bc;ap,bp;q,p,pq,z \\ aq/b;ap/c,bcp; \end{matrix}\right]_n = \frac{[ap,bp;p]_n[cq,aq/bc;q]_n z^n}{[q,aq/b;q]_n\,[ap/c,bcp;p]_n} $$

$$+(1-z)\,{}_4\Phi_3\left[\begin{matrix} cq, aq/bc; ap, bp; q, p; z \\ ap/c; ap/c, bcp \end{matrix}\right]_{n-1},\quad |z|<1 \qquad (3.9)$$

As $n\to\infty$, (3.9) yields the following transformations

$${}_6\Phi_5\left[\begin{matrix} c; aq/bc; ap, bp; q, p, pq, z \\ aq/b; ap/c, bcp; \end{matrix}\right] = (1-z)\,{}_4\Phi_3\left[\begin{matrix} cq, aq/bc; ap, bp; q, p; z \\ ap/c; ap/c, bcp \end{matrix}\right] \qquad (3.10)$$

(v) Taking $\delta_m = \dfrac{[adpq;pq]_m \left[\frac{b}{d}\frac{p}{q};\frac{p}{q}\right]_m [a,b;p]_m \left[c,\frac{ad^2}{bc};q\right]_m q^m}{[a;pq]_m \left[\frac{b}{d};\frac{p}{q}\right]_m [q,aq/b;q;q]_m \left[\frac{adp}{c},\frac{bcp}{d};p\right]_m}$

In (1.2) and making use of (2.11)

$${}_7\Phi_6\left[\begin{matrix} c\ ; ad^2/bc, q; a, b; adpq; b/dq; q, p, pq, p/q; zq \\ dq, adq/b; adp/c, bcp/d; ad; b/d \end{matrix}\right]_n$$

$$= \frac{z^n(1-a)(1-b)(1-c)(1-ad^2/bc)[ap,bp;p]_n[cq,ad^2q;q]_n}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)[dq,adq/b;q]_n[adp/c;bcp/d;p]_n}$$

$$-\frac{(1-ad)(c-ad)(d-bc)(1-d)}{bcd^2(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}$$

$$+\ (1-z)\frac{(1-a)(1-b)(1-c)(1-ad^2/bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}\times{}_5\Phi_4\left[\begin{matrix} cq; ad^2q/bc; q; ap, bp; q, p; z \\ dq, adq/b, adp/c, bcp/d \end{matrix}\right]_{n-1},\ |z|<1 \qquad (3.11)$$

As $n\to\infty$,(3.11) yields

$${}_7\Phi_6\left[\begin{matrix} c\ ; ad^2/bc, q; a, b; adpq; b/dq; q, p, pq, p/q; zq \\ dq, adq/b; adp/c, bcp/d; ad; b/d \end{matrix}\right]$$

$$= (1-z)\frac{(1-a)(1-b)(1-c)(1-ad^2/bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)}\times{}_5\Phi_4\left[\begin{matrix} cq; ad^2q/bc; q; ap, bp; q, p; z \\ dq, adq/b, adp/c, bcp/d \end{matrix}\right]$$

$$-\frac{(1-ad)(c-ad)(d-bc)(1-d)}{bcd^2(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \qquad (3.12)$$

IV. CONCLUSION

In this paper, an attempt has been made to establish six certain transformation formulae for basic hypergeometric series by making use of the identity (1.2)

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