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# Estimation of Delay and Doppler shift of Weighted OFDM of Radar Targets

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**Abstract:** In this paper, Delay and Doppler shift estimation of moving targets of OFDM based radar system by utilizing Maximum Likelihood Estimation(MLE) strategy has been produced. Taking benefits of OFDM waveform, the range Doppler coupling issues can be overcome for radar applications and complex balance filter is never again fundamentally used to adapt to frequency selective fading channel in view of multipath. Here Weighted Orthogonal Frequency Division Multiplexing (WOFDM) system is utilized, and the Delay and Doppler shift estimation of WOFDM is compared with Constant Envelope OFDM (CEOFDM). The comparing Cramer-Rao limits (CRB) for the parameters are derived. Weighted OFDM waveforms are planned with subject to limitations on peak to average power ratio (PAPR). For the delay estimation, The proposed WOFDM modulation have bring down CRB esteem compared with the great constant envelope OFDM regulation while meeting the prerequisite on the PAPR level.

**Keywords:** OFDM, Cramer-Rao bounds, WOFDM, CEOFDM, PAPR, SNR

## I. INTRODUCTION

The growing technologies in communication has lead to spectrum scarcity problem this becomes main problem due to interference which leads to operation capability limitations the use of spectrum sharing is to make S band and WiFi system to share the same spectrum generally a fixed spectrum is allocated to mitigate interference for communication system and radar systems but due to huge growth in cellular communication system for commercial applications more bands are allocated this leads to difficulty in fixed spectrum allocation scheme. Until the recent improvement in developing of Time Division Duplex transceiver architecture was proposed there is no hardware platform demonstrated. In this method one time period is made in to four equal time slots. To estimate the Doppler shift and range a technique called trapezoidal frequency modulation continuous wave is transmitted and for this, the first three time slots are used. For enabling the transmission of communication signal and its processing the last time slot is allocated. The interference between communication and radar is eliminated by this time domain method, and then the transmission of data which uses radio technique and estimation of parameter. Which uses radar technique can separately implemented. However high data rate transmission in wireless communication cannot be satisfied by time division duplex waveforms. Reconfigurable platforms, such as single RF wireless platform [1] is an alternative approach to get spectrum sharing which can realize radar and communication functions. We have to design a dual-used modulation waveform for both communication and radar because their primary goals are different as radar is sensing devices which is based on transmitting waveforms and detecting target and estimating it and tracking so to achieve a high resolution but communication system is different because modulation and demodulation will accomplish the transfer of information bits. We can use many modulation schemes for both radar and communication on a same radio frequency platform. In the radar, orthogonal frequency division multiplexing and its variations are used. OFDM modulation has two main advantages. Primary is limited hardware is used for both radar detection and communication by reusing the waveform of OFDM. Another advantage is OFDM has robustness to the multipath fading [2]. Generally for hardware realization constant envelope including phase modulation is used. However the waveform of constant envelope OFDM offers greater peak to average power ratio. The subcarrier frequencies are taken as Upper band [3]. So we can limit the estimation performance by individual orthogonal frequency division multiplexing symbols. To estimate delay and Doppler shift of target in this paper we represent a non-linear least squares method which is based on weighted OFDM scheme. The WOFDM is non-constant envelope based method. In WOFDM the weights of symbols are made by optimization of the errors bound of estimator which is subjected to PAPR and energy of total transmission.

## II. OFDM-RADAR SIGNAL MODELS

We consider an OFDM signaling system with  $N$  subcarriers. Each subcarrier is modulated with a data symbol. The transmitted time domain complex envelop signal in the  $m$ -th pulse ( $m = 1, \dots, M$ ) is written as

$$s_m(t) = \sum_{n=0}^{N-1} a_{m,n} e^{j2\pi f_n t} I_m(t, T_{PRI}, T_p) \quad (1)$$

where the  $m$ -th pulse is a rectangular function defined by

$$I_m(t, T_{PRI}, T_p) = \begin{cases} 1 & mT_{PRI} \leq t \leq mT_{PRI} + T_p \\ 0 & mT_{PRI} + T_p < t < (m+1)T_{PRI} \end{cases} \quad (2)$$

The symbol  $a_{m,n}$  represents the complex weights transmitted over the  $n$ -th subcarrier of the  $m$ -th pulse.  $T_{PRI}$  is the pulse repetition interval.  $T_p$  is the OFDM symbol duration. The spacing between the subcarriers is represented as  $\Delta f = 1/T_p = B/N$ , where  $B$  is the bandwidth of the signal. The individual subcarrier frequency is

$$f_n = (f_c - \frac{B}{2}) + n\Delta f, \quad n = 0, 1, \dots, N-1 \quad (3)$$

where  $f_c$  denotes the carrier frequency of the radar. It is in the case of reflecting target is presented at distance  $R$  with relative velocity  $v$ , the corresponding Doppler shift is  $\beta = 2v/c$ . Hence, the induced Doppler frequency at the  $n$ -th subcarrier is

$$f_d = f_n \beta = (f_c + \frac{n}{T_p}) \frac{2v}{c} \quad (4)$$

We assume that the  $f_c \gg B$ , i.e., the carrier frequency is practically larger than the signal bandwidth. Hence it is legal to had the appearance of that the Doppler frequency is a constant by all of respect to the sub-carriers, i.e., the average Doppler frequency is

$$f_d \approx f_c \frac{2v}{c} \quad (5)$$

Incorporating Doppler shift  $f_d$  and the round trip delay  $\tau$  between the radar and the signal is

$$x_m(t) = A \sum_{n=0}^{N-1} a_{m,n} e^{j2\pi(f_n + f_d)(t-\tau)} I_m(t, T_{PRI}, T_p) + w_m(t) \quad (6)$$

where  $A$  is the target reflection coefficient. In general, the parameter  $A$  is complex and varies with different subcarriers. For simplicity, here we assume that the coherence bandwidth of the target channel response is greater than the modulation bandwidth  $B$ , by means of this  $A$  is approximately a deterministic constant.  $w_m(t)$  is the clutter and measurement noise, following a zero mean complex Gaussian distribution. Inserting (3) and (5) to (6), we obtain

$$x_m = x_m^s(t) I_m(t, T_{PRI}, T_p) + w_m(t) \quad (7)$$

where the signal component is

$$x_m^s(t) = \sum_{n=0}^{N-1} a_{m,n} e^{j2\pi(f_n + n\Delta f)(t-\tau)} e^{j2\pi f_d(t-\tau)} \quad (8)$$

The corresponding baseband signal is

$$y_m(t) = x_m^s(t) I_m(t, T_{PRI}, T_P) e^{-j2\pi f_c t} + w_m(t) \quad (9)$$

$$= \sum_{n=0}^{N-1} A a_{m,n} e^{j2\pi(n\Delta f)(t-\tau)-f_c\tau} e^{j2\pi f_d(t-\tau)} I_m(t, T_{PRI}, T_P) + w_m(t) \quad (10)$$

Note that, for an OFDM system, the sampling interval  $T_s$  is typically chosen such that  $\Delta f T_s = 1/N$ . Hence, the sampled version of  $y_m(l)$  with  $(l = 0, \dots, N-1)$  becomes

$$y_m(l) = A \sum_{n=0}^{N-1} a_{m,n} e^{j2\pi(n\Delta f)(lT_s + mT_{PRI} - \tau) - f_c\tau} e^{j2\pi f_d(lT_s + mT_{PRI} - \tau)} + w_m(l) \quad (11)$$

$$= A \sum_{n=0}^{N-1} a_{m,n} e^{j2\pi \frac{n}{N} l} e^{-j2\pi(n\Delta f + f_c)\tau} e^{j2\pi f_d(T_s(l + mQN) - \tau)} + w_m(l) \quad (12)$$

where  $Q = T_{PRI}/T_P$ . Next, we define

$$y_{m,n}^s \triangleq A a_{m,n} e^{-j2\pi(n\Delta f + f_c)\tau} e^{j2\pi f_d(T_s mQN)} \quad (13)$$

By the definition of inverse DFT and using  $T_s = \frac{1}{N\Delta f}$ , we re-write (12) as

$$y_m(l) = N \{ IDFT[Y_{m,n}^s] \} e^{j2\pi \frac{f_d}{\Delta f} \frac{l}{N}} e^{-j2\pi f_d \tau} + w_m(l) \quad (14)$$

where the term  $e^{j2\pi \frac{f_d}{\Delta f} \frac{l}{N}}$  represents the modulation in time domain. Hence the DFT of  $y_m(l)$  has a corresponding frequency shift  $\frac{f_d}{\Delta f}$ . Thus, applying DFT to (14) yields

$$Y'_{m,n} = N Y_{m,n - \frac{f_d}{\Delta f}}^s e^{-j2\pi f_d \tau} + W'_{m,n} \quad (15)$$

where  $W'_{m,n}$  is the DFT of the noise term  $w_m(l)$ . In radar parameter estimation, we consider a that  $f_d \leq \Delta f$ , thus the only term specifically influenced by modulation is  $e^{-j2\pi(n\Delta f + f_c)\tau}$ . By re-writing  $n = n - \frac{f_d}{\Delta f}$ , we obtain

$$Y'_{m,n} = NAa_{m,n} e^{-j2\pi((n-\frac{f_d}{\Delta f})\Delta f + f_c)\tau} e^{j2\pi f_d(T_s m QN)} e^{-j2\pi f_d \tau} + W'_{m,n} \quad (16)$$

$$= A' a_{m,n} e^{j2\pi(mf_d T_s QN - n\Delta f \tau)} + W'_{m,n} \quad (17)$$

where  $A' = AN e^{-j2\pi f_c \tau}$  and is invariant term respect to  $m$  and  $n$ . In order to remove the impact of OFDM symbols  $a_{m,n}$ , one approach is to divide the known OFDM symbols on both sides, which yields

$$Y_{m,n} = A' e^{j2\pi(mf_d T_s QN - n\Delta f \tau)} + W_{m,n} \quad (18)$$

where  $Y_{m,n} = Y'_{m,n} a_{m,n}$  and  $W_{m,n} = W'_{m,n} a_{m,n}$ . Hence, the problem of interest is to estimate delay  $\tau$  and Doppler  $f_d$  from the exponential terms  $e^{j2\pi n\Delta f \tau}$  and  $e^{j2\pi m f_d T_s QN}$  of the discrete data  $Y = \{Y_{m,n}\}, m = 1, \dots, M; n = 1, \dots, N$ , respectively.

### III. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

We consider  $A' = be^{j\phi_A}$  be the target response with  $b$  and  $\phi_A$  are representing the magnitude and phase of target response. The parameter vector of interest is

$$\theta = [b, \phi_A, f_d, \tau] \quad (19)$$

From the above signal parameter vector model (18), the nonlinear least-squares error function is,

$$L_n(Y; \theta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |W_{m,n}^a|^2 \quad (20)$$

$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |Y_{m,n} - be^{j\phi_A} e^{j2\pi(mf_d T_s QN - n\Delta f \tau)}|^2$$

where  $M$  is the no of pulses in the coherent processing interval. Hence, we get

$$L_n(Y; \theta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (|Y_{m,n}|^2 + b^2 - 2\Re\{Y_{m,n}^* be^{j\phi_A} e^{j2\pi(mf_d T_s QN - n\Delta f \tau)}\}) \quad (21)$$

The parameters  $\theta$  can be determined by minimizing the error function, i.e.,

$$\text{find } \hat{\theta} = \arg \min_{\theta} L_n(Y; \theta) \quad (22)$$

Since the vector  $\theta$  contains four parameters, a sequence of optimization steps are taken. We start by, the error function is partial derivated with respect to the unknown phase  $\phi_A$ , which yields

$$\frac{\partial L_n(Y; \theta)}{\partial \phi_A} = 2b \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} (2\Re\{-jY_{m,n} be^{-j\phi_A} e^{-j2\pi(mf_d T_s QN - n\Delta f \tau)}\}) = 0 \quad (23)$$

Note that the 2-D discrete time Fourier transform of  $Y_{m,n}$  is

$$Z(f_d, \tau) = \sum_{m=0}^{M-1} \left( \sum_{n=0}^{N-1} Y_{m,n} e^{j2\pi n \Delta f \tau} \right) e^{-j2\pi m f_d T_s Q N} \quad (24)$$

Inserting (24) in (23) we obtain

$$\Re\{-jbe^{-j\phi_A} Z(f_d, \tau)\} = 0 \quad (25)$$

Since  $b$  and  $|Z(f_d, \tau)|$  are positive real values, only the real component of the phase term must be zero. Hence we have

$$\cos\left(\frac{\pi}{2} + \phi_A - \angle Z(f_d, \tau)\right) = 0 \quad (26)$$

This is a simple trigonometric equation and the solution is

$$\phi_A = k\pi + \angle Z(f_d, \tau) \quad (27)$$

where  $k$  is an integer. Using this estimate equation (27) in the error function (21) and avoiding the terms which are independent of  $b$ , we obtain

$$L_n(Y; \hat{\phi}_A, b, f_d, \tau) = 2b \Re\{e^{-\angle Z(f_d, \tau)} Z(f_d, \tau)\} - MNb^2 \quad (28)$$

which simplifies to

$$L_n(Y; \hat{\phi}_A, b, f_d, \tau) = 2b|Z(f_d, \tau)| - MNb^2 \quad (29)$$

By equating the partial derivative with respect to  $b$  to zero yields

$$\hat{b} = \frac{|Z(f_d, \tau)|}{MN} \quad (30)$$

Inserting the estimate (30) in (29) we get

$$L_n(Y; \hat{\phi}_A, b, f_d, \tau) = \frac{|Z(f_d, \tau)|^2}{MN} \quad (31)$$

which is a scaled the periodogram of the 2D-DTFT in (24). Hence, we obtain the final optimization problem for unknown delay and doppler frequency parameters as

$$\text{find } \hat{f}_d, \hat{\tau} = \arg \max_{f_d, \tau} \frac{|Z(f_d, \tau)|^2}{MN} \quad (32)$$

The solution to (32) can be determined by finding the peak of un-windowed two dimensional (2D) periodogram of the signal  $Y_{m,n}$ . This estimation can be accomplished by a two-dimensional search [3].

#### IV. CRAMER-RAO LOWER BOUNDS

Note that if  $w_{m,n}$  in is a Gaussian random process,  $\hat{\theta}$  is a large sample realization of the maximum-likelihood estimator of  $\theta$ . Since the maximum-likelihood estimator is required to accomplish the Cramer-Rao Low Bound (CRLB) as  $M$  or  $N$  increases, it takes after that under the Gaussian assumption no other estimator could perform better in large samples than  $\hat{\theta}$ . In this segment, we determine the Cramer-Rao lower bound of  $\hat{\theta}$ . The probability model of the measurements  $\{Y_{m,n}\}$  is

$$P(Y|\theta) = \prod_{m=0}^{M-1} \prod_{n=0}^{N-1} \frac{|a_{m,n}|^2}{\pi \sigma_{\omega}^2} \exp \left( -\frac{|a_{m,n}|^2}{\pi \sigma_{\omega}^2} |Y_{m,n} - b e^{j\phi_A} e^{j2\pi(mf_d T_s QN - n\Delta f \tau)}|^2 \right) \quad (33)$$

The log-likelihood function is

$$l(Y|\theta) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left( |Y_{m,n}|^2 + b^2 |a_{m,n}|^2 - 2 \Re \{ Y_{m,n}^* b e^{j\phi_A} a_{m,n} e^{j2\pi(mf_d T_s QN - n\Delta f \tau)} \} \right) \quad (34)$$

The Cramer-Rao limits for the evaluating parameter  $\hat{\theta}_i$  are given by the diagonal components of inverse of the  $4 \times 4$  Fisher information J [5],

$$\text{var}(\hat{\theta}_i) \geq [J^{-1}]_{ii} \quad (35)$$

where  $\theta_i$  are the four scalar parameters in the vector  $\theta$  in (19) with  $i = 1, 2, 3, 4$ . Due to space limitations, The determinant of Fisher information matrix J is

$$\det(J) = \frac{256b^6}{\sigma_{\omega}^8} (Ma\pi^2 T_s QN \Delta f)^2 (MQ_M - P_M^2)(aQ_N - P_N^2) \quad (36)$$

where

$$\begin{aligned} P_M &= \sum_{m=0}^{M-1} m = M(M-1)/2 \\ Q_M &= \sum_{m=0}^{M-1} m^2 = M(M-1)(2M-1)/6 \\ Q_N &= \sum_{n=0}^{N-1} n^2 |a_{m,n}|^2 \\ P_N &= \sum_{n=0}^{N-1} n |a_{m,n}|^2 \end{aligned}$$

The Cramer-Rao bound of the estimate of  $b$  is the first element along the diagonal of  $J^{-1}$  given by

$$\text{CRB}_b = \frac{1}{\det(J)} (MQ_M - P_M^2)(Q_N - P_N^2) \frac{128b^6}{\sigma_{\omega}^6} Ma\pi^4 (T_s QN)^2 (\Delta f)^2 \quad (37)$$

The diagonal elements of the  $J^{-1}$  are evaluated using the adjoint method. Next, using (36) and upon simplification we obtain

$\text{CRB}_{\phi_A} = \frac{\sigma_{\omega}^6}{2M_a}$ . The CRLB on  $\phi_A$  is the second element along the diagonal of  $J^{-1}$  and is obtained as

$$\text{CRB}_{\phi_A} = \frac{\sigma_{\omega}^6}{b^2} \frac{MaQ_M Q_N - P_M^2 P_N^2}{2M_a (MQ_M - P_M^2)(aQ_N - P_N^2)} (MQ_M - P_M^2)(Q_N - P_N^2) \quad (38)$$

The CRLB on  $f_d$  is the third component along the diagonal of  $J^{-1}$  given by

$$CRB_{f_d} = \frac{1}{\det(J)} \frac{8b^6}{\sigma_\omega^6} \frac{M^3 a}{b^2} (2\pi\Delta f)^2 (aQ_N - P_N^2) \quad (39)$$

Again upon simplification we obtain

$$CRB_{f_d} = \frac{M\sigma_\omega^2}{8ab^2\pi^2(T_s QN)^2(MQ_M - P_M^2)} \quad (40)$$

Finally, the CRLB on  $\tau$  is the forth element along the diagonal of  $J^{-1}$  defined as,

$$CRB_\tau = \frac{1}{\det(J)} \frac{8b^6}{\sigma_\omega^6} \frac{M^3 a}{b^2} 4\pi^2(T_s QN)^2(MQ_M - P_M^2) \quad (41)$$

Hence, upon simplification we obtain

$$CRB_\tau = \frac{a\sigma^2}{2(aQ_N - P_N^2)b^2M(2\pi\Delta f)^2} \quad (42)$$

## V. WEIGHTED OFDM SYMBOL DESIGN

There have been distinctive ways to deal with outline OFDM symbols. For example to enhance signal detection, eigen-value decomposition strategy is used for augmenting the non-centrality parameter of complex Wishart distributed segments of the test measurement keeping in mind the end goal to optimize waveform with a constraint on PAPR. In low-grazing angle tracking, a maximum common data approach is utilized for OFDM symbol design. Different methodologies incorporate limiting the contrast between the wideband ambiguity function and the idealized ambiguity function to enhance delay resolution. In this paper, we will likely plan weighted OFDM waveform balance to enhance the accuracy of the estimation of radar target parameters. Specifically, we consider the classic Cramer-Rao bound (CRB) on the accuracy of the maximum likelihood estimation. Note that the efficiency of classical methods such as ambiguity function based radar parameter estimator is also evaluated against the CRB. The go for weighted OFDM is to outline OFDM symbols to limit the inverse of  $\det(J)$  subject to proper constraints. ie

$$\text{Find} \quad a_{m,n} = \arg \min_{a_{m,n}} \frac{1}{aQ_N - P_N^2} \quad (43)$$

$$\text{s.t} \quad \begin{cases} (C1) & \min(|a_{m,n}|) > \eta_0 \max(|a_{m,n}|) \\ (C2) & PAPR < \eta_1 \\ (C3) & \sum_{n=0}^{N-1} |a_{m,n}|^2 \leq a \end{cases} \quad (44)$$

Condition (C1) is to impose a constraint on the lower bound of the cost function  $\frac{1}{aQ_N - P_N^2}$  by

$$\frac{\min(|a_{m,n}|)}{\max(|a_{m,n}|)} < \eta_0 \leq 1 \quad (45)$$

The purpose of this constraint is to avoid bandwidth loss. Otherwise, OFDM symbols  $a_{m,n}$  would allocate zero power in most of sub-bands  $n$ , result in bandwidth loss in the transmission signal. Thus, the range determination could be extremely debased. Condition (C2) is to set an upper bound  $\eta_1 \ll N$  on the peak to average power ratio (PAPR) defined by

$$\text{PAPR} = a^{-1} \max_{0 < t < T_p} |s_m(t)|^2 \quad (46)$$

since a high PAPR forces extreme weight on the transmitter because of restricted amplification range of the RF amplifier. For instance, when phase shift keying (PSK) is used, the upper bound on PAPR is  $N$ . Finally, Condition (C3) is the total power constraint. The improvement issue defined in (43) and (44) is understood numerically utilizing convex optimization by the dynamic set obliged nonlinear strategy [6].

#### A. Results

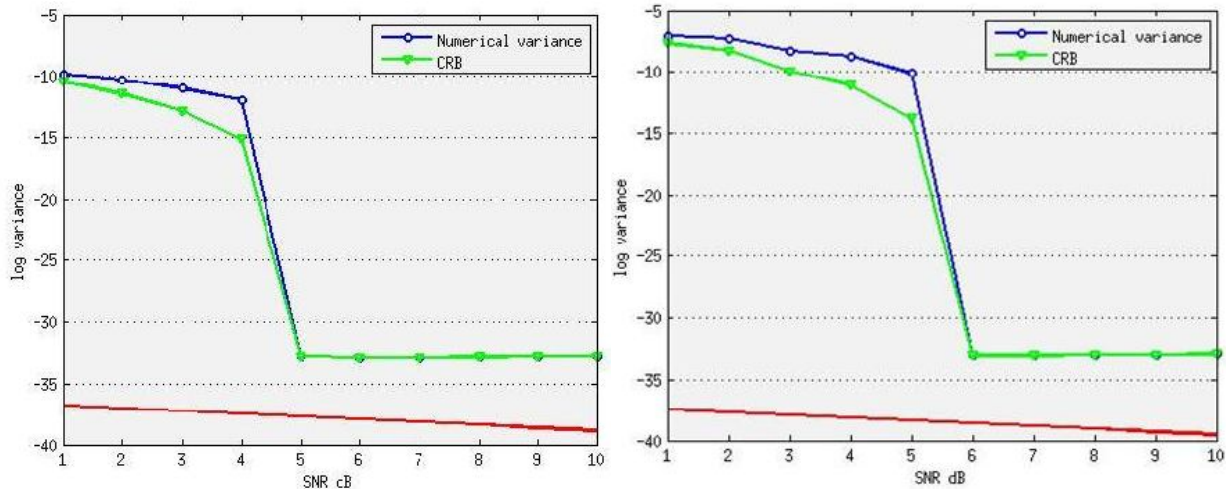


Figure1(a): Delay estimate for CEOFDM

(b) Delay estimate for WOFDM

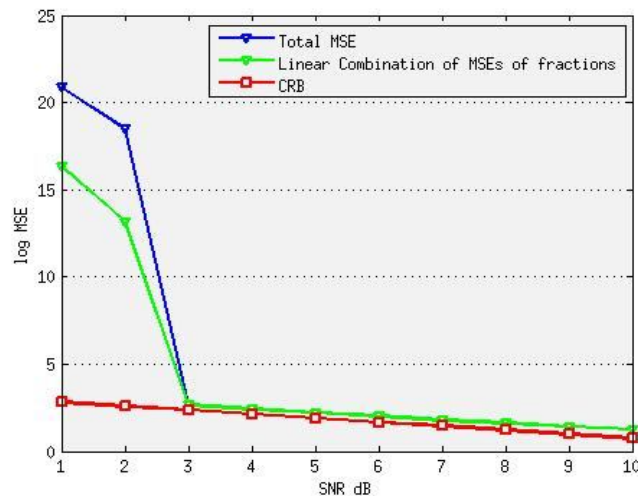


Fig2: Doppler frequency estimation

Figure1(a) and (b) depicts the Delay estimates with respect to CEOFDM and WOFDM modulations respectively. Theoretical CRLB and the numerical values for the delay and Doppler estimates are derived. WOFDM has smaller CRLB than CEOFDM. It is due to we decrease the estimation of the  $\max(|s_m(t)|)$  i.e., conveying it nearer to the mean estimation of  $|s_m(t)|$ . Consequently we accomplish a higher value for  $\det(J)$ , hence prompting a lower CRLB for delay estimate.

For Doppler estimate in Figure2, the CRLBs are the same for the two modulation schemes. Note that the variance plots of the parameter estimates by the WOFDM method closely follows the variance plots of CEOFDM after the SNR goes above 3 dB. There is usually a range of SNR in which the mean-squared error (MSE) rises rapidly as SNR decreases, but here we decreased the MSE value at low SNR say 3 dB, and at even low SNR, the computed SNR follows the CRLB closely.

## VI. CONCLUSION

In this paper we develop the maximum likelihood estimation method for radar target's Delay and Doppler shift using OFDM modulation. The Cramer-Rao bounds are derived for the estimators and the estimator accuracy reaches the CRB after the threshold SNR. Next, a weighted OFDM modulation based on minimizing the CRB is derived. The CRB for Doppler (i.e., velocity) estimate does not change with modulation, however, the CRB of delay (i.e., range) estimate is improved with the weighted symbols. The proposed WOFDM scheme provides a promising means to achieve co-existence between radar and communications via a reconfigurable RF platform

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