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Bit Error Rate Performance in Coherent Optical Systems

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Abstract: The probability of a transmitted bit error is generally termed as BER. It is calculated by the ratio of incorrectly transmitted bits to total transmitted bits. Regardless of the data destination, an optical transport system (OTS) must provide the predefined bit-error rate (BER) performance. Optical data links also face a tradeoff between optical power and unrepeated distance. Although bit error rates are typically on the order of 10^{-6} to 10^{-9}at 155 Mb/s, this is adequate for some applications such as voice and video transmission; recent experiments have shown that error rates as low as 10^{-12} can be obtained at 200 Mb/s using different protocols on free-space optical links. This paper provides enough background material for calculating the bit-error rate (BER) of coherent light wave systems. However, the BER, and hence the receiver sensitivity, depend on the modulation format as well as on the demodulation scheme used by the coherent receiver. This paper explains each system separately.

Key Words: Bit Error Rate, ASK, FSK, PSK, DPSK, SNR

To achieve a target BER regardless of the data destination, the future OTS should be able to adjust the Forward Error Correction strength according to the optical channel conditions. Data communication systems must maintain very low bit error rates, typically between10^{-12} and 10^{-15}, since the consequences of a single bit error can be very serious in a computer system; by contrast, background static in voice communications, such as cellular phones, can often be tolerated by the listener

1 Synchronous ASK Receivers

Consider first the case of heterodyne detection. The signal used by the decision circuit is given by Eq. (1(a)).

The resulting baseband signal is

$$I_d = \langle (I_f Cos(\omega_{IF} t)) \rangle = \frac{1}{2}\left(I_p\, Cos\emptyset + i_c\right) \qquad 1(a)$$

The phase generally varies randomly because of phase fluctuations associated with the transmitter laser and the local oscillator. The effect of phase fluctuations can be made negligible by using semiconductor lasers whose line width is a small fraction of the bit rate. Assuming this to be the case and setting =0 in Eq. (2), the decision signal is given by

$$I_d = \frac{1}{2}(I_p + i_c) \qquad 1(b)$$

Where $I_p \equiv 2R(P_s P_{LO})^{1/2}$ takes values I_1 or I_0 depending on whether a 1 or 0 bit is being detected. Consider the case $I_0 = 0$ in which no power is transmitted during the 0 bits. Except for the factor of $\frac{1}{2}$ in Eq. (1) the factor of $\frac{1}{2}$ does not affect the BER since both the signal and the noise are reduced by the same factor, leaving the SNR unchanged.

$$\text{BER} = \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \qquad (2)$$

Where Q can be written as

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} \approx \frac{I_1}{2_{\sigma_1}} = \frac{1}{2}(SNR)^{\frac{1}{2}} \qquad (3)$$

In relating Q to SNR, we used I_0 =0 and set$\sigma_1 \approx \sigma_0$. The latter approximation is justified for most coherent receivers whose noise is dominated by the shot noise induced by local-oscillator power and remains the same irrespective of the received signal power. The SNR of such receivers can be related to the number of photons received during each 1 bit by the simple relation SNR = 2η$N\,p$ [see Eq. (14)]. Equations (2) and (3) then provide the following expression for the BER:

$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}(\sqrt{\eta N_p/4})$ [ASK heterodyne] (4)

One can use the same method to calculate the BER in the case of ASK homodyne receivers. Equations (2) and (3) still remain applicable. However, the SNR is improved by 3 dB for the homodyne case, so that SNR = 4η*Np* and

$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}(\sqrt{\eta N_p/2})$ [ASK homodyne] (5)

Equations (4) and (5) can be used to calculate the receiver sensitivity at a specific BER. We can define the receiver sensitivity $\bar{P}_{rec}$ as the average received power required for realizing a BER of 10^{-9} or less. From Eqs. (2) and (3), BER = 10^{-9} when $Q \approx 6$ or when SNR = 144 (21.6 dB). For the ASK heterodyne case we can use Eq. (14) to relate SNR to $\bar{P}_{rec}$ if we note that $\bar{P}_{rec} = \frac{P_s}{2}$ simply because signal power is zero during the 0 bits. The result is

$\bar{P}_{rec} = \frac{2Q^2 h\nu f}{\eta} = \frac{72 h\nu\Delta f}{\eta}$ (6)

For the ASK homodyne case, $\bar{P}_{rec}$ is smaller by a factor of 2 because of the 3-dB homodyne-detection advantage. As an example, for a 1.55- μm ASK heterodyne receiver with η = 08 and Δ*f* = 1 GHz, the receiver sensitivity is about 12nW and reduces to 6nW if homodyne detection is used. The receiver sensitivity is often quoted in terms of the number of photons *Np* using Eqs. (4) and (5) as such a choice makes it independent of the receiver bandwidth and the operating wavelength. Furthermore, η is also set to 1 so that the sensitivity corresponds to an ideal photodetector. It is easy to verify that for realizing a BER of = 10^{-9}, *Np* should be 72 and 36 for the heterodyne and homodyne cases, respectively. It is important to remember that *Np* corresponds to the number of photons within a single 1 bit. The average number of photons per bit, $\bar{N}_p$, is reduced by a factor of 2 if we assume that 0 and 1 bits are equally likely to occur in a long bit sequence.

2 SYNCHRONOUS PSK RECEIVERS

Consider first the case of heterodyne detection. The signal at the decision circuit is given by Eq. (1(a)) or by

$I_d = \frac{1}{2}(I_p Cos\emptyset + i_c)$ (7)

The main difference from the ASK case is that *Ip* is constant, but the phase takesvalues 0 or π depending on whether a 1 or 0 is transmitted. In both cases, *Id* is a Gaussian random variable but its average value is either *Ip*/2 or −*Ip*/2, depending on the received bit. The situation is analogous to the ASK case with the difference that $I_0 = -I_1$ in place of being zero. In fact, one can use Eq. (2) for the BER, but *Q* is now given by

$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} \approx \frac{2I_1}{2\sigma_1} = (SNR)^{\frac{1}{2}}$ (8)

Where $I_0 = -I_1$ and $\sigma_1 = \sigma_0$ was used. By using SNR = 2η*Np* from Eq. (14), the BER is given by

$\mathrm{BER} = 1/2\ \mathrm{erfc}(\sqrt{\eta N_p})$ [PSK heterodyne] (9)

As before, the SNR is improved by 3 dB, or by a factor of 2, in the case of PSK homodyne detection, so that

$\mathrm{BER} = 1/2\ \mathrm{erfc}(\sqrt{2\eta N_p})$ [PSK homodyne] (10)

The receiver sensitivity at a BER of 10^{-9} can be obtained by using *Q* = 6 and Eq. (14) for SNR. For the purpose of comparison, it is useful to express the receiver sensitivity in terms of the number of photons *Np*. It is easy to verify that *Np* = 18 and 9 for the cases of heterodyne and homodyne PSK detection, respectively. The average number of photons/bit, $\bar{N}_p$, equals *Np* for the PSK format because the same power is transmitted during 1 and 0 bits. A PSK homodyne receiver is the most sensitive receiver, requiring only 9 photons/bit. It should be emphasized that this conclusion is based on the Gaussian approximation for the receiver noise [1].

It is interesting to compare the sensitivity of coherent receivers with that of a direct detection receiver. Table 1 shows such a comparison. An ideal direct-detection receiver requires 10 photons/bit to operate at a BER of ≤ 10^{-9}. This value is only slightly inferior to the best case of a PSK homodyne receiver and considerably superior to that of heterodyne schemes. However, it is never achieved in practice because of thermal noise, dark current, and many other factors, which degrade the sensitivity to the extent that $\bar{N}_p$ ≈ 1000 is usually required. In the case of coherent receivers, $\bar{N}_p$ below 100 can be realized simply because shot noise can be made dominant by increasing the local-oscillator power.

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Modulation Format	Bit-Error Rate	N_p	$\bar{N}_p$
ASK heterodyne	$\frac{1}{2}\text{erfc}(\sqrt{\eta N_p/4})$	72	36
ASK homodyne	$\frac{1}{2}\text{erfc}(\sqrt{\eta N_p/2})$	36	18
PSK heterodyne	$\frac{1}{2}\text{erfc}(\sqrt{\eta N_p})$	18	18
PSK homodyne	$\frac{1}{2}\text{erfc}(\sqrt{2\eta N_p})$	9	9
FSK heterodyne	$\frac{1}{2}\text{erfc}(\sqrt{\eta N_p/2})$	36	36
Direct detection	$\frac{1}{2}\exp(-\eta N_p)$	20	10

Table 1 Sensitivity of synchronous receivers

3 SYNCHRONOUS FSK RECEIVERS

Synchronous FSK receivers generally use a dual-filter scheme similar to that shown in Fig. 1(a) for the asynchronous case. Each filter passes only 1 or 0 bits.

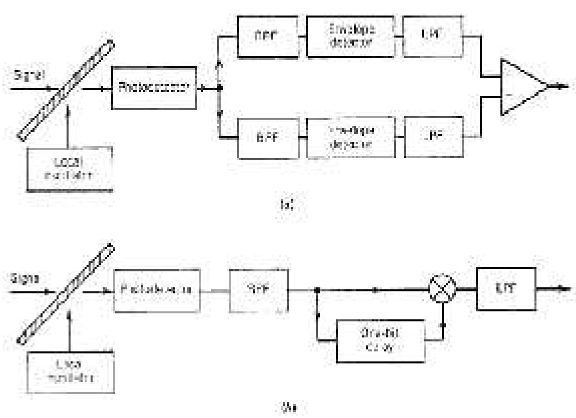

Figure 1: (a) Dual-filter FSK and (b) DPSK asynchronous heterodyne receivers

The scheme is equivalent to two complementary ASK heterodyne receivers operating in parallel. This feature can be used to calculate the BER of dual-filter synchronous FSK receivers. Indeed, one can use Eqs. (2) and (3) for the FSK case also. However, the SNR is improved by a factor of 2 compared with the ASK case. The improvement is due to the fact that whereas no power is received, on average, half the time for ASK receivers, the same amount of power is received all the time for FSK receivers. Hence the signal power is enhanced by a factor of 2, whereas the noise power remains the same if we assume the same receiver bandwidth in the two cases. By using SNR = 4η*Np* in Eq. (3), the BER is given by

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\sqrt{\eta Np/2}\right) \quad \text{. [FSK heterodyne] (11)}$$

The receiver sensitivity is obtained from Eq. (6) by replacing the factor of 72 by 36. In terms of the number of photons, the sensitivity is given by N_p = 36. The average number of photons/bit, $\bar{N}_p$, also equals 36, since each bit carries the same energy. A comparison of ASK and FSK heterodyne schemes in Table 1 shows that $\bar{N}_p$ = 36 for both schemes. Therefore even though the ASK heterodyne receiver requires 72 photons within the 1 bit, the receiver sensitivity (average received power) is the same for both the ASK and FSK schemes. Figure 2 plots the BER as a function of *Np* for the ASK, PSK, and FSK formats by using Eqs. (4), (9), and (11). The dotted curve shows the BER for the case of synchronous PSK homodyne receiver discussed in Section 2. The dashed curves correspond to the case of asynchronous receivers discussed in the following subsections.

4 ASYNCHRONOUS ASK RECEIVERS

The BER calculation for asynchronous receivers is slightly more complicated than for synchronous receivers because the noise statistics does not remain Gaussian when an envelope detector is used. The reason can be understood from, which shows the signal received by the decision circuit. In the case of an ideal ASK heterodyne receiver without phase fluctuations, can be set to zero so that (subscript *d* is dropped for simplicity of notation)

$$I_d = [(I_p + i_c)^2 + i_s^2]^{1/2} \quad (12)$$

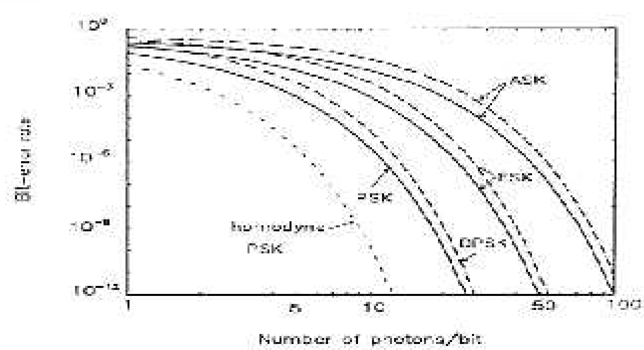

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Figure 2: Bit-error-rate curves for various modulation formats. The solid and dashed lines correspond to the cases of synchronous and asynchronous demodulation, respectively.

Even though both *Ip+ic* and *is* are Gaussian random variables, the probability density function (PDF) of *I* is not Gaussian. It can be calculated by using a standard technique [2] and is found to be given by [3]

$$P(I,Ip) = \frac{I}{\sigma^2}\exp\left(-\frac{I^2+I_p{}^2}{2\sigma^2}\right) I_0\left(\frac{I_p I}{\sigma^2}\right) \qquad (13)$$

where *I0* represents the modified Bessel function of the first kind. Both i_c and i_s are assumed to have a Gaussian PDF with zero mean and the same standard deviation σ ,where σ is the RMS noise current. The PDF given by Eq. (13) is known as the *Rice distribution* [3]. Note that *I* vary in the range 0 to ∞∞, since the output of an envelope detector can have only positive values. When *Ip* = 0, the Rice distribution reduces to the *Rayleigh distribution*, well known in statistical optics [2].

The BER calculation follows the analysis with the only difference that the Rice distribution needs to be used in place of the Gaussian distribution. The BER is given by

$$BER = \frac{1}{2}\left[p\left(\tfrac{0}{1}\right) + P\left(\tfrac{1}{0}\right)\right] \qquad (14)$$

where

$$P\left(\tfrac{0}{1}\right) = \int_0^{I_D} P(I,I_1)dI, \qquad P\left(\tfrac{1}{0}\right) = \int_{I_D}^{\infty} P(I,I_0)dI, \qquad (15)$$

The notation is the same as in particular, *ID* is the decision level and *I*1 and *I*0 are values of *Ip* for 1 and 0 bits. The noise is the same for all bits ($\sigma_0 = \sigma_1 = \sigma$) because it is dominated by the local oscillator power. The integrals in Eq. (15) can be expressed in terms of Marcum's *Q* function defined as [4]

$$Q(\alpha,\beta) = \int_\beta^\infty x\exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x)dx \qquad (16)$$

The result for the BER is

$$BER = \frac{1}{2}\left[1 - Q\left(\frac{I_1}{\sigma},\frac{I_D}{\sigma}\right) + Q\left(\frac{I_0}{\sigma},\frac{I_D}{\sigma}\right)\right] \qquad (17)$$

The decision level I_D is chosen such that the BER is minimum for given values of I_1, I_0, and σ . It is difficult to obtain an analytic expression of *ID* under general conditions. However, under typical operating conditions, $I_0 \approx 0$, $I_1/\sigma >> 1$, and *ID* is well approximated by *I*1/2. The BER then becomes

$$BER \approx \frac{1}{2}\exp\left(-{I_1}^2/8\sigma^2\right) = \frac{1}{2}\exp(-SNR/8) \qquad (18)$$

When the receiver noise σ is dominated by the shot noise, the SNR is given by Eq.(14). Using SNR = 2η*Np*, we obtain the final result,

$$BER = \frac{1}{2}\text{erfc}\left(\sqrt{\eta Np/4}\right) \qquad (19)$$

which should be compared with Eq. (4) obtained for the case of synchronous ASK heterodyne receivers. Equation (19) is plotted in Fig.2 with a dashed line. It shows that the BER is larger in the asynchronous case for the same value of η*Np*. However, the difference is so small that the receiver sensitivity at a BER of 10−9 is degraded by only about 0.5 dB. If we assume thatη = 1, Eq. (19) shows that BER = 10^{-9} for *Np* = 80 (*Np* = 72 for the synchronous case). Asynchronous receivers hence provide performance comparable to that of synchronous receivers and are often used in practice because of their simpler design.

5 ASYNCHRONOUS FSK RECEIVERS

Although a single-filter heterodyne receiver can be used for FSK, it has the disadvantage that one-half of the received power is rejected, resulting in an obvious 3-dB penalty. For this reason, a dual-filter FSK receiver [see Fig. 1] is commonly employed in which 1 and 0 bits pass through separate filters. The output of two envelope detectors is subtracted, and the resulting signal is used by the decision circuit. Since the average current takes values *I p* and −*Ip* for 1 and 0 bits, the decision threshold is set in the middle (*ID* = 0). Let *I* and *I*_ be the currents generated in the upper and lower branches of the dual filter receiver, where both of them include noise currents through Eq. (12). Consider the case in which 1 bit are received in the upper branch. The current *I* is then given by Eq. (12) and follows a Rice distribution with *I p* = *I*1 in Eq. (13). On the other hand, *I* _ consists only of noise and its distribution is obtained by setting *Ip* = 0 in Eq. (13). An error is made when *I'* > *I*, as the signal is then below the decision level, resulting in

$$\left(\tfrac{0}{1}\right) = \int_0^\infty P(I,I_1)\left[\int_1^\infty p(I',0)dI'\right]dI, \qquad (20)$$

where the inner integral provides the error probability for a fixed value of *I* and the outer integral sums it over all possible values of *I*. The probability *P*(1/0) can be obtained similarly. In fact, *P*(1/0) = *P*(0/1) because of the symmetric nature of a dual-filter receiver.

The integral in Eq. (20) can be evaluated analytically. By using Eq. (13) in the inner integral with *Ip* = 0, it is easy to verify that

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$\int_1^\infty p(I', 0)dI' =$
$\exp\left(-\frac{I^2}{2\sigma^2}\right)$

(21)

By using Eqs. (14), (20), and (21) with $P(1/0) = P(0/1)$, the BER is given by

$$= \int_0^\infty \frac{}{_2} \exp\left(-\frac{^2 + ^2}{2 ^2}\right) I_0\left(\frac{}{^2}\right) \exp\left(\frac{-^2}{2 ^2}\right) dI$$

(22)

where $p(I, Ip)$ was substituted from Eq. (13). By introducing the variable $x=\sqrt{2}I$, Eq. (22) can be written as

$$= \frac{1}{2}\exp\left(-\frac{^2}{4 ^2}\right) \int_0^\infty \frac{}{_2} \exp\left(-\frac{^2 + ^2/2}{2 ^2}\right) I_0\left(\frac{}{\sqrt{2}}\right)$$

(23)

The integrand in Eq. (23) is just $p(x, I1/\sqrt{2})$ and the integral must be 1. The BER is thus simply given by

$$= \frac{1}{2}\exp\left(-\frac{^2}{4 ^2}\right) = \frac{1}{2}\exp(\text{-SNR}/4) \qquad (24)$$

By using SNR = 2 Np from Eq. (14), we obtain the final result

$$= \frac{1}{2}\exp\left(-\frac{\eta N_p}{2}\right) \qquad (25)$$

Which should be compared with Eq. (11) obtained for the case of synchronous FSK heterodyne receivers. Figure 2 compares the BER in the two cases. Just as in the ASK case, the BER is larger for asynchronous demodulation. However, the difference is small and the receiver sensitivity is degraded by only about 0.5 dB compared with the synchronous case. If we assume that $\eta = 1$, $Np = 40$ at a BER of 10^{-9} ($Np = 36$ in the synchronous case). Also equals 40, since the same number of photons are received during 1 and 0 bits. Similar to the synchronous case, is the same for both the ASK and FSK formats.

Modulation Format	Bit-Error Rate	N_p	$\bar{N}_p$
ASK heterodyne	$\frac{1}{2}\exp(-\eta N_p/4)$	80	40
FSK heterodyne	$\frac{1}{2}\exp(-\eta N_p/2)$	40	40
DPSK heterodyne	$\frac{1}{2}\exp(-\eta N_p)$	20	20
Direct detection	$\frac{1}{2}\exp(-\eta N_p)$	20	10

Table 2 Sensitivity of asynchronous receivers

6 ASYNCHRONOUS DPSK RECEIVERS

Asynchronous demodulation cannot be used for PSK signals. A variant of PSK, known as DPSK, can be demodulated by using an asynchronous DPSK receiver [see Fig. 1(b)]. The filtered current is divided into two parts, and one part is delayed by exactly one bit period. The product of two currents contains information about the phase difference between the two neighboring bits and is used by the decision current to determine the bit pattern. The BER calculation is more complicated for the DPSK case because the signal is formed by the product of two currents. The final result is, however, quite simple and is given by [5]

$$= 1/2 \exp(-\eta\, \text{Np}) \qquad (26)$$

It can be obtained from the FSK result, Eq. (24), by using a simple argument which shows that the demodulated DPSK signal corresponds to the FSK case if we replace $I1$ by $2I1$ and σ_2 by $2\sigma_2$ [6]. Figure 2 shows the BER by a dashed line (the curve marked DPSK). For $\eta = 1$, a BER of 10^{-9} is obtained for $Np = 20$. Thus, a DPSK receiver is more sensitive by 3 dB compared with both ASK and FSK receivers. Table 2 lists the BER and the receiver sensitivity for the three modulation schemes used with asynchronous demodulation. The quantum limit of a direct-detection receiver is also listed for comparison. The sensitivity of an asynchronous DPSK receiver is only 3 dB away from this quantum limit.

CONCLUSION

This paper provided enough background material for calculating the bit-error rate (BER) of coherent light wave systems. However, the BER, and hence the receiver sensitivity, depend on the modulation format as well as on the demodulation scheme used by the coherent receiver. This paper explained each system separately.

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