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Vague generalized b continuous mappings

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Abstract: The aim of this paper is to introduce and investigate a new class of continuous mapping in vague topological spaces namely vague generalized b continuous mapping, vague generalized b irresolute mapping and their properties are discussed.
Keywords: Vague topology, vague generalized b continuous mappings, vague generalized b irresolute mappings.

I. INTRODUCTION

In 1970, Levine [8] initiated the study of generalized closed sets. The concept of fuzzy sets was introduced by Zadeh [12] in 1965. The theory of fuzzy topology was introduced by C.L.Chang [6] in 1967.Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2] as a generalization of fuzzy sets.

The theory of vague sets was first proposed by Gau and Buehre [7] as an extension of fuzzy set theory and vague sets are regarded as a special case of context dependent fuzzy sets. In this paper we introduce the concept of vague generalized b continuous mapping and vague generalized b irresolute mappings and also obtained their properties and relations with counter examples.

II. PRELIMINARIES

Definition 2.1: [3] A vague set A in the universe of discourse X is characterized by two membership functions given by:

1) A true membership function $t_A : X \to [0,1]$ and

2) A false membership function $f_A : X \to [0,1]$.

where $t_A(x)$ is lower bound on the grade of membership of x derived from the "evidence for x ", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x "and $t_A(x)+ f_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a subinterval $[t_A(x),1- f_A(x)]$ of $[0, 1]$. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq f_A(x)$ The vague set A is written as, A= $\{\langle x,[t_A(x),1-f_A(x)]\rangle / x \in X\}$ where the interval $[t_A(x),1- f_A(x)]$ is called the "vague value of x in A and is denoted by $V_A(x)$.

Definition 2.2: [3] Let A and B be vague sets of the form $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle / x \in X\}$ and $B=\{\langle x,[t_B(x),1-f_B(x)]\rangle / x \in X\}$. Then

$A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1-f_A(x) \leq 1-f_B(x)$.

A= B if and only if $A \subseteq B$ and $B \subseteq A$.

$$A^c = \{\langle x,[f_A(x),1-t_A(x)]\rangle / x \in X\}$$

$$A \cap B=\{\langle x,[(t_A(x) \wedge t_B(x)),((1-f_A(x)) \wedge (1-f_B(x)))]\rangle / x \in X\}.$$

$$A \cup B=\{\langle x,[(t_A(x) \vee t_B(x)),((1-f_A(x)) \vee (1-f_B(x)))]\rangle / x \in X\}.$$

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For the sake of simplicity, we shall use the notion A= $\{\langle x,[t_A(x),1-f_A(x)]\rangle\}$ instead of

A= $\{\langle x,[t_A(x),1-f_A(x)]\rangle / x\in X\}$

Definition 2.3:[9] Let (X,τ) be a topological space. A subset A of X is called:
i) generalized closed (briefly, g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
ii) generalized semi closed (briefly gs-closed) if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
iii) α-generalized closed (briefly αg-closed) if α cl(A) ⊆ U whenever A ⊆ U and U is open in X.
iv) generalized pre closed (briefly gp-closed) if pcl(A) ⊆ U whenever A ⊆ U and U is open in X.
v) generalized b closed set (briefly gb-closed) if bcl(A) ⊆ U whenever A ⊆ U and U is open in X .

Definition 2.4:[9] A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

$0, 1 \in \tau$
$G_1 \cap G_2 \in \tau$
$\cup G_i \in \tau$ for any family $\{G_i / i\in J\} \subseteq \tau$

In this case the pair (X,τ) is called vague topological space (VTS in short) and vague set in τ is known as vague open set (VOS in short) in X. The complement A^C of VOS in (X,τ) is called vague closed set (VCS in short) in X .

Definition 2.5:[9] A Vague set A= $\{\langle x,[t_A,1-f_A]\rangle\}$ in a VTS is said to be a
vague semi closed set (VSCS in short) if vint(vcl(A)) ⊆ A.

vague semi open set (VSOS in short) if A ⊆ vcl(vint(A)).

vague pre-closed set (VPCS in short) if vcl(vint(A)) ⊆ A.

vague pre-open set (VPOS in short) if A ⊆ vint(vcl(A)).

vague α-closed set (VαCS in short) if vcl(vint(vcl(A))) ⊆ A.

vague α-open set (VαOS in short) if A ⊆ vint(vcl(vint(A))).

vague regular open set (VROS in short) if A = vint(vcl(A)).

vague regular closed set (VRCS in short) if vcl(vint(A)) = A.

Definition 2.6:[9] A vague set A of VTS (X,τ) is said to be

1) vague generalized closed set (VGCS in short) if vcl(A) ⊆ U whenever A ⊆ U and U is VOS in X.
2) vague generalized semi closed set (VGSCS in short) if vscl(A) ⊆ U whenever A ⊆ U and U is VOS in X.
3) vague alpha generalized closed set (Vα GCS in short) if vαcl(A) ⊆ U whenever A ⊆ U and U is VOS in X.
4) vague generalized pre-closed set (VGPCS in short) if vpcl(A) ⊆ U whenever A ⊆ U and U is VOS in X .

Definition 2.10:[11] Let (X, τ) be an VTS and A = $\{\langle x,[t_A,1-f_A]\rangle\}$ be a vague set in X. Then the vague b closure of A (vbcl(A) in short) and vague b interior of A (vbint(A) in short) are defined as

vbint(A) = ∪ { G / G is an VbOS in X and G ⊆ A}, vbcl (A) = ∩ { K / K is VbCS in X and A ⊆ K}
Definition 2.11:[11] A vague set A in VTS (X, τ) is said to be vague generalized b closed set (VGbCS short) if vbcl(A) ⊆ U whenever A ⊆ U and U is VOS in (X, τ).The family of all VGbCS of a VTS (X, τ) is denoted by VGbC(X).

Definition 2.12:[10]Let (X, τ) and (Y,σ) be any two vague topological spaces. A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be

1) vague continuous (V continuous in short) if $f^{-1}(V)$ is vague closed set in (X, τ) for every vague closed set V of (Y,σ).
2) vague semi-continuous (VS continuous in short) if $f^{-1}(V)$ is vague semi-closed set in (X, τ) for every vague closed set V of (Y,σ).
3) vague pre-continuous (VP continuous in short) if $f^{-1}(V)$ is vague pre-closed set in (X, τ) for every vague closed setV of (Y,σ).
4) vague α-continuous (Vα-continuous in short) if $f^{-1}(V)$ is vague α-closed set in (X, τ) for every vague closed setV of (Y,σ).
5) vague regular continuous (VP continuous in short) if $f^{-1}(V)$ is vague regular closed set in (X, τ) for every vague closed set V of (Y,σ).
6) vague generalized continuous (VG continuous in short) if $f^{-1}(V)$ is vague generalized closed set in (X, τ) for every vague closed set V of (Y,σ).
7) vague generalized semi-continuous (VGS continuous in short) if $f^{-1}(V)$ is vague generalized semi-closed set in (X, τ) for every vague closed set V of (Y,σ).
8) vague α-generalized continuous (VαG continuous in short) if f $f^{-1}(V)$ is vague α generalized closed set in (X, τ) for every vague closed set V of (Y,σ).
9) vague generalized pre-continuous (VGP continuous in short) mapping if $f^{-1}(V)$ is VGPCS in (X, τ) for every vague closed set of V of (Y,σ).

Definition 2.13:[11] A VTS (X, τ) is called

1) vague $_bT_{1/2}$ space ($V_bT_{1/2}$ space in short) if every VbCS in X is VCS in X.
2) vague $_{gb}T_{1/2}$ space ($V_{gb}T_{1/2}$ space in short) if every VGbCS in X is VCS in X.
3) vague $_{gb}T_b$ space ($V_{gb}T_b$ space in short) if every VGbCS in X is VbCS in X.

III. VAGUE GENERALIZED b CONTINUOUS MAPPINGS

Definition 3.1: A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be vague b continuous (Vb continuous in short) if $f^{-1}(V)$ is vague b closed set in (X, τ) for every vague closed set V of (Y,σ).

Definition 3.2:A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be vague generalized b continuous (VGb continuous in short) mapping if $f^{-1}(V)$ is VGbCS in (X, τ) for every vague closed set V of (Y,σ).

Theorem 3.3:Let (X, τ) and (Y,σ) be any two vague topological spaces. For any vague continuous function $f:(X,\tau)\rightarrow(Y,\sigma)$ we have the following:

1) Every V continuous mapping is Vb continuous mapping.
2) Every V continuous mapping is VGb continuous mapping.
3) Every Vα continuous mapping is Vb continuous mapping.
4) Every Vα continuous mapping is VGb continuous mapping.
5) Every VαG continuous mapping is VGb continuous mapping.
6) Every Vb continuous mapping is VGb continuous mapping.
7) Every VP continuous mapping is VGb continuous mapping.
8) Every VR continuous mapping is VGb continuous mapping
9) Every VS continuous mapping is VGb continuous mapping.
10) Every VGP continuous mapping is VGb continuous mapping.

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11) Every VP continuous mapping is Vb continuous mapping.
12) Every VR continuous mapping is Vb continuous mapping.
13) Every VS continuous mapping is Vb continuous mapping.
14) Every VWG continuous mapping is VGb continuous mapping.
Proof: It is obvious.

Remark 3.4:The converse of the above theorem need not be true as shown by the following examples.

Example 3.5:Let $X = \{a,b\}$, $Y = \{u,v\}$, $G_1 = \{\langle x,[0.3,0.8],[0.5,0.7]\rangle\}$ and $G_2 = \{\langle y,[0.2,0.3],[0.6,0.8]\rangle\}$ then τ = {0,G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*. Then f is a Vb continuous mapping but not V continuous, since $G_2^c = \{\langle y,[0.7,0.8],[0.2,0.4]\rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VCS in X.

Example 3.6: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.5,0.7],[0.5,0.8]\rangle\}$ and $G_2 = \{\langle y,[0.1,0.2],[0.8,0.9]\rangle\}$ Then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*.Then f is a VGb continuous mapping but not V continuous, since $G_2^c = \{\langle y,[0.8,0.9],[0.1,0.2]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VCS in X.

Example 3.7: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.5,0.7],[0.5,0.8]\rangle\}$, $G_2 = \{\langle y,[0.4,0.7],[0.5,0.7]\rangle\}$ then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*.Then f is a Vb continuous mapping but not Vα continuous, since $G_2^c = \{\langle y,[0.3,0.6],[0.3,0.5]\rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VαCS in X

Example 3.8: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.2,0.4],[0.6,0.8]\rangle\}$, $G_2 = \{\langle y,[0.7,0.9],[0.3,0.5]\rangle\}$ then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*. Then f is a VGb continuous mapping but not Vα continuous, since $G_2^c = \{\langle y,[0.1,0.3],[0.5,0.7]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VαCS in X.

Example 3.9: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.5],[0.4,0.6]\rangle\}$, $G_2 = \{\langle y,[0.2,0.5],[0.3,0.6]\rangle\}$ then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*. Then f is a VGb continuous mapping but not VαG continuous, since $G_2^c = \{\langle y,[0.5,0.8],[0.4,0.7]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VGαCS in X.

Example 3.10: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.5,0.7],[0.6,0.8]\rangle\}$ $G_2 = \{\langle y,[0.6,0.7],[0.5,0.7]\rangle\}$ then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b) = v*. Then f is a VGb continuous mapping but not Vb continuous, since $G_2^c = \{\langle y,[0.3,0.4],[0.3,0.5]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VbCS in X.

Example 3.11: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.2,0.3],[0.1,0.5]\rangle\}$ $G_2 = \{\langle y,[0.3,0.8],[0.2,0.6]\rangle\}$ then τ = {0, G1, 1} and σ = { 0, G2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a) = u* and *f(b)* = *v*. Then f is a VGb continuous mapping but not VP continuous, since $G_2^c = \{\langle y,[0.2,0.7],[0.4,0.8]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VPCS in X.

Example 3.12: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.5],[0.4,0.8]\rangle\}$ $G_2 = \{\langle y,[0.2,0.5],[0.3,0.6]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a VGb continuous mapping but not VR continuous, since $G_2^c = \{\langle y,[0.5,0.8],[0.4,0.7]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VRCS in X.

Example 3.13: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.7,0.9],[0.1,0.2]\rangle\}$ $G_2 = \{\langle y,[0.3,0.4],[0.6,0.7]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a VGb continuous mapping but not VS continuous, since $G_2^c = \{\langle y,[0.6,0.7],[0.3,0.4]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VSCS in X.

Example 3.14: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.2,0.3],[0.1,0.4]\rangle\}$ $G_2 = \{\langle y,[0.3,0.6],[0.4,0.5]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a VGb continuous mapping but not VGP continuous, since $G_2^c = \{\langle x,[0.4,0.7],[0.5,0.6]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VGPCS in X.

Example 3.15: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.2,0.3],[0.1,0.5]\rangle\}$ $G_2 = \{\langle y,[0.3,0.8],[0.2,0.6]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a Vb continuous mapping but not VP continuous, since $G_2^c = \{\langle y,[0.2,0.7],[0.4,0.8]\rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VPCS in X.

Example 3.16: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.5],[0.4,0.8]\rangle\}$ $G_2 = \{\langle y,[0.2,0.5],[0.3,0.6]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a Vb continuous mapping but not VR continuous, since $G_2^c = \{\langle y,[0.5,0.8],[0.4,0.7]\rangle\}$ is VbCS in Y but $f^{-1}(G_2^c)$ is not VRCS in X.

Example 3.17:Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.7,0.9],[0.1,0.2]\rangle\}$ $G_2 = \{\langle y,[0.3,0.4],[0.6,0.7]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a Vb continuous mapping but not VS continuous, since $G_2^c = \{\langle y,[0.6,0.7],[0.3,0.4]\rangle\}$ is a VbCS in Y but $f^{-1}(G_2^c)$ is not VSCS in X.

Example 3.18: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \{\langle x,[0.3,0.4],[0.6,0.7]\rangle\}$ $G_2 = \{\langle y,[0.5,0.7],[0.8,0.9]\rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by *f(a)* = *u* and *f(b)* = *v*. Then f is a VWG continuous mapping but not VGb continuous, since $G_2^c = \{\langle y,[0.3,0.5],[0.1,0.2]\rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VWGCS in X.

Remark 3.19: From the above theorem and examples we have the following diagrammatic representation

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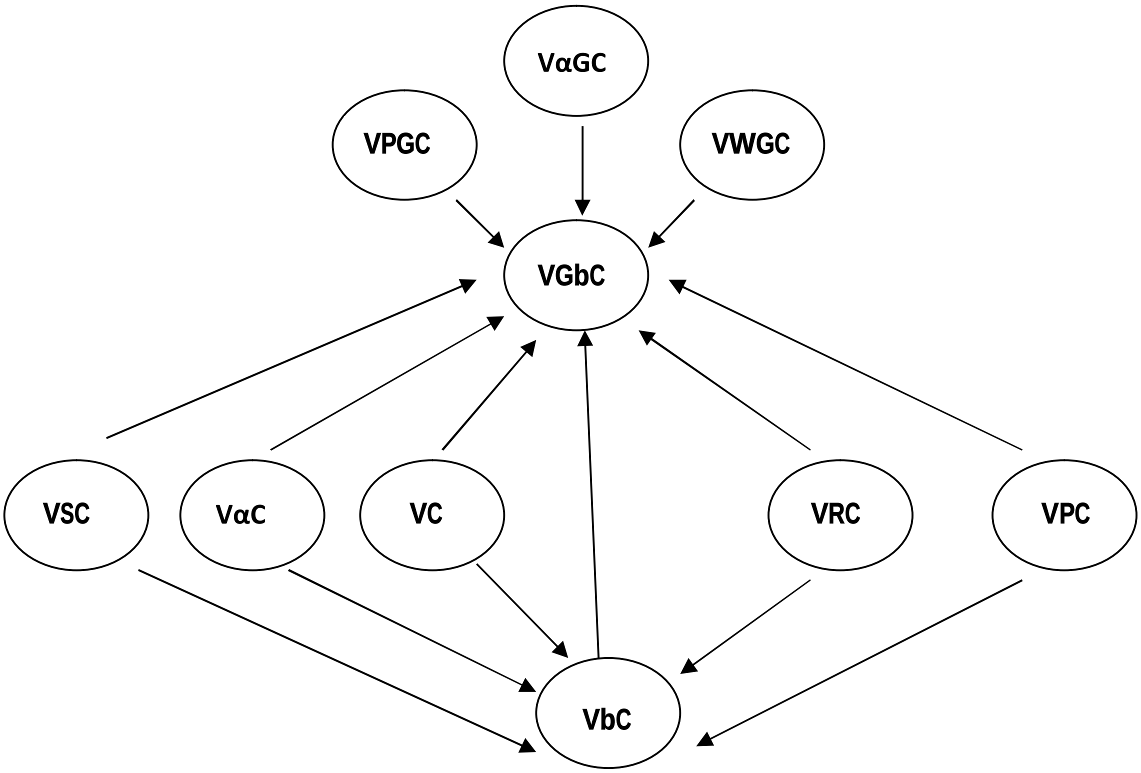

Theorem 3.20: A mapping $f:(X,\tau)\to(Y,\sigma)$ is VGb continuous mapping if and only if the inverse image of each VOS in Y is VGbOS in X.

Proof: Necessity Let A be VOS in Y. This implies A^c is VCS in Y. Since f is VGb continuous mapping, □ $f^{-1}(A^c)$ is VGbCS in X. Since $f^{-1}(A^c)=\left(f^{-1}(A)\right)^c$, $f^{-1}(A)$ is VGbOS in X.

Sufficiency: It is obvious.

Theorem 3.21: Let $f:(X,\tau)\to(Y,\sigma)$ be mapping and let $f^{-1}(A)$ is VRCS in X for every VCS A in Y. Then f is VGb continuous but not conversely.

Proof: Let A be VCS in Y. Then $f^{-1}(A)$ is VRCS in X. Since every VRCS is VGbCS, $f^{-1}(A)$ is VGbCS in X. Hence f is VGb continuous mapping.

Theorem 3.22: Let $f:(X,\tau)\to(Y,\sigma)$ be VGb continuous mapping, then f is vague continuous mapping if X is $V_{gb}T_{1/2}$ space.

Proof: Let A be VCS in Y. Then $f^{-1}(A)$ is VGbCS in X, by hypothesis. Since X is $V_{gb}T_{1/2}$ space, $f^{-1}(A)$ is VCS in X. Hence f is vague continuous mapping.

Theorem 3.23: Let $f:(X,\tau)\to(Y,\sigma)$ be VGb continuous mapping, then f is Vb continuous mapping if X is $V_{gb}T_b$ space.

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Proof: Let A be VCS in Y.Then $f^{-1}(A)$ is VGbCS in X, by hypothesis. Since X is $V_{gb}T_b$ space, $f^{-1}(A)$ is VbCS in X. Hence f is Vb continuous mapping.

Theorem 3.24: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from VTS X into VTS Y. Then the following conditions are equivalent if X is $V_{gb}T_b$ space.

i) f is VGb continuous mapping.
ii) $f^{-1}(B)$ is VGbCS in X for every VCS B in Y.
iii) $vcl\left(vint(f^{-1}(A))\right)\cap vint(vcl(f^{-1}(A)))\subseteq f^{-1}(vcl(A))$ for every vague set A in Y.

Proof: i) $\Rightarrow$ ii): It is obvious.

ii) $\Rightarrow$ iii): Let A be vague set in Y Then $vcl(A)$ is VCS in Y. By hypothesis, $f^{-1}(vcl(A))$ is a VGbCS in X. Since X is $V_{gb}T_b$ space, $f^{-1}(vcl(A))$ is VbCS. Therefore $vcl\left(vint(f^{-1}(vcl(A)))\right)\cap vint(vcl(f^{-1}(vcl(A)))\subseteq f^{-1}(vcl(A))$ $vcl\left(vint(f^{-1}(A))\right)\cap vint(vcl(f^{-1}(A)))\subseteq vcl\left(vint\left(f^{-1}(vcl(A))\right)\right)\cap vint(vcl(f^{-1}(vcl(A))))\subseteq f^{-1}(vcl(A))$. Hence $vcl\left(vint(f^{-1}(A))\right)\cap vint(vcl(f^{-1}(A)))\subseteq f^{-1}(vcl(A))$.

iii) $\Rightarrow$ i): Let A be VCS in Y. By hypothesis $vcl\left(vint(f^{-1}(A))\right)\cap vint(vcl(f^{-1}(A)))\subseteq f^{-1}\left(vcl(A)\right)=f^{-1}(A)$. This implies $f^{-1}(A)$ is VbCS in X and hence it is VGbCS. Thus f is VGb continuous mapping.

Theorem 3.25: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from VTS X into VTS Y. Then the following conditions are equivalent if X is $V_{gb}T_b$ space

i) f is VGb continuous mapping.
ii) $f^{-1}(A)$ is VGbOS in X for every VOS A in Y.
iii) $f^{-1}(vint(A))\subseteq vcl(vint(f^{-1}(A))\cup vint(vcl(f^{-1}(A)))$ for every VS A in Y.

Proof: i)$\Rightarrow$ii):It is obvious.

ii)$\Rightarrow$iii): Let A be vague set in Y. Then $vint(A)$ is VOS in Y. By hypothesis, $f^{-1}(vint(A))$ is VGbOS in X. Since X is $V_{gb}T_b$ space, $f^{-1}(vint(A))$ is VbOS in X. Therefore $f^{-1}(vint(A))\subseteq vcl(vint(f^{-1}(vint(A)))\cup vint(vcl(f^{-1}(vint(A)))$ $f^{-1}(vint(A))\subseteq vcl(vint(f^{-1}(vint(A)))\cup vint\left(vcl(f^{-1}(vint(A))\right)\subseteq vcl(vint(f^{-1}(A))\cup vint(vcl(f^{-1}(A)))$

iii)$\Rightarrow$i): Let A be VCS in Y. Then its complement, say A^c is VOS in Y, then $vint(A^c)=A^c$. Now by hypothesis $f^{-1}(vint(A^c))$ $\subseteq vcl(vint(f^{-1}(vint(A^c)))\cup vint(vcl(f^{-1}(vint(A^c)))$ .This implies $f^{-1}(A^c)\subseteq vcl(vint(f^{-1}(A^c))\cup vint\left(vcl(f^{-1}(A^c))\right)$. Hence $f^{-1}(A^c)$ is VGbOS in X. Thus $f^{-1}(A)$ is VGbCS in X. Hence f is VGb continuous mapping.

Theorem 3.26: Let $f:(X,\tau)\to(Y,\sigma)$ be VGb continuous mapping and $g:(Y,\sigma)\to(Z,\mu)$ is vague continuous mapping, then $g\circ f:(X,\tau)\to(Z,\mu)$ is VGb continuous mapping.

Proof: Let A be VCS in Z. Then $g^{-1}(A)$ is VCS in Y, by hypothesis. Since f is VGb continuous mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X. Hence $g\circ f$ is VGb continuous mapping.

Remark 3.27: The composition of two VGb continuous mapping need not be VGb continuous mapping.

Example 3.28: Let $X=\{a,b\}$, $Y=\{u,v\}$ and Z = {p,q} vague sets G_1,G_2 and G_3 defined as follows: $G_1=\{\langle x,[0.3,0.8],[0.5,0.7]\rangle\}$, $G_2=\{\langle y,[0.2,0.3],[0.6,0.8]\rangle\}$ and $G_3=\{\langle z,[0.1,0.2],[0.8,0.9]\rangle\}$ then $\tau=\{0,G_1,1\}$, σ

= { 0, G_2, 1} and μ = { 0, G_3 , 1} be vague topologies on X,Y and Z respectively.Let the mapping $f: (X, \tau) \to (Y, \sigma)$ by *f(a)* = *u* and *f(b)* = *v*, $g: (Y, \sigma) \to (Z, \mu)$ by *g(x)* = *p* and *g(y)* = *q*. Then the f and g are VGb continuous mapping but the mapping $g \circ f : (X, \tau) \to (Z, \mu)$ is not VGb continuous mapping.

Definition 3.29: Let (X,τ) be VTS. The vague generalized b closure (vgbcl(A) in short) for any vague set *A* is defined as follows, vgbcl(A) = $\cap$ {K/K is a VGbCS in X and A $\subseteq$ K}.If *A* is VGbCS, then vgbcl(A)= *A*.

Theorem 3.30: Let $f: (X, \tau) \to (Y, \sigma)$ be VGb continuous mapping. Then the following conditions hold.

i) $f(vgbcl(A)) \subseteq vcl(f(A))$ for every vague set A in X.
ii) $vgbcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$ for every vague set B in X.

Proof: i) Since *vcl(f(A))* is VCS in Y and f is VGb continuous mapping, then $f^{-1}(vcl(f(A)))$ is VGbCS in X. That is $vgbcl(A) \subseteq f^{-1}\left(vcl(f(A))\right)$. Therefore $f(vgbcl(A)) \subseteq vcl(f(A))$ for every vague set A in X.

ii) Replacing A by $f^{-1}(B)$ in (i), we get $f\left(vgbcl(f^{-1}(B))\right) \subseteq vcl\left(f(f^{-1}(B))\right) \subseteq vcl(B)$. Hence $vgbcl(f^{-1}(B)) \subseteq f^{-1}(vcl(B))$ for every vague set B in Y.

IV. VAGUE GENERALIZED b IRRESOLUTE MAPPINGS

Definition 4.1: A map $f: (X, \tau) \to (Y, \sigma)$ is said to be vague generalized b irresolute (VGb irresolute in short) mapping if $f^{-1}(A)$ is VGbCS in (X, τ) for every VGbCS A in (Y, σ).

Theorem 4.2: Let $f: (X, \tau) \to (Y, \sigma)$ be VGb irresolute mapping, then f is VGb continuous mapping but not conversely.
Proof: Let f be VGb irresolute mapping. Let *A* be any VCS in *Y* .Since every VCS is VGbCS, *A* is VGbCS in *Y*. Since f is VGb irresolute mapping, by definition $f^{-1}(A)$ is VGbCS in *X* . Hence f is VGb continuous mapping.

Example:4.3: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \{\langle x, [0.5, 0.7], [0.6, 0.8] \rangle\}$ $G_2 = \{\langle y, [0.6, 0.7], [0.5, 0.7] \rangle\}$ then τ = {0, G_1, 1} and σ = { 0, G_2, 1} are VTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by *f(a)* = *u* and

f(b) = *v*. Then f is VGb continuous mapping but not Vb continuous, since $G_2^c = \{\langle y, [0.3, 0.4], [0.3, 0.5] \rangle\}$ is VGbCS in Y but $f^{-1}(G_2^c)$ is not VbCS in X.

Theorem 4.4: A mapping $f: (X, \tau) \to (Y, \sigma)$ is VGb irresolute mapping if and only if the inverse image of each VGbOS in *Y* is VGbOS in *X*.

Proof: Necessity Let *A* be VGbOS in *Y*. This implies A^c is VGbCS in *Y*.Since f is VGb irresolute mapping $f^{-1}(A^c)$ is VGbCS in *X* . Since $f^{-1}(A^c) = \left(f^{-1}(A^c)\right)^c$, $f^{-1}(A)$ is VGbOS in *X* .

Sufficiency: It is obvious.

Theorem 4.5: Let $f: (X, \tau) \to (Y, \sigma)$ be VGb irresolute mapping, then f is vague irresolute mapping if *X* is $V_b T_{1/2}$ space.

Proof: Let *A* be VCS in *Y* . Then *A* is VGbCS in *Y* . Since f is VGb irresolute mapping, $f^{-1}(A)$ is VGbCS in *X*, by hypothesis. Since *X* is $V_b T_{1/2}$ space, $f^{-1}(A)$ is VCS in *X*. Hence f is vague irresolute mapping.

Theorem 4.6: Let $f: (X, \tau) \to (Y, \sigma)$ be VGb irresolute mapping, then f is Vb irresolute mapping if *X* is $V_{gb} T_b$ space.

Proof: Let A be VbCS in Y. Then A is VGbCS in Y. Since f is VGb irresolute mapping, $f^{-1}(A)$ is VGbCS in X, by hypothesis. Since X is $V_{gb}T_b$ space, $f^{-1}(A)$ is VbCS in X. Hence f is Vb irresolute mapping.

Theorem 4.7: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be VGb irresolute mapping, where X, Y and Z are VTS, then $g\circ f$ is VGb irresolute mapping.

Proof: Let A be VGbCS in Z. Since g is VGb irresolute mapping, $g^{-1}(A)$ is VGbCS in Y. Since f is VGb irresolute mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X. Hence $(g\circ f)^{-1}$ is VGbCS in X.Therefore $g\circ f$ is VGb irresolute mapping.

Theorem 4.8: Let $f:(X,\tau)\to(Y,\sigma)$ be VGb irresolute mapping and $g:(Y,\sigma)\to(Z,\mu)$ be VGb continuous mapping, where X,Y and Z are VTS, then $g\circ f$ is VGb continuous mapping.

Proof: Let A be VCS in Z. Since g is VGb continuous mapping, $g^{-1}(A)$ is VGbCS in Y. Since f is VGb irresolute mapping, $f^{-1}(g^{-1}(A))$ is VGbCS in X. Hence $(g\circ f)^{-1}$ is VGbCS in X. Therefore $g\circ f$ is VGb continuous mapping.

Theorem 4.9: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from VTS X into VTS Y. Then the following conditions are equivalent if X and Y are $V_{gb}T_b$ space.

i) f is VGb irresolute mapping.
ii) $f^{-1}(B)$ is VGbOS in X for each VGbOS B in Y.
iii) $f^{-1}(vbint(B))\subseteq vbint(f^{-1}(B))$ for each VS B of Y.
iv) $vbcl(f^{-1}(B))\subseteq f^{-1}(vbcl(B))$ for each VS B of Y.

Proof: i)⇒ii) It is obvious.

ii)⇒iii) Let B be VS in Y and $vbint(B)\subseteq B$. Also $f^{-1}(vbint(B))\subseteq f^{-1}(B)$ Since $vbint(B)$ is VbOS in Y, it is VGbOS in Y. Therefore $f^{-1}(vbint(B))$ is VGbOS in X , by hypothesis. Since X is $V_{gb}T_b$ space $f^{-1}(vbint(B))$ is VbOS in X.
Hence $f^{-1}(vbint(B))= vbint\left(f^{-1}(vbint(B))\right)\subseteq \mathrm{vbint}(f^{-1}(B))$
iii)⇒iv) It is obvious by taking complement in (iii).

iv)⇒i) Let B be VGbCS in Y. Since Y is $V_{gb}T_b$ space, B is VbCS in Y and $vbcl(B) = B$. Hence $f^{-1}(B) = f^{-1}(vbcl(B))\supseteq vbcl(f^{-1}(B))$. Therefore $vbcl(f^{-1}(B)) = f^{-1}(B)$.This implies $f^{-1}(B)$ is VbCS and hence it is VGbCS in X. Thus f is VGb irresolute mapping.

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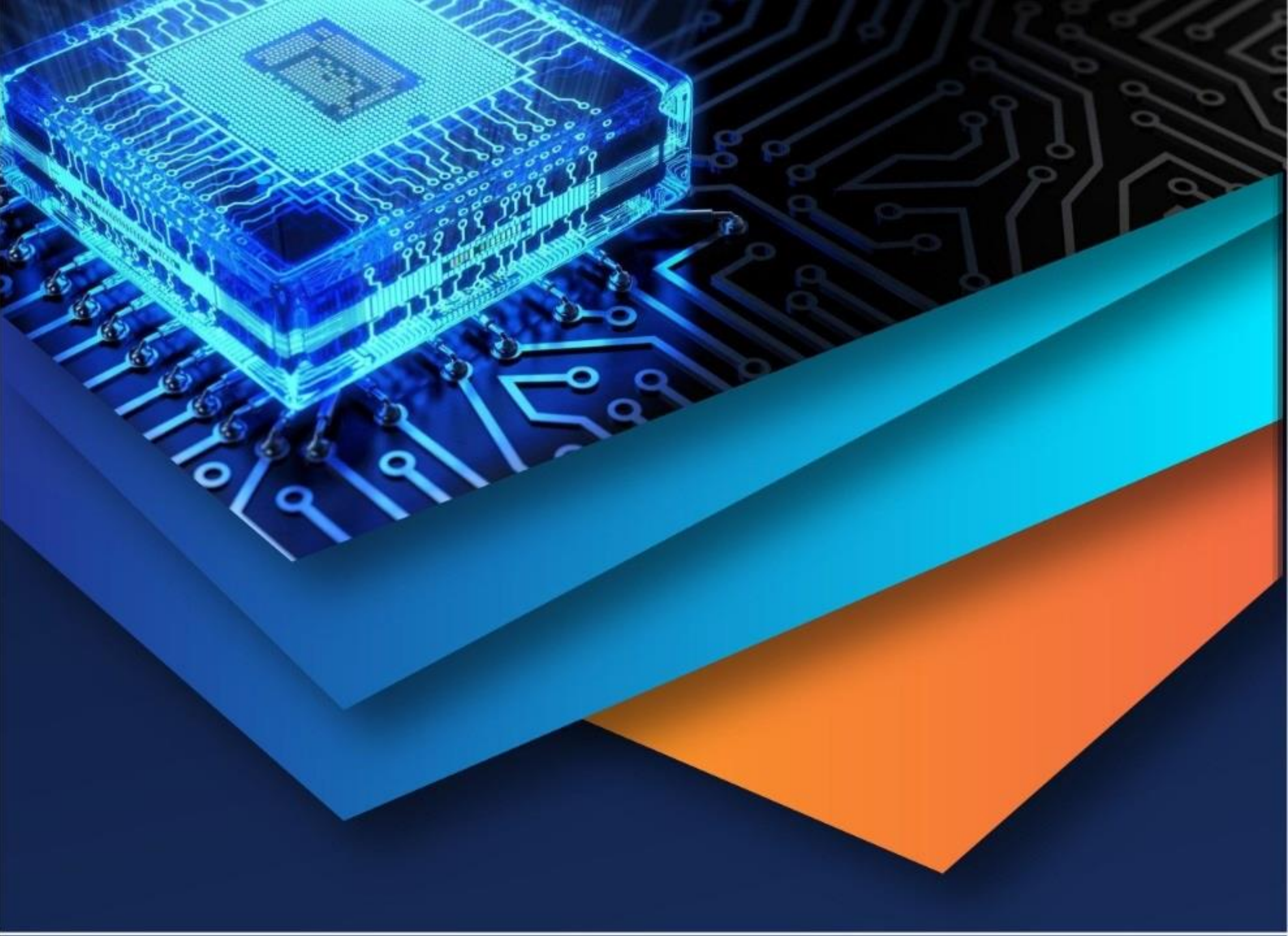

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