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Product Cordiality of Barycentric Subdivision of C_n

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Abstract: Graph labelling is an important area of research in Graph theory. There are many kinds of graph labelling. We investigate some new results in product cordial labelling of $C_n(C_n)$. We investigate that even number of copies of $C_n(C_n)$, each consecutive copies joined by same length of path is product cordial graph.
Keywords: Cordial graph, Product cordial graph, Path union, Barycentric subdivision

I. INTRODUCTION

Graph theory plays a important role in various fields. Graph labelling is one of the important area of graph theory. One of the productive and famous labelling of graph theory is cordial labelling. The concept of cordial graph was introduced by Cahit [1]. Motivated through the concept of cordial labelling the product cordial labelling was introduced by Sundaram [5].

In present work we consider finite, connected and undirected graph without multiple edges and loops. We consider graph $G = (V(G), E(G))$ having set of vertices V and set of edges E respectively. Our work mainly roam around C_n means cycle and P_n means path of length $n-1$. We refer Gross and Yellen [3] for all kind of definitions and notations. And base for this work is taken from Vaidya and Kanani [6]. We will discuss some basic terminology and definitions which is useful to present work.

A. *Definitions 1.1*

If the vertices of graph are assigned by some values or numbers subject to certain rules is known as graph labelling.

B. *Definition 1.2*

Labelling of vertices of graph by binary numbers, 0's and 1's under certain condition is called binary vertex labelling.

Notations:

$v_f(0)$ = Number of vertices with label 0.

$v_f(1)$ = Number of vertices with label 1.

$e_f(0)$ = Number of edges with label 0.

$e_f(1)$ = Number of edges with label 1.

C. *Definition 1.3*

A binary vertex labelling of graph G with induced edge labelling $f^*: E \to \{0,1\}$ defined by $f^*(e = uv) = |f(u) - f(v)|$ is called a cordial labelling if $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. A graph which admits cordial labelling is called cordial graph.

D. *Definition 1.4*

A binary vertex labelling of graph G with induced labelling $f^*: E \to \{0,1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a product cordial labelling if $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. A graph which admits product cordial labelling is called product cordial graph.

E. *Definition 1.5*

(Shee and Ho[4]) Let $G_1, G_2, \dots, G_n, n \ge 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for $i = 1, 2, \dots, n-1$ is called the path union of G.

F. *Definition 1.6*

Barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of the original graph.

G. *Observation*

Present work is motivated by some results of Vaidya and Kanani [6] in product cordial labelling which are given below:

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1) The path union k copies of $C_n(C_n)$ is a product cordial graph except for odd k.
2) The graph obtained by joining two copies of $C_n(C_n)$ by a path of arbitrary length is a product cordial graph.

The present work is done to investigate product cordial labelling for the path union of arbitrary length of copies of $C_n(C_n)$.

II. RESULTS

Theorem 2.1 k copies of $C_n(C_n)$ joined by a path where each consecutive copies joined by a path of same length is product cordial except for odd k.

Proof: Let $G_1, G_2, G_3, \dots, G_k$ be k copies of cycle C_n and $G'_1, G'_2, \dots, G'_k$ be k copies of cycle C_n which are obtained by joining each newly inserted vertices of adjacent edges by an edge.

So G_1 and G'_1 together makes a graph which is denoted by H_1 ; G_2 and G'_2 together makes a graph which is denoted by H_2 .

Same way we get graphs $H_3, H_4, \dots, H_k$.

Now join H_1 and H_2 by a path P_l of length $l-1$, join H_2 and H_3 by a path P_l of length $l-1$, and same way each consecutive H_i and H_{i+1} joined by a path P_l of same length $l-1$.

The graph obtained by joining each consecutive H_i and H_{i+1} is denoted by G.

Let $S_{i1}, S_{i2}, \dots, S_{in}$ be the vertices of G_i and $S'_{i1}, S'_{i2}, \dots, S'_{in}$ be the corresponding vertices of the G'_i , for $i = 1,2, \dots, k$.

Let $w_{j1}, w_{j2}, \dots, w_{jl}$ be the vertices of path P_l joining graph H_j and H_{j+1} for $j = 1,2, \dots, k-1$.

It can be easily see that $S_{i1} = w_{j1}$ for $i = j$ and

$i = 1,2, \dots, k-1$

Also, $w_{jl} = S_{(j+1)1}$ for $j = 1,2, \dots, k-1$,

$w_{jl} = w_{(j+1)1}$ for $j = 1,2, \dots, k-1$.

Note that $|V(G)| = 2nk + (k-1)(l-2)$

$|E(G)| = 3nk + (k-1)(l-1)$.

To define binary vertex labelling $f: V(G) \to \{0,1\}$ we consider following cases.

(i) l is even

(ii) l is odd

As we know that k is even.

For $1 \le i \le k/2$

Define $\left.\begin{matrix} f(s'_{ij})=0 \\ f(s_{ij})=0 \end{matrix}\right\} 1 \le j \le n$

also for $k/2 < i \le k$

Define $\left.\begin{matrix} f(s'_{ij})=1 \\ f(s_{ij})=1 \end{matrix}\right\} 1 \le j \le n$.

A. *Case 1: l is even.*

For $i = 1,2, \dots, \frac{k}{2} - 1$

$f(w_{ij}) = 0$, where $1 \le j \le l$.

For $i = \left(\frac{k}{2} + 1\right), 2, \dots, k-1$.

$f(w_{ij}) = 1$, where $1 \le j \le l$

For $i = k/2$

$f(w_{ij}) = 0, \quad 2 \le j \le l/2$

$f(w_{ij}) = 1, \quad l/2 < j < l$

$v_f(0) = v_f(1)$ and $e_f(0) = e_f(1) + 1$

Hence $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Graph under consideration is a product cordial graph.

B. *Case 2*: l is odd.

For $i = 1,2,\ldots,\frac{k}{2}-1$

$f(w_{ij}) = 0$, where $1 \le j \le l$.

For $i = \left(\frac{k}{2}+1\right),2,\ldots,k-1$.

$f(w_{ij}) = 1$, where $1 \le j \le l$

For $i = {}^{k}/_{2}$

$f(w_{ij}) = 0, \quad 2 \le j \le \frac{l-1}{2}$

$f(w_{ij}) = 1, \quad \frac{l-1}{2} < j < l$

$v_f(0) + 1 = v_f(1)$ and $e_f(0) = e_f(1)$

Hence $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Graph under consideration is a product cordial graph.

Example 2.2 In the figure 1 the product cordial labelling of 4 copies of $C_3(C_3)$ is demonstrated. Where each consecutive copies are joined by path P_3 of length 2.

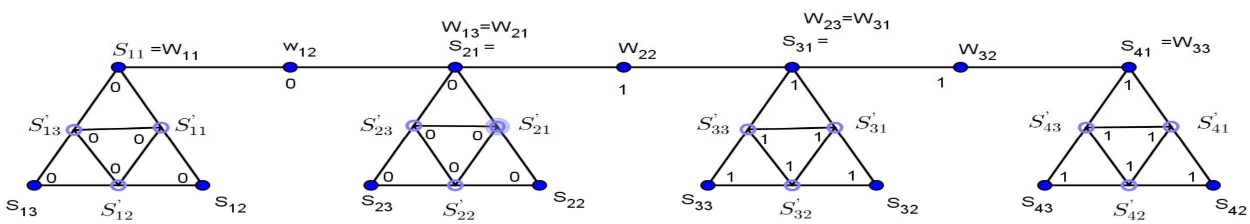


Figure 1: 4 copies of $C_3(C_3)$ with path P_3

Example 2.3 In the figure 2, product cordial labelling of 4 copies of $C_3(C_3)$ is demonstrated. Where each consecutive copies are joined by path P_4 of length 3.

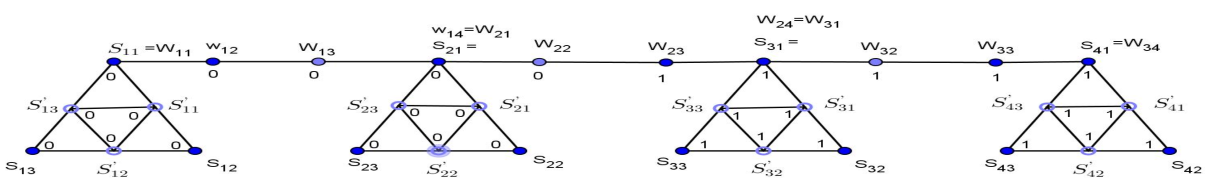


Figure 2: 4 copies of $C_3(C_3)$ with path P_4

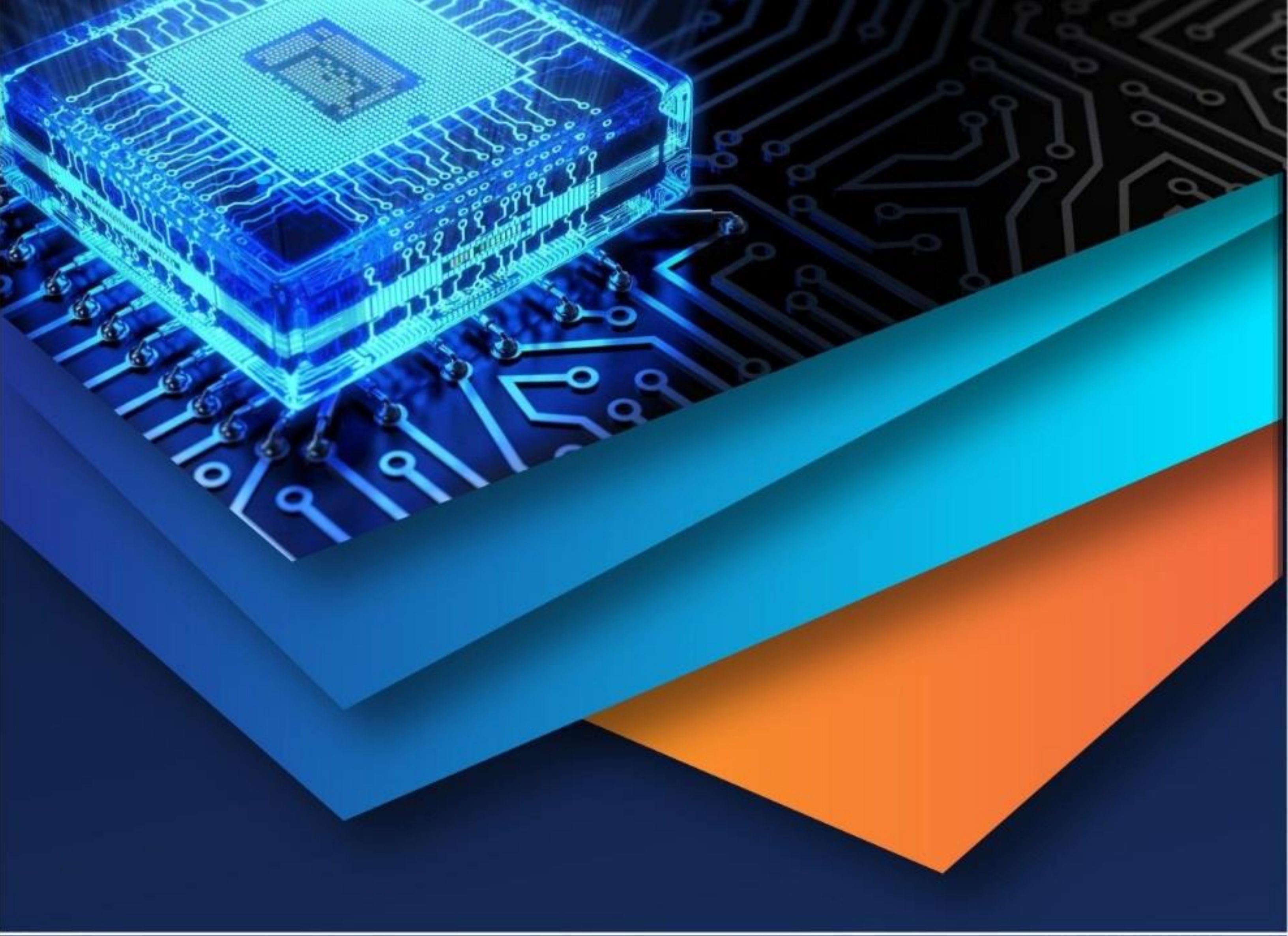
III. CONCLUSIONS

Vaidya and Kanani [6] investigated product cordial labelling for path union of various graph while we have extended one of their results. Whether similar type of result can be true for even number of copies of C_n as well as Petersen graph is an open research problem.

REFERENCES

[1] Cahit, I., (1987), "Cordial Graphs, A weaker version of graceful and Harmonious Graphs", Ars Combinatoria, 23, 201-207.
[2] Gallian, J. A., (2010), "A dynamic survey of graph labelling", The Electronics Journal of Combinatorics, 17, DS6.

[3] Gross, J. and Yellen, J. (1999), “Graph Theory and its applications”, CRC Press
[4] Narsingh Deo, “ Graph Theory with Applications to Engineering and Computer Science”, Paperback – 1979
[5] Shee, S. C. and Ho, Y. S., (1996), “ The cordiality of the path union of n copies of a graph”, Discrete Math., 151, 221-229.
[6] Sundaram, Ponraj, M. and Somsundaram, R., (2004), “Product cordial labelling of graphs”, Bull. Pure and Applied Sciences (Mathematics and Statistics), 23E, 155-163.
[7] Vaidya, S. K. and Kanani, K. K., (2011), “ Some new product cordial graphs”, Mathematics Today Vol.27,64-70
[8] Vaidya, S. K. and Kanani, K. K., (2010), “Some New Results on Cordial Labelling in the Context of Arbitrary Super subdivision of Graph”, Applied Mathematical Sciences, Vol. 4, no. 47, 2323 – 2329



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