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# Modified Gauss Elimination Method

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***Abstract: Gauss elimination method is a direct numerical method to solve a system of equations. In this paper, Gauss elimination method is modified to solve a system of linear equations with any number of variables.***
***Keywords: Gauss elimination method, Modified Gauss elimination method, Gauss Jordan method, Gauss seidel method and Gauss Jacobi method.***

## I. INTRODUCTION

A linear system of equations can be solved by many methods. There are many direct methods and indirect methods. Direct methods give accurate solution without any iteration. Indirect methods give approximate solutions with iterations depending on the accuracy after the decimal places. A system of 'n' equations with 'n' variables can be solved by Gauss elimination method and Gauss Jordan methods. These are direct methods. Gauss seidal and Gauss jacobi are indirect methods. Gauss elimination method requires the system of equations to be written as an augmented matrix and converting the matrix to upper triangular matrix which requires backward substition method to find the solution. Gauss jordan also requires augmented matrix, which is converted to diagonal matrix and hence the solution is obtained directly. Both these methods are efficient numerical procedures and can be implemented on high speed digital computers. the number of arithmetic operations are roughly ($n^3/3$) in gauss elimination and ($n^3/2$) in Gauss jordan method. Moreover, numerical errors can be controlled more easily in Gauss elimination method. Hence Gauss elimination is preferred when solving large system of equations in a computer.

Always direct methods for the solution of linear system are preferred but in the case of matrices with a large number of zero elements, it will be advantageous to use iterative method which preserves these elements. Gauss jacobi starts with arbitrary solution to the variables, say '0' then proceeded as iterations to find the final solutions. Gauss seidal method is similar to Gauss jacobi method with only difference that updated values are used to find the values of variables in next iteration while Gauss jacobi uses only old values. For these 2 methods the system should satisfy the condition that the sum of absolute values of the coefficient ($a_{ij}/a_{ii}$) is almost equal to, or in atleast one equation less than unity, where the system is given by

$$\left.\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \dots\dots\dots\dots + a_{1n}x_n = b_1 \\ &a_{21}x_1 + a_{22}x_2 + \dots\dots\dots\dots + a_{2n}x_n = b_2 \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &a_{n1}x_1 + a_{n2}x_2 + \dots\dots\dots\dots + a_{nn}x_n = b_n \end{aligned}\right\} \quad (1)$$

It can be shown that the Gauss seidal method converges twice as fast as Jacobi method.

In this paper , Gauss elimination method is modified and modified Gauss elimination method is introduced to solve a system of equations. Section 2 discusses the method along with some examples and section 3 concludes the paper.

## II. MODIFIED GAUSS ELIMINATION METHOD

### *A. Procedure*

Let us consider the system of equations (1) for solving. Then the augmented matrix of (1) is

$$\begin{pmatrix} a_{11} & a_{12} & \dots & \dots & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & \dots & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & \dots & \dots & a_{nn} & b_n \end{pmatrix} \quad (2)$$

Augmented matrix (2) is converted to lower triangular matrix. The $n^{th}$ column is made of '0' except $a_{nn}$ by $R_j \rightarrow a_{nn}R_j - a_{jn} R_n$ for $1 \leq j<n$ and $R_j$ is the $j^{th}$ row. Hence (2) becomes

$$\begin{pmatrix} a_{11}' & a_{12}' & \dots & \dots & a_{1(n-1)}' & 0 & b_1' \\ a_{21}' & a_{22}' & \dots & \dots & a_{2(n-1)}' & 0 & b_2' \\ \dots. & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{(n-1)1}' & a_{(n-1)2}' & \dots & \dots & a_{(n-1)(n-1)}' & 0 & b_{n-1}' \\ a_{n1} & a_{n2} & \dots & \dots & a_{n(n-1)} & a_{nn} & b_n \end{pmatrix} \quad (3)$$

where $a_{ij}$' represents changed elements. Similarly proceeding the elements above the diagonal are made of '0' and hence (2) is converted to lower triangular matrix obtained as

$$\begin{pmatrix} A_{11} & 0 & 0 & \dots & 0 & 0 & B_1 \\ A_{21} & A_{22} & 0 & \dots & 0 & 0 & B_2 \\ \dots. & \dots & \dots & \dots & \dots & \dots & \dots \\ A_{(n-1)1} & A_{(n-1)2} & \dots & \dots & A_{(n-1)(n-1)} & 0 & B_{n-1} \\ a_{n1} & a_{n2} & \dots & \dots & a_{n(n-1)} & a_{nn} & b_n \end{pmatrix} \quad (4)$$

It is clear that forward substitution method can be used to find the solutions $x_1, x_2, \dots, x_n$. It is clear that the method will fail if one of the elements $A_{11}$, $A_{22}$ or $A_{33}$ vanishes. In such a case, the method can be modified by rearranging the rows so that the pivot is non-zero. This procedure is called partial pivoting and can be easily implemented on a computer. If this is impossible, the matrix is singular and (1) has no solution. it is found that the number of arithmetic operations are roughly ($n^3/3$).

*1) Examples:*

*a) Example 1:* Modified gauss elimination method can be used for solving 5x+4y=15; 3x+7y=12. Augmented matrix is

$$\begin{pmatrix} 5 & 4 & 15 \\ 3 & 7 & 12 \end{pmatrix}$$

Corresponding Lower triangular matrix is

$$\begin{pmatrix} 23 & 0 & 57 \\ 3 & 7 & 12 \end{pmatrix}$$

Hence x=57/23 and y=15/23.

*b) Example 2:* Consider the system 2x+y+z=10; 3x+2y+3z=18; x+4y+9z=16.
Then the augmented matrix will be

$$\begin{pmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{pmatrix}$$

Hence Corresponding lower triangular matrix is

$$\begin{pmatrix} 6 & 0 & 0 & 42 \\ 8 & 2 & 0 & 38 \\ 1 & 4 & 9 & 16 \end{pmatrix}$$

Thus modified gauss elimination method gives after forward substitution as x=7, y=-9, z=5.

*c) Example 3:* Consider a system of equations as x+y+z+w=1; 3x+2y+3z+4w=7; 2x-y+2z-w=-5; x-2y-3z+2w=5.

Augmented matrix is $$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 3 & 4 & 7 \\ 2 & -1 & 2 & -1 & -5 \\ 1 & -2 & -3 & 2 & 5 \end{pmatrix}$$

This has a lower triangular matrix as $$\begin{pmatrix} 12 & 0 & 0 & 0 & -21 \\ 22 & -21 & 0 & 0 & -21 \\ 5 & -4 & 1 & 0 & -5 \\ 1 & -2 & -3 & 2 & 5 \end{pmatrix}$$

And hence giving the solution x=-7/4, y=-5/6, z=5/12, w=19/6.

## III. CONCLUSION

A system of linear equations can be solved by many direct and indirect methods. Gauss elimination method is modified in this paper to solve a system of equations. The method is elaborated first and the examples are given to illustrate the method.

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