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Pythagorean Triangle with Area/Perimeter as a 4-digit consecutive Sphenic numbers

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Abstract: ***We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the 4-digit consecutive Sphenic numbers namely 2013, 2014 and 2015. Also we present the number of primitive and non-primitive Pythagorean triangles. A few interesting relations among the sides are also given.***
Keywords: Pythagorean triangles, 4-digit consecutive Sphenic numbers, Primitive and non-primitive Pythagorean triangles.
Notations

Tri_n = ***Triangular number of rank*** n.

Hex_n = ***Hexagonal number of rank*** n.

I. INTRODUCTION

Number theory is a broad and diverse part of Mathematics that developed from the study of the integers. Mathematicians all over the ages have been fascinated by Pythagorean Theorem and are solving many problems related to it thereby developing Mathematics. Pythagorean triangles which were first studied by the Pythagoreans around 400 B.C., remains one of the fascinated topics for those who just adore the numbers.

A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Nasty number, Dhuruva numbers and Jarasandha numbers. These numbers have presented in **[1-5]**. In **[6-14]**, special Pythagorean triangles connected with polygonal numbers and Nasty numbers are presented.

In this communication, we search for patterns of Pythagorean triangles, such that in each of which the ratio Area/Perimeter is represented by the 4-digit consecutive Sphenic numbers 2013, 2014 and 2015.

II. BASIC DEFINITIONS

A. *Definition 1*

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x, y, z)$.

Also, in Pythagorean triangle $T(x, y, z)$: $x^2 + y^2 + z^2$, x and y are called its legs and z its hypotenuse.

B. *Definition 2*

The most cited solutions of the Pythagorean equation is $x = m^2 - n^2$, $y = 2mn$, $z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m, n are of opposite parity and $\gcd(m, n) = 1$.

C. *Definition 3*

Sphenic number is a positive integer that is the product of three distinct prime numbers.

III. METHOD OF ANALYSIS

Denoting the Area and Perimeter of the triangle by A and P respectively.

The assumption

$$\frac{A}{P} = \text{4-digit Consecutive Sphenic number.}$$

The above relation leads to the equation

$$\frac{v(u-v)}{2} = \text{4-digit consecutive Sphenic number.} \quad (1)$$

Case 1

When $$\frac{v(u-v)}{2} = 2013 \quad (2)$$

which implies $$v(u-v) = 4026 \quad (3)$$

On computation, the values of the generators u, v satisfying (3) are given in the following table.

S.No	v	$u-v$	u	x	y	z	A	P	$\frac{A}{P}$
1	1	4026	4027	8054	16216728	16216730	65304763660	32441512	2013
2	2	2013	2015	8060	4060221	4060229	16362690630	8128510	2013
3	3	1342	1345	8070	1809016	1809034	7299379560	3626120	2013
4	6	671	677	8124	458293	458365	1861586166	924782	2013
5	11	366	377	8294	142008	142250	588907176	292552	2013
6	22	183	205	9020	41541	42509	187349910	93070	2013
7	33	122	155	10230	22936	25114	117317640	58280	2013
8	61	66	127	15494	12408	19850	96124776	47752	2013
9	122	33	155	37820	9141	38909	172856310	85870	2013
10	183	22	205	75030	8536	75514	320228040	159080	2013
11	366	11	377	275964	8173	276085	1127726886	560222	2013
12	671	6	677	908534	8088	908570	3674111496	10002192	2013
13	1342	3	1345	3609980	8061	3609989	14550024390	7228030	2013
14	2013	2	2015	8112390	8056	8112394	32676706920	16232840	2013
15	4026	1	4027	32425404	8053	32425405	130560889200	64858862	2013

From the above table, it is seen that there are 15 Pythagorean triangles. Out of these, 7 triangles are primitive and remaining 8 triangles are non-primitive triangles.

Case 2:

When $$\frac{v(u-v)}{2} = 2014 \quad (4)$$

which implies $$v(u-v) = 4028 \quad (5)$$

On computation, the values of the generators u, v satisfying (5) are given in the following table.

S.No	v	$u-v$	u	x	y	z	A	P	$\frac{A}{P}$
1	1	4028	4029	8058	16232840	16232842	65402112360	32473740	2014
2	2	2014	2016	8064	4064252	4064260	16387064060	8136576	2014
3	4	1007	1011	8088	1022105	1022137	4133392620	2052330	2014
4	19	212	231	8778	53000	53722	232617000	115500	2014
5	38	106	144	10944	19292	22180	105565824	52416	2014
6	53	76	129	13674	13832	19450	94569284	46956	2014
7	76	53	129	19608	10865	22417	106520460	52890	2014
8	106	38	144	30528	9500	31972	145008000	72000	2014
9	212	19	231	97944	8417	98305	412197324	204666	2014
10	1007	4	1011	2036154	8072	2036170	8217917544	4080396	2014
11	2014	2	2016	8120448	8060	8120452	32725405440	16248960	2014
12	4028	1	4029	32457624	8057	32457625	130755538300	64923306	2014

From the above table, it is seen that there are 12 Pythagorean triangles. Out of these, 5 triangles are primitive and remaining 7 triangles are non-primitive Pythagorean triangles.

Case 3:

When $$\frac{v(u-v)}{2} = 2015 \quad (6)$$

which implies $$v(u-v) = 4030 \quad (7)$$

On computation, the values of the generators u, v satisfying (7) are given in the following table.

S.No	v	$u-v$	u	x	y	z	A	P	$\frac{A}{P}$
1	1	4030	4031	8062	16248960	16248962	65499557760	32505984	2015
2	2	2015	2017	8068	4068285	4068293	16411461690	8144646	2015
3	5	806	811	8110	657696	657746	2666957280	1323552	2015

4	10	403	413	8260	170469	170669	1408073940	349398	2015
5	13	310	323	8398	104160	104498	437367840	217056	2015
6	26	155	181	9412	32085	33437	150992010	74934	2015
7	31	130	161	9982	24960	26882	124575360	61824	2015
8	62	65	127	15748	12285	19973	193464180	48006	2015
9	65	62	127	16510	11904	20354	196535040	48768	2015
10	130	31	161	41860	9021	42821	188809530	93702	2015
11	155	26	181	56110	8736	56786	245088480	121632	2015
12	310	13	323	200260	8229	200429	823969770	408918	2015
13	403	10	413	332878	8160	332978	1358142240	674016	2015
14	806	5	811	1307332	8085	1307357	5284889610	2622774	2015
15	2015	2	2017	8128510	8064	8128514	32774152320	16265088	2015
16	4030	1	4031	32489860	8061	32489861	130950380700	64987782	2015

From the above table, it is seen that there are 16 Pythagorean triangles. Out of these, 8 triangles are primitive and remaining 8 triangles are non-primitive Pythagorean triangles.

IV. OBSERVATIONS

1) $z-x,\quad \frac{1}{2}(y+z)$ are perfect squares.

2) The expressions $6(z-x),\quad 12(y+z)$ represents a nasty numbers.

3) $x \equiv 0 \bmod(2).$

4) $A \equiv 0 \bmod(10).$

5) $y+z+2u = Tri_u.$

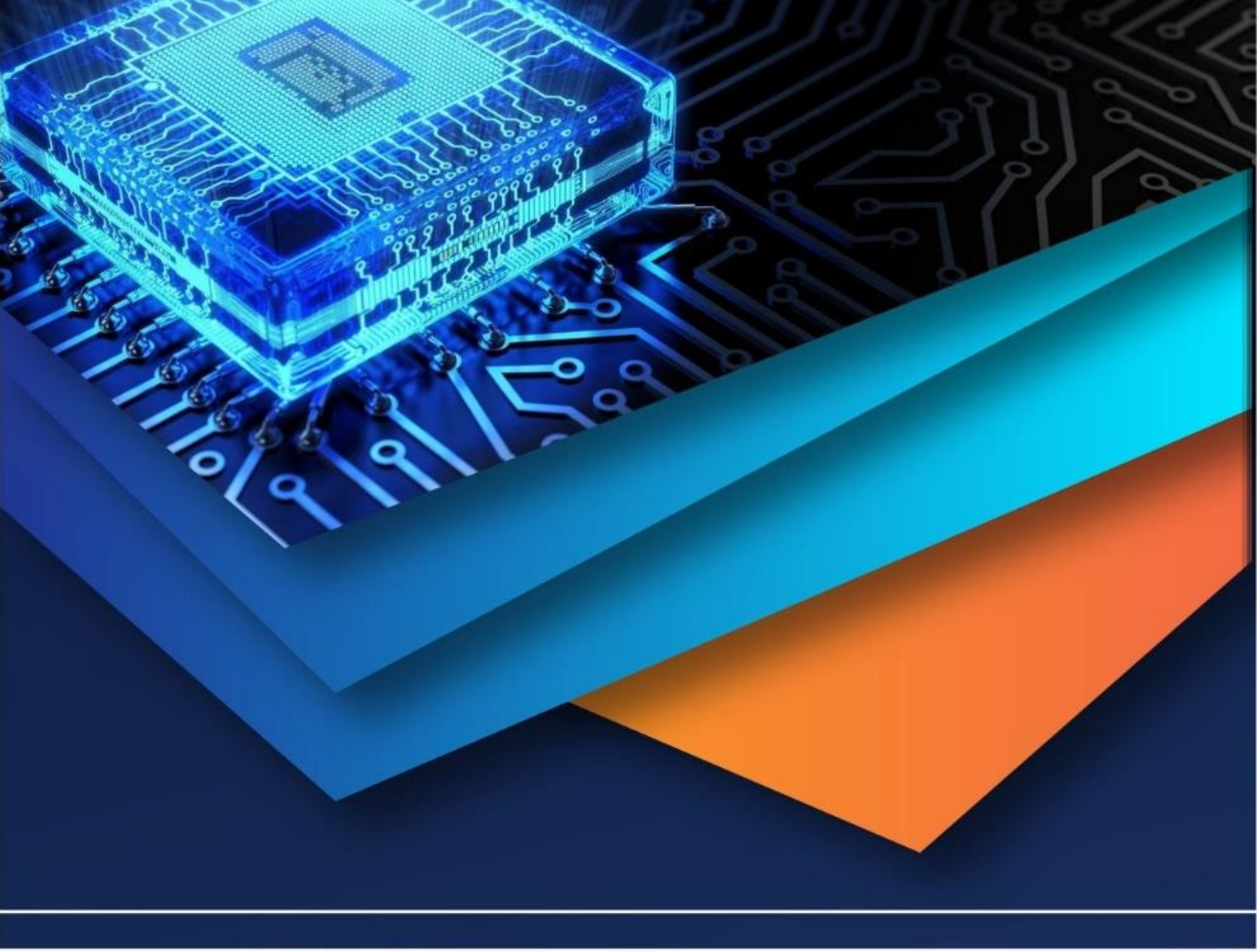
6) $y+z-2u = Hex_{u.}.$

V. CONCLUSION

One may search for the special Pythagorean triangles which are connected with the other Sphenic numbers.

REFERENCES

[1] W. Sierpinski, " Pyhtagorean triangles", Dover publications, INC, New York, 2003.
[2] J. N. Kapur, "Dhuruva Numbers", Fascinating world of Mathematics and Mathematical Sciences, Trust Society, Vol 17, 1997.
[3] Bert Miller, "Nasty Numbers", The mathematics teacher, No.9, Vol 73, 649, 1980.
[4] K.Charles Brown, "Nasties are primitives", The mathematics teacher, No.9, Vol 74, 502-504, 1981.
[5] P.S.N.Sastry, "Jarasandha numbers", The mathematics teacher, No.9, Vol 37, issue 3 and 4, 2001.
[6] M.A.Gopalan, A.Gnanam and G.Janaki, " A remarkable Pythagorean problem", Acta Ciencia Indica, Vol. XXXIII M, No.4, 1429-1434, 2007.
[7] M.A.Gopalan and G.Janaki, "Pyhtagorean triangle with area/perimeter as a special polygonal number", Bulletin of Pure and Applied Sciences, Vol 27E(No.2), 393-402, 2008.
[8] M.A.Gopalan and G.Janaki, " Pythagorean triangle with perimeter as pentagonal number ", Antartica J.Math, Vol 5(2), 15-18, 2008.
[9] M.A.Gopalan and G.Janaki, "Pythagorean triangle with nasty number as a leg", Journal of Applied Mathematical Analysis and Applications, Vol 4, No.1-2, 13-17, 2008.
[10] Dr. Mita Darbari, " A connection between Hardy-Ramanujan number and special pythagorean triangle", Bulletin of society for Mathematical services and standards, Vol 3, N0.2, 71-73, 2014.
[11] Dr.G.Janaki and S.Vidhya "Special pairs of Rectangles and Sphenic number", International Journal for Research Applied Science & Engineering Technology, Volume 4, Issue II, February 2016. Pg No:376-378.
[12] Dr.G.Janaki and S.Vidhya, "Special pairs of Pythagorean triangles and 2-digit consecutive Sphenic numbers", International Journal of Academic Research and Development, Volume 1, Issue 11, Feb 2016, Pg.No: 29-31.
[13] Dr.G.Janaki and S.Vidhya, "Special pairs of Pythagorean Triangles and 2-digit Sphenic numbers", American International Journal of Science, Technology, Engineering & Mathematics, Volume 15, Issue 1, June-August 2016. Pg.No:19-22.
[14] Dr.G.Janaki and C.Saranya, "Pythagorean Triangle with Area/Perimeter as a Jarasandha numbers of order 2 & 4", International Research Journal of Engineering and Technology, Volume 3, Issue 7, July 2016, Pg.No: 1259-1264.



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