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A Deterministic Inventory Model with Two Warehouses and a Finite Rate of Replacement for Deteriorating Products

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Abstract: When demand is uniform, shortages are permitted, and the replenishment rate is finite, a deterministic inventory model for degrading items with two warehouses is created. It is anticipated that there may be differences in the rates of item deterioration between the two warehouses. An analysis of the model is conducted for the continuous release pattern situation. The variance in the ideal inventory level and optimum cost for changing shortage costs is illustrated with a numerical example and discussion of several unique instances.
Keywords: Production, demand, deterioration, inventory, and warehouse.

I. INTRODUCTION

The deterministic demand scenario involving a single storage facility is the primary scenario for which the classical inventory models are constructed. However, when considering the matter more practically, if a considerable supply needs to be held and the available facility, like the own warehouse (OW), has a limited capacity, then extra storage space may be needed. One possible option for this extra storage space may be a leased warehouse (RW), like a central warehouse equipped with advanced preservation technology. Hartley [3] provides an early explanation of an inventory model with two storage facilities. The holding cost in the RW is typically thought to be higher than the same in the OW. As a result, only extra stock is kept in the RW; all other products are kept in the OW. Additionally, the RW goods are released first, followed by the OW things.

A number of writers have recently thought about expanding on the fundamental two-storage inventory model covered. Constructed a deterministic inventory model with an infinite production rate and two layers of storage, with and without shortages. A case study of an extension to the finite production rate without shortages has been studied. The study is done for the scenario of bulk release pattern in the two models mentioned above. In the event of an unlimited pace of replenishment with shortages, explored expanding his previous model, assuming that the goods degrade in both warehouses. We create an order-level inventory model with two storage facilities for decaying items in this research. We presume that the deterioration rate of the products held might be different in the two warehouses due to the difference in the environmental conditions or preserving settings. Even in the event that the rate of deterioration in both warehouses is constant, the model is still relevant. For the situation of continuous release pattern, we formulate and assess the model assuming uniform demand and shortages are tolerated and the production rate is finite. We also infer an accurate form of the cost equation for the model without making any approximations, and we also make some observations on the 'single storage' version of this model.

II. NOTATIONS

R is the demand rate per time unit, which remains constant during the duration of the analysis.

P is the limited output rate.

There is no supply lead time and the scheduling period T is a set constant.

There is a backlog of shortages, and the cost per unit of time is pi for each shortage.

The unit holding cost is expressed as C_1 per unit cost, where C_1 denotes items in the OW as H and items in the RW as F.

The rates of OW and RW deterioration are a and b, respectively.

The inventory level in the OW at time point t is shown by $Q_o(t)$, and the inventory level in the RW at time point r is indicated by $Q_r(t)$.

The RW has an infinite capacity, but the OW has a limited capacity of W units.

C is the estimated cost of a degraded unit, which takes salvage value and disposal costs into account.

S is a decision variable that represents the amount of inventory in the RW at which production is halted.

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III. ANALYSIS AND DESCRIPTION OF THE MODEL

This model is based on the assumption that, in the production stage, requests are satisfied immediately and that inventory is taken into account in the OW until it is full to capacity W. After that, the inventory is kept in the RW until a level S is achieved, at which point manufacturing is halted. The cost of moving generated goods to the warehouses is thought to be minimal. Following production halt, needs are satisfied from the RW until it is empty, at which point the OW's inventory is used to satisfy the demand. Furthermore, it is presumable that subpar inventory goods are not swapped out for superior ones. In order to find the level S O in RW that minimises the overall average cost, we evaluate the model. We refer to the system with one warehouse as a L_l-System and the system above as an L_2-system.

As seen in Figure 1, the inventory position can be illustrated. There is no stock in either of them. The warehouses when all backlogs are cleared, at time t = 0. When $t = t_1$, the OW is filled to capacity W; surplus products are then sent to the RW once production demands are satisfied. The RW's inventory level hits S at $t = t_2$, at which point manufacturing is ceased. Due to degradation and demand, the RW's inventory continuously drops during (t_2, t_3), reaching zero at $t = t_3$. In the OW, only degradation was the cause of the inventory decline during (t_1, t_3) while both deterioration and demand were the cause of the inventory decrease during (t_3, t_4). Inventory in the OW reaches zero at $t = t_4$, after which shortages start to happen and build up until $t = t_5$. At this time, production begins, and at t = T, the backlog is cleared. After a time T, this cycle repeats again.

The differential equation that characterises the OW's inventory level at $(0, t_1)$ is

$$\frac{dQo(t)}{dt} + \alpha Qo(t) = p - R \quad \text{for} \quad 0 \le t \le t1$$

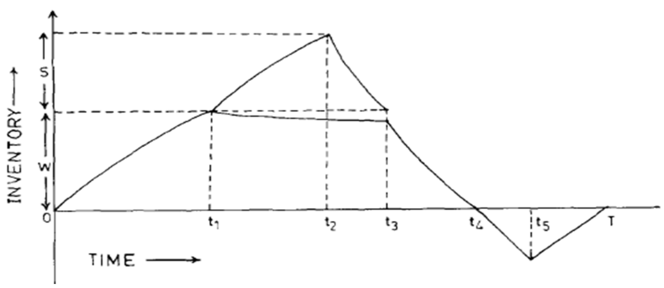

Figure 1 shows the two storage facilities' inventory levels.

and using the initial condition $Q_o(0) = 0$ the solution is

$$Q_o(t) = (p - R)[1 - e^{-\alpha t}]/\alpha \quad \text{for } 0 \leqslant t \leqslant t_1. \tag{1}$$

For the situation in the RW during (t_1, t_3) the differential equations are

$$\frac{dQ_r(t)}{dt} + \beta Q_r(t) = p - R \quad \text{for } t_1 \leqslant t \leqslant t_2,$$
$$\frac{dQ_r(t)}{dt} + \beta Q_r(t) = -R \quad \text{for } t_2 \leqslant t \leqslant t_3.$$

Using $Q_r(t_1) = 0$ and $Q_r(t_2) = S$, the solutions are

$$Q_r(t) = \begin{cases} (p - R)[1 - e^{-\beta(t - t_1)}]/\beta & \text{for } t_1 \leqslant t \leqslant t_2, \quad (2) \\ (S + R/\beta)\, e^{-\beta(t - t_2)} - R/\beta & \text{for } t_2 \leqslant t \leqslant t_3. \quad (3) \end{cases}$$

Further, the differential equations describing the state of inventory in the OW during (t_1, t_4) are

$$\frac{dQ_o(t)}{dt} + \alpha Q_o(t) = 0 \quad \text{for } t_1 \leqslant t \leqslant t_3,$$
$$\frac{dQ_o(t)}{dt} + \alpha Q_o(t) = -R \quad \text{for } t_3 \leqslant t \leqslant t_4,$$

and with $Q_o(t_1) = W$ we get,

$$Q_o(t) = W\, e^{-\alpha(t - t_1)} \quad \text{for } t_1 \leqslant t \leqslant t_3 \tag{4}$$
$$Q_o(t) = R[e^{-\alpha(t - t_3)} - 1]/\alpha + W\, e^{-\alpha(t - t_1)} \quad \text{for } t_3 \leqslant t \leqslant t_4. \tag{5}$$

The following time points are obtained in terms of S using $Q_o(t_1) = W$ in (1), $Q_r(t_2) = S$ in (2), $Q_r(t_3) = 0$ in (3) and $Q_o(t_4) = 0$ in (5):

$$t_1 = [\log\{(p - R)/(p - R - W\alpha)\}]/\alpha, \tag{6}$$
$$t_2 = t_1 + [\log\{(p - R)/(p - R - S\beta)\}]/\beta, \tag{7}$$
$$t_3 = t_1 + [\log\{(R + S\beta)(p - R)/[R(p - R - S\beta)]\}]/\beta, \tag{8}$$
$$t_4 = [\log\{e^{\alpha t_3} + W\alpha(p - R)/[R(p - R - W\alpha)]\}]/\alpha. \tag{9}$$

In the quantity deteriorated in RW is $(t_2 - t_l)$ p - $(t_3 - t_1)$ R. In the quantity deteriorated in RW is $Pt_1 - R(t_4 - t_3 + t_l)$ and the total quantity deteriorated is $pt_2 - Rt_4$. Since the quantity deteriorated in the OW is α times the entire inventory kept in the OW. The entire inventory kept in the OW is $[pt_1 - R(t_4 - t_3 + t_1)]/\alpha$ a similar vein, $[(t_2 - t_l)P - (t_3 - t_l)R]/\beta$ represents the total inventory kept in the RW.

The overall shortage amount is

$$\tfrac{1}{2}R(t_5 - t_4)^2 + \tfrac{1}{2}(p-R)(T-t_5)^2 = \tfrac{1}{2}R(1-R/p)(T-t_4)^2, \tag{10}$$

since $t_5 = (1-R/p)T + (R/P)t_4$.

The total average cost function for the system is thus given by

$$\begin{aligned} C_2(S) &= \Big[C(pt_2 - Rt_4) + F([t_2-t_1]p - [t_3-t_1]R)/\beta + H[pt_1 - R(t_4 - t_3 + t_1)]/\alpha \\ &\qquad + \tfrac{1}{2}\pi R(1-R/p)(T-t_4)^2\Big]/T \\ &= \Big[(R-p)(F/\beta - H/\alpha)t_1 + p(C+F/\beta)t_2 - R(F/\beta - H/\alpha)t_3 - R(C+H/\alpha)t_4 \\ &\qquad + \tfrac{1}{2}\pi R(1-R/p)(T-t_4)^2\Big]/T \end{aligned} \tag{11}$$

Let $u = R + S\beta$, $v = p - R - S\beta$ and

$$y = \frac{p-R}{p-R-W\alpha}\left(\frac{p-R}{R}\right)^{\alpha/\beta}(u/v)^{\alpha/\beta} + \frac{W\alpha(p-R)}{R(p-R-W\alpha)}. \tag{12}$$

The equation $\mathrm{d}C_2(S)/\mathrm{d}S = 0$ reduces to

$$\begin{aligned} &-\frac{1}{uy}\left(y - \frac{W\alpha(p-R)}{R(p-R-W\alpha)}\right)\{R(C+H/\alpha) + \pi R(1-R/p)(T - [\log y]/\alpha)\} \\ &- R(F/\beta - H/\alpha)/u + (C+F/\beta) = 0. \end{aligned} \tag{13}$$

The condition $\mathrm{d}C_2(S)/\mathrm{d}S$ at $S=0$ is negative is equivalent to

$$\pi > \frac{(C+H/\alpha)W\alpha/R}{(1-R/p)\left(T - \dfrac{1}{\alpha}\log\left(\dfrac{p-R}{R}\right)\left(\dfrac{R+W\alpha}{p-R-W\alpha}\right)\right)} = \pi_c \quad \text{(say)} \tag{14}$$

When other parameters are fixed, Where Π_c is the critical value of Π that directs the selection between the L_1-system and L_2-system. Hence, the L_2-system is optimal if $\Pi > \Pi_c$, and solving yields the ideal inventory level. We consider the L_1-system for $\Pi < \Pi_c$.

The single-warehouse variant (L_1-system with limitless capacity), although their cost function and solution are heavily approximated. The accurate cost function for this model is shown here.
By taking S = 0 and W = S_1 then the total average cost function for the Ll-system may be inferred from the L2-system. Next,

$$t_1 = t_2 = t_3 = \frac{1}{\alpha}\log\frac{p-R}{p-R-\alpha S_1} = t'_1, \tag{15}$$

$$t_4 = \frac{1}{\alpha}\log\frac{(p-R)(R+S_1\alpha)}{R(p-R-S_1\alpha)} = t'_2, \tag{16}$$

and

$$t_3 = (1-R/p)T + (R/p)t_4 = t'_3.$$

The total average cost function $C_1(S_1)$ of the L_1-system is

$$C_1(S_1) = \left[(C + H/\alpha)(pt_1' - Rt_2') + (\pi R/2)(1 - R/p)(T - t_2')^2\right]/T, \tag{17}$$

and the equation $\mathrm{d}C_1(S_1)/\mathrm{d}S_1 = 0$ reduces to

$$(C\alpha + H)S_1/R - \pi(1 - R/p)(T - t_2') = 0. \tag{18}$$

Thus, the solution of equation yields the L_l-system's optimal inventory level. The cost function for the single-storage model is precisely expressed, which is acquired without the need for any assumptions.

The ideal level is W with optimum cost $C_1(W)$ if $\Pi > \Pi_c$; otherwise, it is defined optimum cost is provided if we take the single-storage model with restricted capacity into consideration.

Assume that the decision-maker must select from a number of warehouses that have varying deterioration rates and holding costs (that is, warehouses with infinite capacity whose holding costs exceed OW). The warehouse with the lowest holding costs would be the apparent choice to rent when the rates of deterioration are the same in all of the warehouses. It is distinct, though, with differing rates of degradation. Therefore, selecting a rented warehouse with the best cost considerations is possible using the current model.

IV. NUMERICAL EXAMPLE

As we mentioned other factors staying fixed, the shortfall cost determines which system to use. We provide a numerical example to show how the optimum cost and order-level might vary in capacity-limited systems and L_2-systems with fluctuating shortage costs. It should be noted that if $\Pi < \Pi_c$, there is no need for a rented warehouse because the own warehouse can accommodate the optimum order level. Table 1 also shows that it is more advantageous to think about the L2-system rather than the limited-capacity L_1-system when the shortfall cost is larger.

With R = 600 units, p = 1000 units, C = 25 rupees, H = 0.5 rupees, F = 1 rupee, α = 0.01, β = 0.005, W = 11000 units, and T = 60 days, we have a look at a numerical example. Depending on the value of Π we calculate: S_1 is the L_1-System's optimal inventory level when capacity is restricted. S_2 is the ideal level of inventory in the RW of the L_2-system. For a limited capacity L_1-system, $C_1(S_1)$ = optimal cost. $C_2(S_2)$ = L_2-system optimal cost.

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